
ECONtribute
Discussion Paper No. 261

**Strategic Complementarities in a Model of
Commercial Media Bias**

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October 2023

www.econtribute.de



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October 25, 2023

Abstract

Media content is an important privately supplied public good. While it has been shown that contributions to a public good crowd out other contributions in many cases, the issue has not been thoroughly studied for media markets yet. We show that in a standard model of commercial media bias, qualities of media content are strategic complements, whereby investments into quality *crowd in* further investments and engage competitors in a race to the top. Therefore, financially strong public service media can mitigate commercial media bias: the content of commercial media can be more in line with the preferences of the audience and less advertiser-friendly in a dual (mixed public and commercial) media system than in a purely commercial media market.

Key words: commercial media bias, public service media, advertising, two-sided markets, supermodular games, strategic complements, public goods

JEL Codes: C70, H41, L13, L51, L82

1 Introduction

Media content belongs to the most important cases of privately supplied public goods. Its consumption is non-rival, and in many cases like free TV or freely available Internet

*We thank Lara Mai for excellent research assistance.

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content no exclusion is taking place. Media content differs markedly from other public goods, though, because of the importance of advertising revenue for media markets, and the economic analysis of the private supply of these public goods must take the multi-sided nature of media markets into account (Anderson and Coate 2005). Recent literature has made major progress in this research area (see Anderson and Jullien 2015 and Jullien, Pavan, and Rysman 2021).

One important result from the theory of private public good supply is that, under fairly general conditions, private contributions to a public good are strategic substitutes, whereby high contributions are crowding out others (see Batina and Ihori, 2005, Chapter 6, and Bucholz and Sandler, 2021, Finding F9, for overviews). Surprisingly, this issue has not been thoroughly examined for media markets, even though it is highly relevant for the welfare analysis of media policy. E.g., in discussions about the proper role and scope of public service media (PSM), one crucial question is whether raising the program quality of a regulated (public) broadcaster will increase or decrease the program quality of its commercial competitors.¹

There are two conflicting views. On the one hand, PSM could crowd out private investment and innovation in media markets. E.g., the existence of PSM may lead to less entry of commercial media; see Berry and Waldfogel (1999) for empirical evidence. Similarly, Armstrong and Weeds (2007a) show that in a duopoly where a public and a commercial broadcaster compete, raising the program quality of PSM partially crowds out the commercial broadcaster and lowers its program quality. This reasoning is echoed by regulation authorities like Ofcom in the UK and the Scientific advisory board at the Federal Ministry of Finance in Germany (Ofcom 2004, Wissenschaftlicher Beirat beim Bundesministerium der Finanzen 2014.)

However, PSM might also foster a “competition for quality”, whereby public and commercial media compete for audiences. This reasoning goes back to Coase (1947), pondering that PSM might induce a “natural rivalry to furnish the most attractive programs” (p.197). Indeed, recent empirical evidence suggests that in countries where PSM invest in high-quality media content, program quality of commercial media tends to be high, too (Simon 2013). Similarly, Sehl et al. (2020) find that, controlling for GDP, per capita revenues of PSM and commercial broadcasters are positively correlated across EU countries.²

¹In this paper, "commercial media" refers are all profit-maximizing media outlets.

²These correlations are in line with a *crowding in* effect of PSM. They might, however, also be driven by unobserved confounding factors such as high preferences for television, and do not allow to infer

In this paper, we demonstrate that in a model of commercial media bias, program qualities in terms of the media’s reporting accuracy are strategic complements rather than strategic substitutes. Reporting accuracy here refers to media content that fully and truthfully reports facts as opposed to hiding information or dumbing down content.³ Viewers prefer high, while advertisers prefer lower reporting accuracy. The strategic complementarity stems from the media’s fundamental trade-off in these models: Increasing reporting accuracy increases the value of the media content for the audience but decreases the willingness to pay of the advertisers to reach consumers. The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to increase reporting accuracy are higher. Thus, in a media market with both public and commercial media, raising the PSM’s reporting accuracy reduces the commercial media’s audiences and thereby also their implicit cost of increasing their own reporting accuracy. As a result, the PSM *crowd in* reporting accuracy and engage the commercial media in a race to the top.

Our main model focuses on media content that is freely available. We show that our results generalize to a model featuring both pay media and free media, however, if reporting accuracy is about revealing information that the media already possess. We also discuss conditions under which our findings generalize to multidimensional strategy spaces, spillover effects of reporting accuracy on advertising revenue of other media outlets, different demand functions, income effects of taxes or license fees used to finance the PSM, endogenous entry and exit, and biases of Public Service Media.

Our paper contributes to four strands of literature. First, we contribute to the literature on commercial media bias. Several empirical papers document the effect of advertising on media coverage in terms of mutual fund recommendations (Reuter and Zitzewitz 2006), product mentions (Gambaro and Puglisi 2015), coverage of government scandals (Di Tella and Franceschelli 2011) and climate change (Beattie, 2020). We present a fairly standard model of commercial media bias. Our model is in many

causality. Weeds (2020) reviews the literature on the question whether Public Service Broadcasters crowd out or crowd in private programming and concludes that “further research is needed in this area before firm conclusions can be drawn“ (Weeds 2020, p. 10). Similarly, Nielsen et al. (2016) review academic publications and studies funded by stakeholders such as government agencies and public or private media organizations. They point out that there is little research on the market impact of public service media, and conclude that “existing studies provide little evidence for a negative market impact of public service media upon domestic private sector media” (p. 17).

³E.g., advertisers might prefer the media not to report critically about their products. Moreover, studies from marketing have shown that advertisers prefer light genres like comedy that put consumers in a more ad-receptive mood, whereas consumers prefer action and news. See Ellman and Germano (2009) and Kerkhof and Münster (2015) for extensive discussion.

ways similar the models studied by Ellman and Germano (2009), Germano and Meier (2013), and Kerkhof and Münster (2015); it captures bias through distorted reporting accuracy that caters to the preferences of advertisers rather than consumers. Our paper is especially close to Germano and Meier (2013) who show that competition on media market mitigates commercial bias, and to Kerkhof and Münster (2015) who find that competition between media outlets makes it more likely that a cap on advertising quantities is welfare enhancing. Relatedly, Blasco et al. (2016) find that if the media can raise their audience share through increasing their reporting accuracy, then competition in the market may also increase the expected accuracy of reports.⁴ These predictions are in line with the empirical results of Beattie et al. (2021) who find that newspapers provide less coverage of car recalls by their advertisers, but competition for readers raises reporting accuracies and thus mitigates bias. Similarly, Focke et al. (2016) show that commercial media bias is likely mitigated by reputational concerns on behalf of the media, e.g., if they face a demanding audience.

Second, we advance the broad research on the private supply of public goods (Bergstrom et al., 1986, Batina and Ihuri, 2005). The provision of public goods via advertising is studied by Luski and Wettstein (1994) and Anderson and Coate (2005). These papers do not study media bias, however.

Third, we add to the literature on supermodular games, i.e., games in which the best response of any player is increasing in the actions of its competitors (Topkis 1979, Milgrom and Roberts, 1990, Vives 1985, 1990, 2005a, 2005b, Van Zandt and Vives, 2007, Frankel et al., 2003). Leveraging the theory of supermodular games allows us to obtain fairly general results in a model with many asymmetric media outlets. Specifically, we show that reporting accuracies are strategic complements rather than strategic substitutes.

Fourth, as a consequence of strategic complementarities, public investments into program quality induce commercial media to provide high quality, too. Hence, our results support media policies that advocate financially strong PSM. In this way, we also contribute to the economics literature on public service media (see Armstrong and Weeds 2007b, Strömberg 2015, and Weeds 2020 for surveys). To the best of our knowledge, the issue how public media affect the content of commercial media has not been studied yet in the literature on commercial media bias. Other aspects of this debate have of course been analyzed; in addition to the empirical literature referenced above,

⁴Blasco and Sobbrío (2012) provide a survey on competition and commercial media bias.

several theoretical studies on the market impact of public media exist. Armstrong and Weeds (2007a) study investments in a vertical quality dimension. Richardson (2006) investigates how a publicly-provided radio station offering local content affects the provision of local content by commercial stations. Garcia Pires (2016) compares media diversity in commercial versus mixed public and private duopolies. Our paper complements this line of research by studying commercial media bias.

The remainder of this paper is structured as follows. Section 2 introduces our theoretical framework. In Section 3, we demonstrate that program qualities in terms of the media’s reporting accuracy are strategic complements, which is our main finding, and describe the implications for *crowding in* effects of Public Service Media. Section 4 considers the case where some commercial media are pay media. Section 5 discusses several extensions of our model. Section 6 concludes.

2 Model

In this section we introduce a fairly standard model of commercial media bias (Ellman and Germano 2009, Germano and Meier 2013, Kerkhof and Münster 2015, see Blasco and Sobbrío 2012 for a survey). Consider a model with n commercial media denoted by $1, \dots, n$ and m public service media (PSM) denoted $n + 1, \dots, n + m$. The set of commercial media is denoted by $C = \{1, \dots, n\}$, the set of PSM is $P = \{n + 1, \dots, n + m\}$. Each media outlet $i \in C \cup P$ chooses a reporting accuracy $v_i \in V_i \subseteq \mathbb{R}_+$. (An extension to multidimensional strategy spaces is considered in Section 5.) Reporting accuracy v_i is about fully and truthfully reporting facts, as opposed to hiding information or dumbing down content. The audience prefers higher reporting accuracy, whereas advertisers prefer lower reporting accuracy. We assume that the strategy sets V_i are compact and contain $v_i = 0$.

A consumer’s utility from consuming outlet i is $u_i = f_i(v_i)$, where f_i is continuous, strictly increasing, and satisfies $f_i(0) = 0$. Unless otherwise noted, we simply assume $f_i(v_i) = v_i$. In Sections 2 and 3, nothing is lost in setting $u_i = v_i$; the distinction between utility u_i and reporting accuracy v_i becomes important when considering pay media or multidimensional strategies. For a commercial outlet $i \in C$, let $u_{-i}^C = (u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n)$ denote the vector of the utilities of i ’s commercial competitors, $u^P = (u_{n+1}, \dots, u_{n+m})$ the vector of utilities of the public service media, and $u_{-i} = (u_{-i}^C, u^P)$.

The size of the audience of a media outlet is denoted by s_i . We impose the following assumptions.⁵

Assumption (1) For all $i \in C$, s_i is positive, continuous, increasing in u_i , and decreasing in u_j for all $j \in P \cup C \setminus \{i\}$.

