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Abstract

This paper studies trade in endogenously evolving markets exhibiting few traders at any given point in time. Traders arrive in the market and bargain until they complete a trade. We find that, unlike large markets, small markets feature trade delay and price dispersion, even when sellers and buyers are homogeneous and matching frictions are small. We characterize transaction prices as a function of the endogenous evolution of the market composition and economic conditions, providing several novel comparative statics results. Our analysis highlights the need to incorporate sub-market structures into the theoretical study of job, real estate, and rental markets, where trade opportunities are typically constrained by both the geographical location and individual characteristics of each trader.

Keywords: Small dynamic markets, decentralized bargaining, trade delay.
JEL Classifications: C73, C78, D53, G12.

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1 Introduction

Many dynamic markets are small: while traders come and go over time, there are few active traders at any given date. For example, there is increasing evidence that many job seekers are locked into small job markets determined by their commuting areas and specific skills.[1] Similarly, people looking to rent or buy housing units typically focus on geographically reduced areas and a narrow range of characteristics (number of rooms, garden, etc.).

Two features make small markets qualitatively different from large markets. First, because of the arrival and exit of traders, small markets are in permanent evolution. This makes their state endogenous and stochastic, both when the environment is stationary and when economic conditions (such as the state of the economy, idiosyncratic market hotness, or legislative changes) evolve dynamically. Second, each active trader in a small market has some market power at each point in time. This means her actions will affect the timing and pricing of other transactions in the market; for example, her not trading today may induce other traders to trade. She will therefore condition her decision to trade on the effect of such transactions on future market dynamics.

The current paper studies how the endogenous evolution of a small market shapes the timing and pricing of transactions. We develop a new, fully dynamic model of a small decentralized market and obtain that the trade outcome differs qualitatively from that of a large decentralized market. We show that prices are determined mainly by the endogenous evolution of the future excess demand, not by the particular bargaining protocol used. Notably, small dynamic markets are shown to exhibit trade delay and price dispersion, even when traders are homogeneous and matching frictions are small. Our analysis illustrates that modeling a market as the sum of small dynamic markets, rather than as a single large static market—which may be more appropriate in certain economic sectors—may lead to theoretical predictions with significantly different qualitative features.

We propose a general model of a small dynamic market, which can be seen as a version of the large decentralized market studied in Gale (1987). At any given moment in time, the market is composed of a finite number of sellers, each of whom owns one unit of a homogeneous indivisible good, and a finite number of homogeneous buyers with unit demand. There is also a multi-dimensional state capturing the economic conditions, with both exogenously evolving components (e.g., the economic cycle, legislation) and endogenously evolving components (e.g., market hotness, local government policies). Once in the market, each trader (buyer or seller) is repeatedly matched at random with traders from the other side of the market until she completes a trade. Within each match, one of the traders is randomly chosen to make a take-it-or-leave-it offer. The other trader either accepts the offer,

[1] See, for example, Artuç et al. (2010), Dix-Carneiro (2014), Artuç and McLaren (2015), Marinescu and Rathelot (2018), Manning and Petrongolo (2017), and Azar et al. (2018).

in which case both traders leave the market, or rejects it, in which case both traders remain in the market. We allow the arrival of traders, the matching rate, and the changes in economic conditions to depend on the market composition and economic conditions. We study Markov perfect equilibria, using the market composition and economic conditions as the state variable.

Our first result establishes that equilibrium offers are sometimes rejected: unlike in previous models (reviewed below), a seller and a buyer may fail to agree to trade, even though sellers and buyers are homogeneous and have no private information. The result relies on the observation that if a trader decides not to trade, the state of the market changes faster if she belongs to the long side of the market than if she belongs to the short side—in the first case, matches involving other traders are more likely to occur. Consequently, if traders on the long side of the market expect their position to improve after a transaction takes place, their reservation value from not trading is high. Similarly, if traders on the short side of the market expect their position to worsen after a transaction takes place, such a possibility does not lower their reservation value by much. The gains from trade may then be lower than the sum of the equilibrium reservation values, so trade delay may occur. We prove that, nonetheless, there is never a "market breakdown"; that is, there is a strictly positive probability of trade in every match.

The second result shows that when matching frictions are small, transaction prices depend only on the evolution of the market imbalance. We first show that when traders are matched very often, the price is independent of whether the offer is made by a seller or buyer. We call this state-dependent price the *market price*. We argue that the market price corresponds to the competitive price for the traders on the long side of the market, as it makes them indifferent between trading now and trading later. When the market is balanced, the market price is obtained from a Rubinstein bargaining game with randomly arriving outside options. Overall, the market price at a given date is approximated by the expected discounted length of time in the future during which the market exhibits excess demand under a modified law of motion for the state of the market. Such a modified law of motion corresponds to the evolution of the state of the market if, at any time, a trader on the long side of the market deviates to not trading. Our characterization is novel in the literature and potentially helpful in applied work, as it expresses prices in terms of the observed evolution of the market composition, and hence does not require knowledge about the bargaining protocol, the entry process, or the law of motion of the evolution of market conditions. It also establishes that since the expected future market imbalance changes over time, price dispersion remains sizable even if the matching frictions are small, despite the homogeneity of sellers and buyers.

Our simple characterization of the price dynamics permits us to obtain novel testable predictions. First, even if the market composition does not drift toward being balanced, the market price always increases in expectation when there is excess supply, and it always decreases in expectation when there is excess demand. Second, as in the standard results for large decentralized markets, trade delay disappears in the limit as the rate of arrival of traders becomes large. As the arrival rate increases, it becomes increasingly cheap for each trader to wait to trade until she is on the short side of the market, but because both buyers and sellers can do so, waiting becomes increasingly useless. In the limit, the price depends on the economic conditions and not on the current market composition; it equals the future expected discounted time with excess demand. Finally, we provide conditions that ensure that there is no equilibrium trade delay; these conditions take the form of bounds on the effect that individual transactions may have on the arrival rates. We show that, under these conditions, raising the interest rate results in a mean-preserving spread of the distribution of market prices.

We examine some extensions of our model. We argue that our characterization of price dynamics persists when arrival into the market is endogenized through an entry cost or when traders may exogenously or endogenously exit the market without trading. We also discuss ways in which the bargaining protocol can be changed so that our results continue to hold. On the other hand, we argue that allowing gains from trade to change over time could lead to market breakdown in some states, while allowing trader heterogeneity could lead to additional market inefficiencies.

The rest of our paper is organized as follows: after the literature review, Section 2 introduces our model, and Section 3 provides the equilibrium analysis. Section 4 analyzes the equilibrium outcome when the matching frictions are small. Section 5 discusses some extensions of the model and concludes. The appendix provides the proofs of the results and some additional derivations.

1.1 Literature review

Our paper contributes to the literature on small markets with arrival and exit of traders. The paper closest to ours, Taylor (1995), analyzes a market where sellers and buyers arrive over time at a fixed rate. In every period, traders on the short side of the market make price offers; when the market is balanced