Assumption (2) For all $i \in C$, s_i has increasing differences in (u_i, u_{-i}) .

Assumptions (1) to (2) are fulfilled by several standard demand formulations, including linear demand functions, quadratic demand functions where the coefficients of the interaction terms are positive, and the Hotelling, Salop, and Spokes (Chen and Riordan 2007) model in the relevant range where all market shares are interior. The logit model violates Assumption (2) whenever there are $n \geq 2$ commercial outlets; in Section 5 we give a sufficient condition for our results to hold when Assumption (2) is violated.

Denote the advertising revenue of outlet i , per member of the audience, by R_i . A crucial assumption in models of advertiser bias is that, for a given audience, ad revenue depends negatively on reporting accuracy:

Assumption (3) For all $i \in C$, R_i is positive, continuous, decreasing in v_i , and independent of v_j for all $j \neq i$.

By Assumption (3), R_i is independent from the reporting in other outlets as in Ellman and Germano (2009) and Kerkhof and Münster (2015). Germano and Meier (2013) model spillover effects of reporting accuracy on the advertising revenue of other outlets; we will discuss spillover effects in Section 5.

Each media outlet i has a cost $c_i(v_i)$ that may depend on its reporting accuracy.⁶

Assumption (4) For all media outlets $i \in C \cup P$, c_i is continuous, increasing in v_i , and zero at $v_i = 0$.

We distinguish between two cases. First, reporting accuracy could be about faithfully reporting information that the media already have. In this case, the only cost of reporting accurately is lower advertising revenue, but there is no additional direct

⁵In this paper, unless otherwise stated, “increasing” means “weakly increasing” and “positive” means “non-negative”. A similar remark applies for “decreasing” and “negative”.

⁶The cost c_i does not depend on audience size; it can be thought of as “first copy costs”. In online and broadcast media (radio and TV), once distribution channels are in place, marginal costs of an additional audience is basically zero. In printed newspaper markets the costs for paper, printing and delivery are substantial. Any constant marginal cost of an additional consumer can be thought of as incorporated in the function R_i , which then gives advertising revenue per consumers net of marginal costs per consumer. Note that with this interpretation of R_i , it could become strictly negative for high values of v_i (since advertising revenue might be strictly smaller than marginal costs), in conflict with Assumption (3). Such values of v_i , however, lead to losses and hence are dominated; they can be eliminated from the strategy set V_i , restoring Assumption (3).

cost of obtaining the information in the first case. Formally, in the current model it means that $c_i(v_i)$ is constant in v_i . We refer to this case of withholding information as “dumbing down content”. Second, reporting accuracy could also be about investigative journalism, about establishing new facts and information. Then it seems plausible that $c_i(v_i)$ is strictly increasing in v_i . For example, the media might have to hire more journalists to increase reporting accuracy (see Hamilton 2016 for a detailed description of the economics of investigative journalism). We refer to this case as “investigative journalism”.

Arguably, dumbing down content is highly relevant for the study of commercial media bias; indeed several papers in the literature focus on this case (Ellman and Germano 2009, Germano and Meier 2013, Kerkhof and Münster 2015, Blasco, Pin and Sobbrío 2016). For example, two important topics where advertising has influenced editors are the health risks of smoking (e.g. Bagdikian 2004 Chapter 12) and climate change (Beattie 2020, Boykoff and Boykoff 2004). The scientific facts about these topics were long well established and easily accessible, but media coverage and public perception significantly lagged in time behind the scientific consensus. Moreover, as shown in Beattie (2022), commercial media bias in the *tone* of coverage about climate change, measured based on comparisons of environmental and skeptical texts, can have important behavioral consequences, and merely changing the tone of coverage does not impact its cost.

On the other hand, pressure from advertisers may also deter media from investigative journalism. Our main results on free media do not depend on whether we study dumbing down content or investigative journalism. For pay media, we show that the distinction matters.

Commercial media in our main model are funded by advertising and their content is freely available for consumers; pay media will be considered in Section 4. The profit of a commercial media outlet $i = 1, \dots, n$ is (substituting $v_i = u_i$ and $v_{-i} = u_{-i}$ into s_i)

$$\pi_i(v_i, v_{-i}) = s_i(v_i, v_{-i}) R_i(v_i) - c_i(v_i).$$

Commercial outlet i maximizes $\pi_i(v_i, v_{-i})$ by choosing $v_i \in V_i$. Note that we disregard fixed costs which could be saved by going out of business; we defer a discussion of exit and entry to Section 5.

The public service media (PSM) in our model are not-for-profit and financed in-

dependent of advertising. Their content is freely available for all consumers.⁷ The budget of PSM $i \in P$ is b_i . We assume the PSM spend their budget to maximize consumer utility by choosing $v_i \in V_i$ subject to $c_i(v_i) \leq b_i$. The feasible sets $V_i \subseteq \mathbb{R}_+$ are compact, contain $v_i = 0$, and may depend on the budget b_i . We assume that a larger budget enlarges the feasible set: if $b_i < b'_i$, then $V_i(b_i) \subseteq V_i(b'_i)$. The model allows for inefficiencies of PSM, since different media outlets can have different cost functions and different feasible sets of reporting accuracies. We discuss potential biases of PSM in Section 5.

Some (but not all) of our considerations below impose the additional assumption that a sufficiently high reporting accuracy is necessary for a PSM to attract an audience. To express this formally, for $i \in P$ let $v_{-i}^P = (v_{n+1}, \dots, v_{i-1}, v_{i+1}, \dots, v_{n+m})$ denote the vector of reporting accuracies of the other PSM.

Assumption (5) A PSM outlet with zero reporting accuracy ($v_i = 0$) attracts no audience: $s_i(0, v^C, v_{-i}^P) = 0$ for all $i \in P$ and (v^C, v_{-i}^P) , and demand for the other media outlets is as if outlet i did not exist.

Assumption (5) seems reasonable when the audience has a sufficiently attractive outside option not to consume any media. Note that under Assumption (5), a PSM with an insufficient budget cannot produce a content that attracts any audience; then the game reduces to a game between the remaining media outlets only. We will explicitly indicate where we use Assumption (5).

3 Main results

Consider the PSM first.

Proposition 1 *A PSM i chooses*

$$v_i = \bar{v}_i(b_i) := \max_{v_i \in V_i(b_i)} \{v_i \mid c_i(v_i) \leq b_i\}.$$

Moreover, $\bar{v}_i(b_i)$ is increasing in b_i , and independent of the strategies of the other media outlets.

⁷Consumers have to pay taxes or licence fees that are used to finance the PSM, but these payments are independent of personal media consumption. These payments could affect media demand via income effects. We discuss income effects in Section 5.

Proof. Outlet $i \in P$ solves

$$\max_{v_i \in V_i} v_i \quad \text{s.t.} \quad c_i(v_i) \leq b_i.$$

An increase of b_i relaxes the PSMs budget constraint and enlarges the feasible set V_i , hence \bar{v}_i is increasing in b_i . Moreover \bar{v}_i is unique and independent of the strategies chosen by the other media outlets. ■

We now turn to the commercial media. Proposition 1 allows us to view the game between the commercial media as parameterized by the budgets of the PSM $b := (b_{n+1}, \dots, b_{n+m})$. Let $\bar{v}^P(b) = (\bar{v}_i(b_i))_{i=n+1}^m$ denote the vector of reporting accuracies chosen by the PSMs. For $i \in C$, let

$$\tilde{\pi}_i(v_i, v_{-i}^C, b) := \pi_i(v_i, v_{-i}^C, \bar{v}^P(b))$$

and let $\Gamma_b = (C, (\tilde{\pi}_i)_{i=1}^n, \Pi_{i=1}^n V_i)$ denote the resulting game between the commercial media outlets: the set of players is C , payoff functions are $\tilde{\pi}_i$, and strategy spaces are V_i .

Proposition 2 Γ_b is a parameterized supermodular game.⁸

Proof. The strategy spaces $V_i \subseteq \mathbb{R}_+$ are compact by assumption, hence compact lattices, and the objective functions $\tilde{\pi}_i$ are continuous in v_i for fixed v_{-i} and b .

Next, we show that π_i has increasing differences in (v_i, v_{-i}) . For simplicity of the exposition, we will assume here that the functions R_i , s_i and c_i are differentiable; Appendix A gives the proof without assuming differentiability. From

$$\pi_i(v_i, v_{-i}) = s_i(v_i, v_{-i}) R_i(v_i) - c_i(v_i)$$

we obtain

$$\frac{\partial \pi_i}{\partial v_i} = \frac{\partial s_i(v_i, v_{-i})}{\partial v_i} R_i(v_i) + s_i(v_i, v_{-i}) R_i'(v_i) - c_i'(v_i)$$

and

$$\frac{\partial^2 \pi_i}{\partial v_j \partial v_i} = \frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i})}{\partial v_j} R_i'(v_i) \geq 0, \quad j \neq i \quad (1)$$

where the inequality follows because of $\frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i} \geq 0$ by Assumption (2), $\frac{\partial s_i(v_i, v_{-i})}{\partial v_j} \leq 0$ by Assumption (1), and $R_i'(v_i) \leq 0$ by Assumption (3).

⁸See e.g. Sarver (2023) Chapter 3 for the definition of a parameterized supermodular games.

We have shown that π_i has increasing differences in (v_i, v_{-i}) . Therefore, $\tilde{\pi}_i$ has increasing differences in (v_i, v_{-i}^C) . Moreover, π_i has increasing differences in (v_i, v^P) . It remains to show that $\tilde{\pi}_i$ has increasing differences in (v_i, b) . By Proposition 1, $\bar{v}_k(b_k)$ is increasing in b_k , while $\bar{v}_{k'}$ does not depend on b_k for $k' \neq k$. Since π_i has increasing differences in (v_i, v^P) , it follows that $\tilde{\pi}_i$ has increasing differences in (v_i, b) . ■

Proposition 2 shows that the reporting accuracies are strategic complements. The economics of the result is straightforward. The fundamental trade-off for a commercial outlet in a model of commercial media bias is as follows: providing content in line with the preferences of the audience attracts a bigger audience, but leads to lower advertising revenue per consumer. If the reporting accuracies of competing media increase, the audience of a given outlet is smaller, hence also the implicit cost of increasing its own reporting accuracy. The logic is closely related to the finding in Germano and Meier (2013) that underreporting typically increases with the concentration of ownership on the media market, which has found empirical support in Beattie et al. (2021).

Leveraging the theory of supermodular games (see Vives 2005 or Sarver 2023 for expositions) allows us to generate fairly general results in our model featuring many asymmetric media outlets. Denote a strategy profile in game Γ_b by $v^C = (v_1, \dots, v_n)$. Proposition 2 implies that Γ_b has, for any b , a lowest equilibrium $v^{C,low}$ and a highest equilibrium $v^{C,high}$, such that any equilibrium v^C satisfies $v^{C,low} \leq v^C \leq v^{C,high}$. Since $\tilde{\pi}_i$ is continuous in (v_i, v_{-i}) for all $i \in C$, Milgrom and Roberts (1990) applies and the set strategy combinations that survive iterated elimination of strictly dominated strategies has smallest and largest elements $v^{C,low}$ and $v^{C,high}$.

Turning to comparative statics, the equilibria $v^{C,low}$ and $v^{C,high}$ are monotone increasing in b . When the equilibrium is unique, a stronger monotone comparative static result is available, which we highlight the following Proposition 3.

Proposition 3 *Suppose Γ_b has a unique equilibrium. Then the equilibrium reporting accuracy of each commercial media outlet $i \in C$ is increasing in the budget b_j of any PSM $j \in P$.*

Proposition 3 states that PSM *crowd in* reporting accuracy, in line with the idea that PSM engage commercial media in a race to the top. To illustrate the result, we compare a “dual” (or mixed public and commercial) media market, featuring both PSM and n commercial media outlets, with a purely commercial media market consisting only of the same n commercial outlets, postponing considerations about entry

to Section 5. Under Assumption (5), Proposition 3 implies that in a “dual” media market, the content of the commercial media outlets is more audience-friendly and less advertiser-friendly than when there are only the n commercial media. To see this, recall that when b_i is insufficient, the PSM i cannot attract any audience in our model by Assumption (5), and then the resulting competition between the remaining media is as if PSM i was not on the market. Applying Proposition 3 shows that the reporting accuracies of the commercial media will be lower in this situation than when there are viable PSM.

4 Pay media

This section studies an extension to the model where some commercial media outlets are pay media, i.e. earn revenue from direct payments from consumers. Suppose that media outlets $i \in C^f = \{1, \dots, n_f\}$ are free media: they are funded solely by advertising and ‘free’ in the sense that consumers do not pay a monetary price for consumption. Outlets $i \in C^{pay} \in \{n_f + 1, \dots, n\}$ are pay media. The set of all commercial media is $C = C^f \cup C^{pay}$.⁹ As above, outlets $i \in P = \{n + 1, \dots, m\}$ are PSMs.

A pay media outlet $i \in C^{pay}$ chooses reporting accuracy $v_i \in V_i$ and price $p_i \in P_i \subseteq \mathbb{R}_+$, where P_i contains $p_i = 0$ and is compact.¹⁰ Utility from outlet i is $u_i = v_i - p_i$, utility from an outlet $j \in C^f \cup P$ is simply $u_j = v_j$.

The profit of a pay media outlet $i \in C^{pay}$ is

$$\pi_i(v_i, p_i, u_{-i}) = s_i(u_i, u_{-i}) (R_i(v_i) + p_i) - c_i(v_i),$$

where u_{-i} is the vector of utilities offered by the other outlets. The profit of a free media outlet $i \in C^f$ is

$$\pi_i(u_i, u_{-i}) = s_i(u_i, u_{-i}) R_i(v_i) - c_i(v_i).$$

In all other respects, the model is as in Section 2 above.

⁹We assume $n_f < n$. The case $n_f = n$ is the case without pay media studied above. We can allow for the case where all commercial outlets are pay media ($n_f = 0$). The most relevant case is when pay media and free media co-exist ($0 < n_f < n$). Note we assume that outlets $i \in C^f$ provide their content for free to consumers. Similarly, we assume that outlets in C^{pay} choose strictly positive prices.

¹⁰This implies that there is a highest feasible price that firms cannot exceed, which is without loss of generality if consumers stop buying if the price is too high.

For pay media, results depend on whether we consider dumbing down content or investigative journalism (see the discussion after Assumption (4) above). Results are clear cut in the case of dumbing down content, where $c_i(v_i)$ is constant in v_i . We show that in this case, when some competitor $j \neq i$ increases the utility u_j , ceteris paribus outlet i will also offer a higher utility: pay media keep their reporting accuracy constant but lower their price, while free media increase their reporting accuracy. We briefly discuss the case of investigative journalism towards the end of this section.

Proposition 4 *Assume that $c_i(v_i)$ is constant in v_i for all $i \in C^{pay}$. Then outlet $i \in C^{pay}$ will choose reporting accuracy*

$$v_i = \bar{v}_i := \arg \max_{v_i \in V_i} (R_i(v_i) + v_i).$$

Moreover, \bar{v}_i is independent of price p_i chosen by outlet i , and independent of the strategies of the other media outlets.

Proof. We adapt a technique from Armstrong (2006) and proceed in two steps to solve the maximization problem of $i \in C^{pay}$. The first step maximizes profits by choosing v_i , holding consumer utility $u_i = v_i - p_i$ constant at a given level \bar{u}_i by implicitly adjusting the price. The second step maximizes by choosing the price, or equivalently consumer utility. Formally, the first step is

$$\max_{v_i \in V_i} s_i(\bar{u}_i, u_{-i}) (R_i(v_i) + v_i - \bar{u}_i) - c_i(v_i).$$

In this maximization problem, consumer utility is constant, hence demand s_i is constant as well. Because $c_i(v_i)$ is constant, the profit maximizing v_i does not depend on c_i , and i maximizes $s_i(\bar{u}_i, u_{-i}) (R_i(v_i) + v_i - \bar{u}_i)$. Moreover, s_i is just a multiplicative constant in this objective function that does not change the profit maximizing v_i . Hence the solution is $v_i = \bar{v}_i$ as defined in the proposition. Note that \bar{v}_i is independent of the price p_i and the other firms' strategies. ■

The reporting accuracy \bar{v}_i might be so high that outlet $i \in C^{pay}$ has no advertising revenue and is funded only by payments from its audience. On the other hand, if $R_i(\bar{v}_i) > 0$, then outlet i has two sources of revenue, advertisers and consumers.

Proposition 4 allows us to consider the game as a game where the pay media have only one choice variable, their price. To find the profit maximizing prices, substitute $v_i = \bar{v}_i$ into the profit function of outlet $i \in C^{pay}$. Instead of maximizing profits by

choosing p_i , we can equivalently think of firm i as choosing utility u_i , taking into account that $u_i = \bar{v}_i - p_i$ so $p_i = \bar{v}_i - u_i$. The set of utilities that outlet i can choose from is $U_i := \{u_i \mid u_i = \bar{v}_i - p_i, p_i \in P_i\}$.

The objective function is

$$\hat{\pi}_i(u_i, u_{-i}) := s_i(u_i, u_{-i})(R_i(\bar{v}_i) + \bar{v}_i - u_i) - c_i(v_i).$$

Mirroring our definitions leading to Proposition 2 above, let

$$\tilde{\pi}_i(u_i, u_{-i}^C, b) = \begin{cases} \pi_i(u_i, u_{-i}^C, \bar{v}^P(b)), & \text{if } i \in C^f, \\ \hat{\pi}_i(u_i, u_{-i}^C, \bar{v}^P(b)), & \text{if } i \in C^{pay}. \end{cases}$$

Let $\Gamma_b^{pay} = \left(C, (\tilde{\pi}_i)_{i=1}^n, \Pi_{i=1}^{n_f} V_i \times \Pi_{i=n_f+1}^n U_i\right)$ denote the resulting game between the commercial media outlets: the set of players is $C = C^f \cup C^{pay}$, payoff functions are $\tilde{\pi}_i$, outlets $i \in C^f$ chooses $u_i \in V_i$, outlets $i \in C^{pay}$ choose $u_i \in U_i$ while their reporting accuracy is fixed at \bar{v}_i by Proposition 4, and as above the utilities offered by the PSM are given by $\bar{v}^P(b)$.

Proposition 5 *Assume that $c_i(v_i)$ is constant in v_i for all $i \in C^{pay}$. Then Γ_b^{pay} is a parameterized supermodular game.*

Proof. To begin with, since $P_i \subseteq \mathbb{R}_+$ is compact, the set of feasible utilities $U_i := \{u_i \mid u_i = \bar{v}_i - p, p_i \in P_i\} \subseteq \mathbb{R}$ is compact as well.

For simplicity, we will only give the proof assuming differentiability. For $i \in C^{pay}$ and $j \neq i$,

$$\begin{aligned} \frac{\partial^2 \hat{\pi}_i}{\partial u_j \partial u_i} &= \frac{\partial}{\partial u_j} \left(\frac{\partial s_i(u_i, u_{-i})}{\partial u_i} (R_i(\bar{v}_i) + \bar{v}_i - u_i) - s_i(u_i, u_{-i}) \right) \\ &= \frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} (R_i(\bar{v}_i) + \bar{v}_i - u_i) - \frac{\partial s_i(u_i, u_{-i})}{\partial u_j} \geq 0 \end{aligned}$$

where the inequality follows because of $\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} \geq 0$ by Assumption (2), $R_i(\bar{v}_i) \geq 0$ and $\bar{v}_i - u_i = p_i \geq 0$, and $\frac{\partial s_i(u_i, u_{-i})}{\partial u_j} \leq 0$ by Assumption (1).

The remainder of the proof is similar to the proof of Proposition 2. ■

Proposition 5 shows that Γ_b^{pay} has increasing reaction functions. That is, when some competitor of a commercial media outlet i increases the utility it offers, ceteris paribus outlet i will also offer a higher utility. For a pay media outlet $i \in C^{pay}$, reporting

accuracy is constant by Proposition 4, but i lowers its price. The economics behind the result is straightforward: tougher competitors reduce residual demand, and as a reaction firm i charges a lower price. Proposition 5 also shows that the free media will, as in Section 3 above, ceteris paribus react to increases in an competitor’s utility by increasing their reporting accuracy.

As above, we can leverage the theory of supermodular games to obtain results on Γ_b^{pay} . In particular, Proposition 3 generalizes in the following way:¹¹

Proposition 6 *Assume that $c_i(v_i)$ is constant in v_i for all $i \in C^{pay}$, and suppose that Γ_b^{pay} has a unique equilibrium. Then an increase of the utility u_j of a PSM $j \in P$ will increase the utilities u_i offered by all commercial outlets: the reporting accuracies of free media increase, the reporting accuracies of pay media stay constant but their prices decline.*

To conclude this section, we briefly consider to the case of investigative journalism. We point out that the assumption that $c_i(v_i)$ is constant in v_i for all $i \in C^{pay}$ is crucial for our proofs of Propositions 4 to 6. If c_i is strictly increasing in v_i , then the equilibrium reporting accuracy of firm i can be strictly decreasing in u_{-i} , and the total effect of u_{-i} on u_i is ambiguous, as we show in example in Appendix B below. We leave a full exploration of pay media in the case of investigative journalism for future research.

5 Discussion

Of course, our model abstracts away from several issues that could potentially be relevant. In this section, we discuss multidimensional strategy spaces, spillover effects of reporting accuracy on the advertising revenue of other media outlets, demand functions with decreasing differences, income effects, entry, and potential biases in PSM. We focus on free media, and assume differentiability wherever convenient for expositional simplicity.

¹¹Of course, if $c_j(v_j)$ is identically zero for the public media as well, and V_j does not depend on b_j , then changes in the budgets of PSM have no effects in our model. Even if $c_j(v_j)$ is identically zero for $j \in P$, however, $\bar{v}_j(b_j)$ will be strictly increasing if a larger budget strictly enlarges the feasible set V_j by allowing strictly higher accuracies. Moreover, a larger budget may allow the PSM to increase their attractiveness in other ways that are not related to reporting accuracy.

5.1 Multidimensional strategy spaces

Media outlets may choose different reporting accuracies for different topics, and may choose other dimensions of program quality that are less of a concern for advertisers.

Suppose that outlet i reports about k_i topics, and let $v_{i,k}$ denote reporting accuracy about topic k . Outlet i chooses a vector $v_i \in V_i \subseteq \mathbb{R}_+^{k_i}$ of reporting accuracies. We assume V_i is compact and contains 0. Consumer utility from outlet i is $u_i = f_i(v_i)$, where f_i is continuous, strictly increasing, and satisfies $f_i(0) = 0$. Advertising revenue per consumer is $R_i(v_i)$, where $R_i : V_i \rightarrow \mathbb{R}_+$ is positive, continuous, decreasing in v_i , and independent of v_{-i} . The profit of $i \in C$ is $\pi_i = s_i(u_i, u_{-i}) R_i(v_i) - c_i(v_i)$. Turning to the PSM, suppose that $i \in P$ chooses $v_i \in V_i(b_i)$ subject to $c_i(v_i) \leq b_i$. As above, a higher budget may enlarge the feasible set V_i .

Dumbing down In the case of dumbing down content, where there are no direct costs of raising accuracy so $c_i(v_i)$ is constant in v_i for all $i \in C$, our results generalize in a similar way as in the case of pay media considered above.

To see why, decompose the profit maximization problem of a commercial outlet into two steps. In the first step, the vector of reporting accuracies is chosen to maximize advertising revenue per consumer, subject to the constraint that the utility of the consumer is at least equal to some given u_i . The maximal value of advertising revenue under this constraint is

$$R_i^*(u_i) = \max_{v_i \in V_i} \{R_i(v_i) \mid f_i(v_i) \geq u_i\}.$$

We show in Appendix C.1 that R_i^* has all the features assumed about R_i in our main model (see Assumption (3)): R_i^* is positive, continuous, decreasing in u_i and independent of u_{-i} .

The second step then optimizes over u_i . The choice set is

$$U_i := \{u_i \in \mathbb{R}_+ \mid \exists v_i \in V_i : f_i(v_i) \geq u_i\}.$$

We show in Appendix C.1 that U_i has all the features assumed about the strategy set V_i in our main model.

This two-step procedure allows us to consider the interaction between the commercial outlets as a game where each outlet $i \in C$ has one decision variable u_i , choice set

U_i , and payoff function

$$\pi_i(u_i, u_{-i}) = s_i(u_i, u_{-i}) R_i^*(u_i).$$

From here, the analysis is as in our main model above. In particular, it follows that if the equilibrium is unique, an increase of the budget of a PSM increases the utilities offered by all commercial outlets.

Investigative reporting Consider next the case of investigative reporting. Trivially, as long as the game remains supermodular, our results generalize. A relevant concern is, however, whether the profit of an outlet will be supermodular in its own choice variables.

We illustrate this with a two-dimensional case inspired by Germano and Meier (2013). Instead of denoting the choice variable of outlet i by (v_{i1}, v_{i2}) , we denote it by (v_i, y_i) to avoid notational clutter, slightly abusing notation. Suppose that outlet i chooses reporting accuracy $v_i \in V_i \subseteq \mathbb{R}_+$ and quality $y_i \in Y_i \subseteq \mathbb{R}_+$, where V_i and Y_i are compact and contain 0. The quality y_i does not affect R_i . Consumer utility from outlet i is $u_i = f_i(v_i, y_i)$, where f_i is a continuous and strictly increasing function with $f_i(0, 0) = 0$. The profit of $i \in C$ is

$$\pi_i = s_i(u_i, u_{-i}) R_i(v_i) - c_i(v_i, y_i).$$

PSM $i \in P$ maximizes $u_i = f_i(v_i, y_i)$ subject to $c_i(v_i, y_i) \leq b_i$ by choosing $v_i \in V_i$ and $y_i \in Y_i$. As above, a higher budget may enlarge the feasible sets V_i and Y_i .

We show in Appendix C.2 that π_i has increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$. For a supermodular game, however, π_i also needs to be supermodular in (v_i, y_i) . Whether or not this is the case depends on the cost function c_i and the utility function f_i .

If there are sufficiently strong economies of scope in producing (v_i, y_i) , or if there are sufficiently strong complementarities between v_i and y_i for the consumers, then π_i is supermodular in (v_i, y_i) . If this is the case for all commercial outlets $i \in C$, our results generalize. In particular, Proposition 3 generalizes in the sense that an increase in the budget of a PSM increases both y_i and v_i of all commercial outlets $i \in C$, and the utility u_i of all consumers increases.

On the other hand, π_i will be submodular in (y_i, v_i) if s_i is concave in u_i and there

are no complementarities stemming from the cost function c_i or the utility function f_i . In this case, the effect of a higher budget for the PSM on the utility of the audience of commercial media is ambiguous, as we show in an example in Appendix C.3.

5.2 Spillover effects of reporting accuracies on advertising revenue of other media outlets

Our main model assumes that the advertising revenue of a commercial media outlet depends on its own reporting accuracy, but not on the reporting accuracies of other media outlets. Arguably, advertising revenue of all outlets might be negatively affected when some outlets report about deficiencies of a product, hence there may be spillover effects as in Germano and Meier (2013). Our results are robust when these spillover effects are small compared to the direct effect of an outlet’s own reporting on its advertising revenue. To make this precise, replace Assumption (3) by

Assumption (3’) For all $i \in C$, R_i is positive, continuous, decreasing in (v_i, v_{-i}) , and has increasing differences in (v_i, v_{-i}) .

Note that, since R_i is decreasing in v_i , increasing differences here mean that advertising revenue is not as severely affected by an increase in v_i when other outlets have a high reporting accuracy, which seems a reasonable assumption. We show in Appendix D.1 that, under Assumptions (1), (2), (3’), and (4), a sufficient condition for Γ_b to be a parameterized supermodular game is that spillover effects are small in the sense that

$$\frac{\left| \frac{\partial R_i}{\partial v_j} \right|}{\left| \frac{\partial s_i}{\partial v_j} \right|} \leq \frac{\left| \frac{\partial R_i}{\partial v_i} \right|}{\left| \frac{\partial s_i}{\partial v_i} \right|}$$

for all $i \in C$ and all $j \neq i$.

If spillover effects were as strong as the direct effects, however, the strategic complementarities between the outlets’ reporting accuracies might cease to exist, and reporting accuracies might become independent of each other. We illustrate this in an example in Appendix D.2.

5.3 Demand functions with decreasing differences: logit model

Violations of Assumption (2) do not overturn our results when the elasticity of advertising revenue with respect to reporting accuracy is sufficiently high. To see this, note that the crucial inequality (1) in the proof of Proposition 2 holds if

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{\frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i}}{\frac{\partial s_i(v_i, v_{-i})}{\partial v_j}}.$$

Under Assumption (2), the right hand side is negative hence the above inequality is always satisfied; when Assumption (2) is violated advertising revenue must react sufficiently strongly to reporting accuracy for the inequality to hold.

To illustrate, suppose there is mass of consumers normalized to one, and the market share of outlet i is

$$s_i(v_i, v_{-i}) = \frac{f_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)},$$

where the functions $f_i(v_i)$ are strictly positive and strictly increasing, and v_0 is the utility of the outside option. We allow (but do not require) the functions f_i to differ across media outlets. The logit model is a special case where $f_i(v_i) = \exp(\mu v_i)$ for some exogenous parameter $\mu > 0$.

This demand function satisfies Assumption (1), but in general violates Assumption (2). In particular, if there are two or more commercial media outlets, s_i cannot have increasing differences for all $i \in C$, as we show in Appendix E. There we also prove, however, that a sufficient condition for Γ_b to be a supermodular game is that

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{f'_i(v_i)}{f_i(v_i)}$$

for all $i \in C$. In the logit model, this sufficient condition reduces to $|R'_i(v_i)|/R_i(v_i) \geq \mu$ for all $i \in C$.

This illustration shows that, while Assumption (2) is restrictive, decreasing differences in the demand functions do not necessarily overturn our results when advertising revenue reacts strongly on reporting accuracy.

5.4 Income effects

Consumers in our model have to pay taxes or licence fees to cover the budgets of the PSM. These payments are independent of individual media consumption. They could, however, affect demand via income effects. Such income effects can strengthen our main results, however, when the media are normal goods, i.e. demand increases in income.

Suppose that for $i \in C$, $s_i(v_i, v_{-i}, b)$ is decreasing in b (the higher b , the lower consumers' remaining income; if media are normal goods, demand is lower). Moreover, suppose that s_i has increasing differences in (v_i, b) , i.e. demand reacts more on quality differences when income is lower. The strategic complementarities between the commercial media are not affected by the income effects. For $i \in C$ and $j \in P$, consider the cross-partial

$$\begin{aligned} \frac{\partial^2 \tilde{\pi}_i}{\partial b_j \partial v_i} &= \left(\frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i})}{\partial v_j} R'_i(v_i) \right) \bar{v}'_j(b_j) \\ &+ \frac{\partial^2 s_i(v_i, v_{-i}, b)}{\partial b_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i}, b)}{\partial b_j} R'_i(v_i). \end{aligned}$$

The first line describes the effects studied in our main model above: an increase of b_j increases \bar{v}_j and this has the effects studied above (the terms in the bracket are the same as in inequality (1) in the proof of Proposition 2). The second line stems from the income effect. Note that $\frac{\partial^2 s_i(v_i, v_{-i}, b)}{\partial b_j \partial v_i} \geq 0$ because s_i has increasing differences in (v_i, b) , and $\frac{\partial s_i(v_i, v_{-i}, b)}{\partial b_j} \leq 0$ because good i is normal; hence the second line is positive. This shows that income effects strengthen the strategic complementarities that drive our results.

On the other hand, PSM might lead commercial media to exit the market. Income effects can strengthen this type of crowding out: the PSM do not only offer competing products, but also lower demand for commercial media via income effects. We discuss entry and exit next.

5.5 Entry and exit

Our results on strategic complementarities apply to situations where the PSM do not induce any of the commercial outlets to exit the market. In reality, when the PSM budgets are sufficiently increased, commercial outlets may be driven out of business.

By the same token, if PSM were scaled back, this could trigger entry of additional commercial outlets. The new entrants would have to provide sufficiently high quality in order to overturn the results in Proposition 3, however.

To illustrate, suppose the PSM were abolished in favor of a purely commercial media market. Without additional entry, our results above predict that reporting accuracy of the commercial outlets would decline. Entry of m additional commercial outlets would keep the number of media outlets constant. If these entrants provide lower reporting accuracy than the PSM used to, however, incumbent commercial media will still provide lower reporting accuracy than before the commercialization of the media market, and so all consumers are negatively affected by lower reporting accuracies. Whether there will be sufficiently many entrants with sufficiently high reporting accuracy to overcome this negative effect on consumers will depend, among other things, on barriers to entry, the revenue potential of the market, the PSMs' budgets, and possibly cost advantages or disadvantages of new entrants.

5.6 Biases in PSM

Our model allows PSM to be cost-inefficient but assumes them to be unbiased. Of course, PSM may themselves be biased as well (see e.g. Crawford and Levonyan 2018). Commercial biases can exist in PSM, as they also engage in advertising or product placement. In some countries, governments are major advertisers themselves (Di Tella and Franceschelli 2011, Szeidl and Szucs 2021), and PSM may be especially susceptible to government influence. Moreover, advertising revenue helps against other sources of biases in media content (see for example Besley and Prat 2006 and Petrova 2011).

While such considerations are clearly important, we point out that our results can allow for some biases in PSM. The strategic complementarities between the reporting accuracies of commercial media do not depend on assumptions about the PSM at all. Concerning the impact of PSM budgets on commercial media, the key issue is whether a higher budget of a PSM will translate into more or less severe biases of this PSM outlet. As long as the reporting accuracies of PSM are increasing in their budgets, Propositions 2 and 3 are robust to biases in the PSM.

As in our discussion of entry above, any evaluation of the impact of potentially biased PSM on the content of commercial media outlets crucially depends on what the alternatives to PSM are. PSM are typically not for profit, and they often face tighter

limitations on advertising than commercial outlets (see e.g. Crawford et al. 2017), which may counteract commercial media bias (Kerkhof and Münster 2015). Indeed, PSM typically have a higher share of hard news and socially relevant topics in their program, so their commercial biases may be lower (see Cushion 2017 for a wide ranging review).

6 Conclusion

This paper shows that in a standard model of commercial media bias, program qualities in terms of the media’s reporting accuracy are strategic complements rather than strategic substitutes. The strategic complementarity stems from the media’s fundamental trade-off in these models: Increasing reporting accuracy increases the value of the media content for the audience but decreases the willingness to pay of the advertisers to reach consumers. The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to increase reporting accuracy are higher. Thus, in a media market with both public service media (PSM) and commercial media, raising the PSMs’ reporting accuracy reduces the commercial media’s audiences and thereby also the implicit costs of increasing their own reporting accuracy. As a result, the PSM *crowd in* reporting accuracy and engage the commercial media in a race to the top. This is in line with recent empirical evidence on public and private investments into program quality and on the impact of competition in media markets.

Our finding contributes to recurrent media policy debates about the proper role and scope of PSM. While several regulation authorities fear that raising the program quality of PSM could crowd out private investments into program quality, our results support policies that advocate strong and financially well-equipped PSM.

A Increasing differences without differentiability

In the proof of Proposition 2, we assumed that the functions s_i , R_i , and c_i are differentiable in order to prove that π_i has increasing differences in (v_i, v_{-i}) . In this appendix we give the proof without assuming differentiability. Consider one outlet $j \neq i$, hold all other v_k ($k \neq i, j$) constant and suppress them in the formulas to avoid notational

clutter. Then

$$\pi_i(v_i, v_j) = s_i(v_i, v_j) R_i(v_i) - c_i(v_i).$$

Suppose that $v_i^h > v_i^l$ and $v_j^h > v_j^l$. Then

$$\pi_i(v_i, v_j^h) - \pi_i(v_i, v_j^l) = (s_i(v_i, v_j^h) - s_i(v_i, v_j^l)) R_i(v_i)$$

and

$$\begin{aligned} & \pi_i(v_i^h, v_j^h) - \pi_i(v_i^h, v_j^l) - (\pi_i(v_i^l, v_j^h) - \pi_i(v_i^l, v_j^l)) \\ &= (s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l)) R_i(v_i^h) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)) R_i(v_i^l) \\ &= (s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l))) R_i(v_i^h) \\ &+ (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)) (R_i(v_i^h) - R_i(v_i^l)) \end{aligned}$$

By Assumption (2), s_i has increasing differences in (v_i, v_j) , i.e.

$$s_i(v_i^h, v_j^h) - s_i(v_i^l, v_j^h) \geq s_i(v_i^h, v_j^l) - s_i(v_i^l, v_j^l)$$

or equivalently

$$s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) \geq s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l).$$

Since $R_i(v_i^h) \geq 0$, it follows that

$$(s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l))) R_i(v_i^h) \geq 0.$$

Moreover, by (2) $s_i(v_i^l, v_j^h) \leq s_i(v_i^l, v_j^l)$ and by (3), $R_i(v_i^h) \leq R_i(v_i^l)$, thus

$$(s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)) (R_i(v_i^h) - R_i(v_i^l)) \geq 0.$$

It follows that

$$\pi_i(v_i^h, v_j^h) - \pi_i(v_i^h, v_j^l) \geq \pi_i(v_i^l, v_j^h) - \pi_i(v_i^l, v_j^l)$$

or equivalently

$$\pi_i(v_i^h, v_j^h) - \pi_i(v_i^l, v_j^h) \geq \pi_i(v_i^h, v_j^l) - \pi_i(v_i^l, v_j^l)$$

i.e. π_i has increasing differences in (v_i^h, v_j^h) .

B Pay media and investigative reporting: an example

In this appendix we consider an example of a pay media outlet in the case of investigative journalism. Consider a Hotelling duopoly. Outlet 1 is a pay media outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to u_2 ; for this exercise it does not matter whether outlet 2 is another commercial (pay or free) media outlet or a PSM.

Example 1 *Suppose that $V_1 = \mathbb{R}_+$, $c_1(v_1) = kv_1^2/2$ where k is a parameter, and $R_1(v_1) = \max\{1 - \beta v_1, 0\}$ with $0 < \beta < 1$. The total audience has a fixed size normalized to 1, and the market share of outlet 1 is given by the Hotelling specification*

$$s_1(v_1, p_1, u_2) = \begin{cases} 0, & \text{if } \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \leq 0, \\ \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, & \text{if } 0 < \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} < 1, \\ 1, & \text{otherwise.} \end{cases}$$

The profit of outlet 1 is

$$\pi_1(v_1, p_1, u_2) = s_1(v_1, p_1, u_2)(R_1(v_1) + p_1) - \frac{kv_1^2}{2}.$$

Assume that

$$4k\tau > (1 - \beta)^2 \tag{2}$$

in order that π_1 is strictly concave in (v_1, p_1) in the relevant range. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_1 > 0$, $p_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$.¹²

Note that for this example k has to be sufficiently high for the second order condition to hold, hence the case of dumbing down is not a limit case of this example.

Remark 1 *In Example 1, v_1 and p_1 are strictly decreasing in u_2 . Moreover, $u_1 = v_1 - p_1$ is strictly increasing in u_2 if $2k\tau > (1 - \beta)^2$, and strictly decreasing if $2k\tau < (1 - \beta)^2$.*

¹²Conditions on the fundamentals such that the solution is interior will be given in the proof below.

Proof. In the relevant range,

$$\pi_1(v_1, p_1, u_2) = \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) (1 - \beta v_1 + p_1) - \frac{kv_1^2}{2}.$$

The partial derivatives are

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= -\frac{1}{2\tau} (1 - \beta v_1 + p_1) + \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, \\ \frac{\partial \pi_1}{\partial v_1} &= \frac{1}{2\tau} (1 - \beta v_1 + p_1) - \beta \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) - kv_1. \end{aligned}$$

Moreover,

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial p_1^2} &= -\frac{1}{\tau} < 0, \\ \frac{\partial^2 \pi_1}{\partial v_1^2} &= -\frac{\beta}{\tau} - k < 0, \\ \frac{\partial^2 \pi_1}{\partial p_1 \partial v_1} &= \frac{1 + \beta}{2\tau}. \end{aligned}$$

Hence the determinant of the Hessian is

$$\frac{1}{\tau} \left(\frac{\beta}{\tau} + k \right) - \left(\frac{1 + \beta}{2\tau} \right)^2 > 0$$

iff $4k\tau > (1 - \beta)^2$. This shows π_1 is strictly concave in the relevant range if inequality (2) holds.

The first order conditions for an interior solution are

$$\begin{aligned} \frac{1}{2\tau} (1 - \beta v_1 + p_1) &= \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, \\ \frac{1}{2\tau} (1 - \beta v_1 + p_1) &= \beta \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) + kv_1. \end{aligned}$$

Solving the first order conditions gives

$$\begin{aligned} v_1^*(u_2) &= \frac{(\tau + 1 - u_2)(1 - \beta)}{4k\tau - (1 - \beta)^2}, \\ p_1^*(u_2) &= \frac{(\beta(1 - \beta) + 2k\tau)(\tau - u_2) + 1 - \beta - 2k\tau}{4k\tau - (1 - \beta)^2}. \end{aligned}$$

Differentiate

$$\begin{aligned}\frac{\partial v_1^*(u_2)}{\partial u_2} &= -\frac{1-\beta}{4k\tau - (1-\beta)^2} < 0, \\ \frac{\partial p_1^*(u_2)}{\partial u_2} &= -\frac{\beta(1-\beta) + 2k\tau}{4k\tau - (1-\beta)^2} < 0.\end{aligned}$$

Moreover, from $u_1^*(u_2) = v_1^*(u_2) - p_1^*(u_2)$,

$$\frac{\partial u_1^*(u_2)}{\partial u_2} = \frac{2k\tau - (1-\beta)^2}{4k\tau - (1-\beta)^2}.$$

Therefore, $u_1^*(u_2)$ is strictly increasing in u_2 if $2k\tau > (1-\beta)^2$, and $u_1^*(u_2)$ is strictly decreasing in u_2 if $2k\tau < (1-\beta)^2$.

It remains to check under which parameter constellations an interior solution exists.

Note that $v_1^*(u_2) > 0$ iff

$$\tau + 1 > u_2, \tag{3}$$

and $p_1^*(u_2) > 0$ iff

$$\tau + \frac{1-\beta-2k\tau}{(\beta(1-\beta)+2k\tau)} > u_2. \tag{4}$$

Note that

$$\frac{1-\beta-2k\tau}{(\beta(1-\beta)+2k\tau)} < 1$$

by inequality (2). Thus inequality (3) is implied by inequality (4).

We turn to advertising revenue next. Note that

$$R_1(v_1^*(u_2)) = 1 - \beta \frac{(\tau + 1 - u_2)(1-\beta)}{4k\tau - (1-\beta)^2}$$

is strictly positive iff

$$u_2 > \tau + 1 - \frac{4k\tau - (1-\beta)^2}{\beta(1-\beta)}. \tag{5}$$

Inequalities (4) and (5) hold simultaneously iff

$$\tau + \frac{1-\beta-2k\tau}{(\beta(1-\beta)+2k\tau)} > u_2 > \tau + 1 - \frac{4k\tau - (1-\beta)^2}{\beta(1-\beta)}. \tag{6}$$

By inequality (2), the right hand side is strictly smaller than the left hand side; therefore (6) is satisfied in an nonempty open set of values for u_2 .

Finally, we need to make sure that $0 < s_1(u_1^*(u_2), u_2) < 1$. This is the case iff

$$0 < \frac{1}{2} + \frac{v_1^*(u_2) - p_1^*(u_2) - u_2}{2\tau} < 1,$$

or equivalently

$$-\tau < v_1^*(u_2) - p_1^*(u_2) - u_2 < \tau.$$

We have

$$\begin{aligned} & v_1^*(u_2) - p_1^*(u_2) - u_2 \\ &= \frac{(\tau + 1 - u_2)(1 - \beta)}{4k\tau - (1 - \beta)^2} - \frac{(\beta(1 - \beta) + 2k\tau)(\tau - u_2) + 1 - \beta - 2k\tau}{4k\tau - (1 - \beta)^2} - u_2 \\ &= \tau \frac{2k - 2k\tau + (1 - \beta)^2 - 2ku_2}{4k\tau - (1 - \beta)^2}. \end{aligned}$$

Thus $0 < s_1(u_1^*(u_2), u_2) < 1$ iff

$$-1 < \frac{2k - 2k\tau + (1 - \beta)^2 - 2ku_2}{4k\tau - (1 - \beta)^2} < 1,$$

or equivalently

$$-(4k\tau - (1 - \beta)^2) < 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 < 4k\tau - (1 - \beta)^2. \quad (7)$$

The expression in the middle is a strictly decreasing function of u_2 .

Since $u_2 < \tau + 1$ by (3),

$$\begin{aligned} 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 &> 2k - 2k\tau + (1 - \beta)^2 - 2k(\tau + 1) \\ &= -(4k\tau - (1 - \beta)^2), \end{aligned}$$

thus the first inequality in (7) holds.

Similarly, by (5),

$$\begin{aligned}
& 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 \\
& < 2k - 2k\tau + (1 - \beta)^2 - 2k \left(\tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)} \right) \\
& = \frac{1}{\beta(1 - \beta)} (\beta^2 - \beta + 2k) (-\beta^2 + 2\beta + 4k\tau - 1).
\end{aligned}$$

Therefore, a sufficient condition for the second inequality in (7) is that

$$\begin{aligned}
& (4k\tau - (1 - \beta)^2) - \left(\frac{1}{\beta(1 - \beta)} (\beta^2 - \beta + 2k) (-\beta^2 + 2\beta + 4k\tau - 1) \right) \\
& = 2(\beta(1 - \beta) - k) \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)} > 0,
\end{aligned}$$

which is true iff

$$\beta(1 - \beta) > k. \quad (8)$$

We have established that the problem has an interior solution under the conditions (8), (2), and (6), which we repeat here for convenience:

$$\begin{aligned}
& \beta(1 - \beta) > k, \\
& 4k\tau > (1 - \beta)^2, \\
& \tau + \frac{1 - \beta - 2k\tau}{(\beta(1 - \beta) + 2k\tau)} > u_2 > \tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)}.
\end{aligned}$$

To see they can be satisfied simultaneously, first choose β and k such that the first line holds. Then choose τ such that the second line holds; note that depending on how you choose τ , either $2k\tau > (1 - \beta)^2$ or $2k\tau < (1 - \beta)^2$. Finally, choose u_2 for the last line.

■

A numerical example that satisfies all the constraints may be reassuring. Let $\beta = 0.5$, $\tau = 1.25$, and $u_2 = 1.5$. For $k = 0.11$, $2k\tau = 2 * 0.11 * 1.25 = 0.275 > (1 - \beta)^2 = 0.25$ and $u_1^*(u_2)$ is strictly increasing in u_2 . For $k = 0.09$, $2k\tau = 2 * 0.09 * 1.25 = 0.225 < 0.25 < 4k\tau = 0.45$, and $u_1^*(u_2)$ is strictly decreasing.

Within our parameter restrictions, $u_1^*(u_2)$ is strictly increasing in u_2 if k is large. An economic intuition is that the marginal costs of v_1 are rapidly increasing if k is large, and hence then the falling price dominates the decrease in reporting accuracy.

To give more details, recall that $v_1^*(u_2)$ and $p_1^*(u_2)$ are strictly decreasing in u_2 . If k is large, the effect of u_2 on $v_1^*(u_2)$ becomes less important (smaller in absolute value):

$$\frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} = \frac{4\tau(1-\beta)}{(4k\tau - (1-\beta)^2)^2} > 0.$$

On the other hand, the effect of u_2 on $p_1^*(u_2)$ also becomes less important:

$$\begin{aligned} \frac{\partial}{\partial k} \frac{\partial p_1^*(u_2)}{\partial u_2} &= \frac{\partial}{\partial k} \left(-\frac{\beta(1-\beta) + 2k\tau}{4k\tau - (1-\beta)^2} \right) \\ &= \frac{2\tau(1-\beta^2)}{(4k\tau - (1-\beta)^2)^2} > 0 \end{aligned}$$

But note that $4\tau(1-\beta) - 2\tau(1-\beta^2) = 2\tau(1-\beta)^2 > 0$, thus

$$\frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} > \frac{\partial}{\partial k} \frac{\partial p_1^*(u_2)}{\partial u_2}.$$

That is, if k increases, the change of $v_1^*(u_2)$ in u_2 is vanishing quicker than the change of $p_1^*(u_2)$ in u_2 . For large enough k , $u_1^*(u_2)$ increases in u_2 because the falling price overcompensates for the falling accuracy.

C Multidimensional strategy spaces

C.1 Dumbing down

This appendix considers multidimensional strategy spaces in the case of dumbing down, where $c_i(v_i)$ is constant in v_i for all $i \in C$. As in the main text, let

$$U_i := \{u_i \in \mathbb{R}_+ \mid \exists v_i \in V_i : f_i(v_i) \geq u_i\}.$$

We show that U_i has the properties assumed about the choice set V_i in our main model. That is, we show that $U_i \subseteq \mathbb{R}_+$ is compact, and U_i contains zero. Since $0 \in V_i$ and $f_i(0) = 0$, $0 \in U_i$. Moreover, since V_i is compact and f_i is continuous, by the Weierstrass Theorem a maximum achievable utility exists, thus $U_i = [0, \max_{v_i \in V_i} f_i(v_i)]$ is compact.

As in the main text, let $R_i^* : U_i \rightarrow \mathbb{R}_+$ be defined by

$$R_i^*(u_i) = \max_{v_i \in V_i} \{R_i(v_i) \mid f_i(v_i) \geq u_i\}.$$

We show that the function R_i^* has all the properties required in Assumption (3).

First, R_i^* is positive since R_i is positive by assumption.

Second, we use the Maximum Theorem to show that R_i^* is continuous. R_i is continuous by assumption. It remains to show that the constraint correspondence, which gives for any $u_i \in U_i$ the set of reporting accuracies that achieve utility at least equal u_i , is continuous. Let $g_i : U_i \rightarrow V_i$, $g_i(u_i) = \{v_i \in V_i \mid f_i(v_i) \geq u_i\}$, denote the constraint correspondence. The range of g_i is V_i , which is compact by assumption. Moreover, g_i is upper hemicontinuous by continuity of f_i ,¹³ and g is lower hemicontinuous by standard arguments establishing continuity of the expenditure function via the Maximum Theorem (see e.g. Kreps 2013, page 237).

Third, R_i^* is decreasing in u_i . To see this, suppose to the contrary that $u_i^1 \geq u_i^0$ but $R_i^*(u_i^1) > R_i^*(u_i^0)$. Then there exists $v_i^1 \in V_i$ such that $f_i(v_i^1) \geq u_i^1$ and $R_i^*(u_i^1) = R_i(v_i^1)$. But since $u_i^1 \geq u_i^0$, it is also true that $f_i(v_i^1) \geq u_i^0$, and therefore

$$R_i^*(u_i^0) = \max_{v_i \in V_i} \{R_i(v_i) \mid f_i(v_i) \geq u_i^0\} \geq R_i(v_i^1) = R_i^*(u_i^1),$$

contradicting the assumption that $R_i^*(u_i^1) > R_i^*(u_i^0)$.

Fourth, R_i^* is obviously independent of v_{-i} .

C.2 Investigative reporting: conditions for a supermodular game

This appendix studies conditions for a supermodular game when outlet i chooses reporting accuracy v_i and a quality y_i which does not affect R_i . Recall that consumer utility from outlet i is $u_i = f_i(v_i, y_i)$. For $i \in C$,

$$\pi_i = s_i(u_i, u_{-i}) R_i(v_i) - c_i(v_i, y_i).$$

¹³Since the range of g_i is compact, g_i is upper hemicontinuous if for any two sequences $u_i^m \rightarrow u_i \in U_i$ and $v_i^m \rightarrow v_i$, with $u_i^m \in U_i$ and $v_i^m \in g_i(u_i^m)$ for all m , we have $v_i \in g_i(u_i)$ (see e.g. Mas-Colell, Whinston, and Green 1995, Section M.H). Since $v_i^m \in g_i(u_i^m)$ for all m , $f_i(v_i^m) \geq u_i^m$ for all m , hence $f_i(v_i) \geq u_i$ by continuity of f_i . Moreover, $v_i^m \in g_i(u_i^m)$ for all m implies $v_i^m \in V_i$ for all m , and since V_i is compact, $v_i \in V_i$. This completes the proof that $v_i \in g_i(u_i)$.

We first show that π_i has increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$.

$$\begin{aligned}\frac{\partial \pi_i}{\partial v_i} &= \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + s_i(u_i, u_{-i}) R'_i(v_i) - \frac{\partial c_i(v_i, y_i)}{\partial v_i}, \\ \frac{\partial \pi_i}{\partial y_i} &= \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R_i(v_i) - \frac{\partial c_i(v_i, y_i)}{\partial y_i}.\end{aligned}$$

Note that for all $j \neq i$ and $x_j \in \{v_j, y_j\}$,

$$\frac{\partial}{\partial x_j} \frac{\partial \pi_i}{\partial v_i} = \left(\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + \frac{\partial s_i(u_i, u_{-i})}{\partial u_j} R'_i(v_i) \right) \frac{\partial f_j(v_j, y_j)}{\partial x_j} \geq 0$$

and

$$\frac{\partial}{\partial x_j} \frac{\partial \pi_i}{\partial y_i} = \frac{\partial^2 s_i(f_i(v_i, y_i), u_{-i})}{\partial u_j \partial u_i} \frac{\partial f_j(v_j, y_j)}{\partial x_j} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R_i(v_i) \geq 0,$$

so π_i has increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$.

For a supermodular game, however, π_i also needs to be supermodular in (v_i, y_i) .

To study when this is the case, calculate the cross-partial

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial y_i \partial v_i} &= \frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_i^2} \frac{\partial f_i(v_i, y_i)}{\partial y_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial^2 f_i(v_i, y_i)}{\partial v_i \partial y_i} R_i(v_i) \\ &+ \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R'_i(v_i) - \frac{\partial^2 c_i(v_i, y_i)}{\partial y_i \partial v_i}.\end{aligned}$$

Thus π_i will be supermodular in (v_i, y_i) if there are pronounced economies of scope in producing (v_i, y_i) so that $\frac{\partial^2 c_i(v_i, y_i)}{\partial y_i \partial v_i}$ is sufficiently negative, or if v_i and y_i are strong complements for the consumers so that $\frac{\partial^2 f_i(v_i, y_i)}{\partial v_i \partial y_i}$ is sufficiently positive.

But note that the third term in the above formula for the cross-partial of π_i ,

$$\frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R'_i(v_i)$$

is negative. Moreover the first term

$$\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_i^2} \frac{\partial f_i(v_i, y_i)}{\partial y_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i)$$

is negative if s_i is concave in u_i . Therefore, π_i will be submodular in (y_i, v_i) if there are no complementarities stemming from c_i and f_i , and s_i is concave in u_i . In this case, the effect of a higher budget for the PSM on the utility of the audience of commercial

media is ambiguous, as we show in an example in Appendix C.3.

C.3 Investigative reporting: an example with a submodular profit function

In this appendix, we show by example that π_i may be strictly submodular in the choice variables of outlet i , and that in this case the effect of PSM on the utility of the product offered by outlet i is ambiguous. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to u_2 ; for this exercise it does not matter whether outlet 2 is another commercial (pay or free) media outlet or a PSM.

Example 2 Consider a duopoly where outlet 1 is a commercial outlet. Suppose $V_1 = Y_1 = \mathbb{R}_+$. The audience has a fixed total size of 1, and market share of outlet 1 is given by the Hotelling demand specification

$$s_1(u_1, u_2) = \begin{cases} 0, & \text{if } \frac{1}{2} + \frac{u_1 - u_2}{2\tau} \leq 0, \\ \frac{1}{2} + \frac{u_1 - u_2}{2\tau}, & \text{if } 0 < \frac{1}{2} + \frac{u_1 - u_2}{2\tau} < 1, \\ 1, & \text{otherwise.} \end{cases}$$

for $i = 1, 2$. Consumer utility from the commercial media outlet is $u_1 = f_1(v_1, y_1) = v_1 + y_1$ and the cost function is $c_1(v_1, y_1) = ky_1^2/2$. Advertising revenue per consumer is $R_1(v_1) = \max\{1 - \beta v_1, 0\}$ where $\beta > 0$ is an exogenous parameter. We assume that

$$4k\tau > \beta \tag{9}$$

in order that the π_1 is strictly concave in (v_1, y_1) in the relevant range. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_1 > 0$, $y_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$.¹⁴

Note that in Example 2, there are no complementarities between v_1 and y_1 stemming from the cost function c_1 or the utility function f_1 . Moreover, the demand function is linear in u_1 in the relevant range. As a consequence, π_1 is submodular in (v_1, y_1) in the relevant range.

¹⁴We provide conditions on the fundamentals where this is the case in the proof below.

Remark 2 In Example 2, v_1 is strictly increasing in u_2 , and y_1 is strictly decreasing in u_2 . The utility offered by the commercial outlet, u_1 , is strictly increasing in u_2 if $2k\tau > \beta$, and strictly decreasing in u_2 if $2k\tau < \beta$.

Proof. In the relevant range,

$$\pi_1 = \left(\frac{1}{2} + \frac{v_1 + y_1 - u_2}{2\tau} \right) (1 - \beta v_1) - \frac{k}{2} y_1^2.$$

The first order conditions are

$$\begin{aligned} \frac{\partial \pi_1}{\partial v_1} &= \frac{1}{2\tau} (1 - \beta v_1) - \beta \left(\frac{1}{2} + \frac{v_1 + y_1 - u_2}{2\tau} \right) = 0, \\ \frac{\partial \pi_1}{\partial y_1} &= \frac{1}{2\tau} (1 - \beta v_1) - k y_1 = 0. \end{aligned}$$

The second derivatives are

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial v_1^2} &= -\frac{\beta}{\tau} < 0, \\ \frac{\partial^2 \pi_1}{\partial y_1^2} &= -k < 0, \\ \frac{\partial^2 \pi_1}{\partial v_1 \partial y_1} &= -\frac{\beta}{2\tau} < 0. \end{aligned}$$

The last inequality shows that π_1 is strictly submodular in (v_1, y_1) in the relevant range.

The determinant of the Hessian matrix is

$$\frac{k\beta}{\tau} - \frac{\beta^2}{4\tau^2} = \frac{\beta}{\tau} \left(k - \frac{\beta}{4\tau} \right) > 0$$

iff $4k\tau > \beta$; i.e. π_1 is strictly concave in the relevant range if inequality (9) holds.

Assuming an interior solution, the best reply function is

$$\begin{aligned} v_1^*(u_2) &= \frac{1}{\beta(4k\tau - \beta)} (-\beta + 2k\tau - 2k\beta\tau^2 + 2k\beta\tau u_2), \\ y_1^*(u_2) &= \frac{1}{4k\tau - \beta} (\beta\tau - \beta u_2 + 1). \end{aligned}$$

Note that, since $4k\tau > \beta$ by assumption (9),

$$\begin{aligned}\frac{\partial v_1^*(u_2)}{\partial u_2} &= \frac{2k\tau}{4k\tau - \beta} > 0, \\ \frac{\partial y_1^*(u_2)}{\partial u_2} &= \frac{-\beta}{4k\tau - \beta} < 0.\end{aligned}$$

The utility offered by 1 is $u_1^*(u_2) = v_1^*(u_2) + y_1^*(u_2)$. Thus

$$\frac{\partial u_1^*(u_2)}{\partial u_2} = \frac{2k\tau - \beta}{4k\tau - \beta}$$

which is strictly positive if $2k\tau > \beta$, but strictly negative if $2k\tau < \beta$.

It remains to establish conditions on the fundamentals such that the solution is interior. Note that $v_1^*(u_2) > 0$ if u_2 is sufficiently large, and $y_1^*(u_2) > 0$ when u_2 is sufficiently small. We show that there exists a non-empty open interval of values for u_2 such that both $v_1^*(u_2) > 0$ and $y_1^*(u_2) > 0$. We have $v_1^*(u_2) > 0$ iff

$$u_2 > \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta)$$

and $y_1^*(u_2) > 0$ iff

$$u_2 < \frac{\beta\tau + 1}{\beta}.$$

Moreover,

$$\frac{\beta\tau + 1}{\beta} > \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta),$$

since by inequality (9)

$$\frac{\beta\tau + 1}{\beta} - \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta) = \frac{1}{2k\beta\tau} (4k\tau - \beta) > 0.$$

Therefore, whenever

$$\frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta) < u_2 < \frac{\beta\tau + 1}{\beta}, \quad (10)$$

we have both $v_1^*(u_2) > 0$ and $y_1^*(u_2) > 0$.

We also need to make sure that $R_1(u_1^*(u_2)) > 0$ and $s_1(u_1^*(u_2), u_2) \in (0, 1)$.

$$\begin{aligned} R_1(u_1^*(u_2)) &= 1 - \frac{1}{(4k\tau - \beta)} (-\beta + 2k\tau - 2k\beta\tau^2 + 2k\beta\tau u_2) \\ &= \frac{2k\tau}{4k\tau - \beta} (\beta\tau + 1 - \beta u_2) > 0, \end{aligned}$$

which is strictly positive because $\beta\tau + 1 > \beta u_2$ by (10).

Moreover,

$$s_1(u_1^*(u_2), u_2) = \frac{k(\beta\tau + 1 - \beta u_2)}{\beta(4k\tau - \beta)} > 0$$

by inequality (10). It remains to check whether $s_1(u_1^*(u_2), u_2) < 1$. Note that $s_1(u_1^*(u_2), u_2)$ is strictly decreasing in u_2 . By inequality (10),

$$\frac{k(\beta\tau + 1 - \beta u_2)}{\beta(4k\tau - \beta)} < \frac{k\left(\beta\tau + 1 - \beta\left(\frac{1}{2k\beta\tau}(2k\beta\tau^2 - 2k\tau + \beta)\right)\right)}{\beta(4k\tau - \beta)} = \frac{1}{2\beta\tau},$$

so a sufficient condition for $s_1(u_1^*(u_2), u_2) < 1$ is that

$$2\beta\tau > 1. \tag{11}$$

We have shown that the maximization problem has an interior solution if inequalities (11) (9), and (10) hold, which we repeat here for convenience:

$$\begin{aligned} 2\beta\tau &> 1, \\ 4k\tau &> \beta, \\ \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta) &< u_2 < \frac{\beta\tau + 1}{\beta}. \end{aligned}$$

To see they can be simultaneously satisfied, first choose β and τ to satisfy the first inequality. Then choose k to satisfy the second inequality; note that depending on how you choose k , you can have either $2k\tau > \beta$ or $2k\tau < \beta$. Finally, choose u_2 to satisfy the third inequality. ■

A numerical example that satisfies all the constraints may be reassuring. Let $\beta = 2/3$, $\tau = 1$, and $u_2 = 2.2$. For $k = 1$, $2k\tau = 2 > \beta$ so u_1 is strictly increasing in u_2 . For $k = \frac{2}{10}$, $4k\tau = \frac{8}{10} > \frac{2}{3} = \beta > \frac{4}{10} = 2k\tau$ so u_1 is strictly decreasing.

Within our parameter restrictions, u_1 is strictly increasing in u_2 if k is sufficiently large. An economic intuition for this is that, if k is large, the marginal costs of y_1 are rapidly increasing, hence the positive impact of u_2 on v_1 overcompensates the negative impact of u_2 on y_1 . In more detail, the reaction of v_1^* and y_1^* to changes in u_2 both become smaller in absolute value when k increases:

$$\begin{aligned}\frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} &= \frac{-2\beta\tau}{(4k\tau - \beta)^2} < 0, \\ \frac{\partial}{\partial k} \frac{\partial y_1^*(u_2)}{\partial u_2} &= \frac{4\beta\tau}{(4k\tau - \beta)^2} > 0.\end{aligned}$$

But k affects the reaction of y_1^* to changes in u_2 more than it affects the reaction of v_1^* , so for large enough values of k , the reaction of v_1^* dominates the effect of u_2 on $u_1^*(u_2)$.

D Spillover effects of reporting accuracies on advertising revenue of other media outlets

D.1 Small spillover effects

This appendix proves the claim that under Assumptions (1), (2), (3'), and (4), Γ_b is a parameterized supermodular game if spillover effects are small in the sense that

$$\left| \frac{\partial R_i}{\partial v_j} \right| \leq \frac{\left| \frac{\partial R_i}{\partial v_i} \right|}{\left| \frac{\partial s_i}{\partial v_j} \right|}.$$

Differentiate

$$\pi_i = s_i(v_i, v_{-i}) R_i(v_i, v_{-i}) - c_i(v_i)$$

to obtain

$$\begin{aligned}\frac{\partial \pi_i}{\partial v_i} &= \frac{\partial s_i}{\partial v_i} R_i + s_i \frac{\partial R_i}{\partial v_i} - \frac{\partial c_i}{\partial v_i}, \\ \frac{\partial^2 \pi_i}{\partial v_j \partial v_i} &= \frac{\partial^2 s_i}{\partial v_j \partial v_i} R_i + \frac{\partial s_i}{\partial v_i} \frac{\partial R_i}{\partial v_j} + \frac{\partial s_i}{\partial v_j} \frac{\partial R_i}{\partial v_i} + s_i \frac{\partial^2 R_i}{\partial v_j \partial v_i}.\end{aligned}$$

We have $\frac{\partial^2 s_i}{\partial v_j \partial v_i} \geq 0$ by Assumption (2) and $\frac{\partial^2 R_i}{\partial v_j \partial v_i} \geq 0$ by Assumption (3'). Therefore, $\frac{\partial^2 \pi_i}{\partial v_j \partial v_i} \geq 0$ holds if

$$\frac{\partial s_i}{\partial v_i} \frac{\partial R_i}{\partial v_j} + \frac{\partial s_i}{\partial v_j} \frac{\partial R_i}{\partial v_i} \geq 0.$$

Rearranging completes the proof.

D.2 Large spillover effects: an example

In this appendix, we show by example that π_i may have constant differences in (v_i, v_{-i}) if spillover effects are large. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to v_2 ; for this exercise it does not matter whether outlet 2 is another free commercial media outlet or a PSM.

Example 3 Suppose that $V_1 = \mathbb{R}_+$,

$$R_1(v_1, v_2) = \max\{1 - (\alpha v_1 + \beta v_2), 0\}$$

where $\alpha > 0$ and $\beta > 0$ are exogenous parameters, s_1 is given by a Hotelling specification. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_1 > 0$, $R_1 > 0$, and $0 < s_1 < 1$.¹⁵

Note that R_1 in Example 3 satisfies Assumption (3').

Remark 3 Consider Example 3. If $\alpha > \beta$, then π_1 has strictly increasing differences in (v_1, v_2) in the relevant range. If $\alpha = \beta$, then π_1 has constant differences in (v_1, v_2) in the relevant range.

Proof. In the relevant range,

$$s_1(v_1, v_2) = \frac{1}{2} + \frac{v_1 - v_2}{2\tau},$$

hence

$$\frac{\partial^2 s_1(v_1, v_2)}{\partial v_2 \partial v_1} = 0$$

¹⁵We show by example after the proof that there are parameter constellations where the solution is interior.

and

$$\frac{\partial s_1(v_1, v_2)}{\partial v_1} = -\frac{\partial s_1(v_1, v_2)}{\partial v_2} = \frac{1}{2\tau}.$$

The profit of commercial outlet 1 is

$$\pi_1(v_1, v_2) = s_1(v_1, v_2)(1 - (\alpha v_1 + \beta v_2)) - c_1(v_1).$$

Hence

$$\frac{\partial \pi_1}{\partial v_1} = \frac{1}{2\tau}(1 - (\alpha v_1 + \beta v_2)) - \alpha s_1(v_1, v_2) - \frac{\partial c_1(v_1)}{\partial v_1}$$

and

$$\frac{\partial^2 \pi_1}{\partial v_1 \partial v_2} = \frac{\alpha - \beta}{2\tau}.$$

Therefore, if $\alpha > \beta$, π_1 has strictly increasing differences in (v_1, v_2) . On the other hand, if $\alpha = \beta$, then π_1 has constant differences in (v_1, v_2) . ■

An implication of Remark 3 is that, if the cost function c_1 is strictly convex and twice differentiable, the profit maximizing reporting accuracy $v_1^*(v_2)$ is strictly increasing in v_2 if $\alpha > \beta$, and $v_1^*(v_2)$ is constant in v_2 if $\alpha = \beta$.¹⁶

To conclude this appendix, we assume a quadratic cost function to illustrate that all the assumptions in Example 3 are consistent with each other. Suppose that $c_1(v_1) = kv_1^2/2$, $k > 0$. Then the best reply function is

$$v_1^*(v_2) = \frac{\frac{1}{2\tau} - \frac{\alpha}{2} + \frac{\alpha - \beta}{2\tau}v_2}{\left(\frac{\alpha}{\tau} + k\right)}.$$

Example 3 assumed an interior solution with $v_1 > 0$, $R_1 > 0$ and $s_1 \in (0, 1)$. To see these assumptions are consistent with each other, consider the symmetric case

¹⁶To see this, note the first order condition for an interior solution is

$$\frac{\partial \pi_1(v_1, v_2)}{\partial v_1} = 0.$$

The second order condition holds since

$$\frac{\partial^2 \pi_1(v_1, v_2)}{\partial v_1^2} = -c_1''(v_1) < 0.$$

By the implicit function rule,

$$\text{sign}\left(\frac{dv_1^*(v_2)}{dv_2}\right) = \text{sign}\left(\frac{\partial^2 \pi_1}{\partial v_1 \partial v_2}\right) = \text{sign}(\alpha - \beta).$$

where both firms are commercial media and have the same cost and advertising revenue functions. In a symmetric equilibrium,

$$v_1 = v_2 = \frac{1 - \alpha\tau}{\alpha + \beta + 2k\tau} > 0$$

iff $\alpha\tau < 1$. Moreover, for $i = 1, 2$,

$$R_i(v_i) = 1 - \frac{(\alpha + \beta)(1 - \alpha\tau)}{\alpha + \beta + 2k\tau} = \frac{\tau(\alpha^2 + \alpha\beta + 2k)}{\alpha + \beta + 2k\tau} > 0.$$

Finally, by symmetry $s_1(v_1, v_2) = s_2(v_1, v_2) = 1/2$.

E Demand functions with decreasing differences: the logit model

Suppose that

$$s_i(v_i, v_{-i}) = \frac{f_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)},$$

where the functions $f_i(v_i)$ are strictly positive and strictly increasing, and v_0 is the utility of the outside option. For $k \neq i$,

$$\begin{aligned} \frac{\partial s_i}{\partial v_k} &= \frac{-f_i(v_i) f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2}, \\ \frac{\partial^2 s_i}{\partial v_i \partial v_k} &= \frac{f'_k(v_k) f'_i(v_i) \left(f_i(v_i) - \sum_{j \neq i} f_j(v_j)\right)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^3}. \end{aligned}$$

This implies that, if there are two or more commercial outlets, s_i cannot have increasing differences for all $i \in C$, so Assumption (2) is violated.

Moreover,

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial v_i \partial v_k} &= \frac{f'_k(v_k) f'_i(v_i) \left(f_i(v_i) - \sum_{j \neq i} f_j(v_j) \right)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)^3} R_i(v_i) - \frac{f_i(v_i) f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)^2} R'_i(v_i) \\
&= \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)^2} \left(\frac{f'_i(v_i) \left(f_i(v_i) - \sum_{j \neq i} f_j(v_j) \right)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)} R_i(v_i) - f_i(v_i) R'_i(v_i) \right) \\
&> \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)^2} \left(f'_i(v_i) \frac{\left(-\sum_{j \neq i} f_j(v_j) \right)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)} R_i(v_i) - f_i(v_i) R'_i(v_i) \right) \\
&> \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j) \right)^2} \left(-f'_i(v_i) R_i(v_i) - f_i(v_i) R'_i(v_i) \right)
\end{aligned}$$

so a sufficient condition for π_i to have increasing differences in (v_i, v_{-i}) is that

$$-f_i(v_i) R'_i(v_i) \geq f'_i(v_i) R_i(v_i)$$

or equivalently

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{f'_i(v_i)}{f_i(v_i)}.$$

In the logit model, $f_i(v_i) = \exp(\mu v_i)$ and hence $f'_i(v_i)/f_i(v_i) = \mu$.

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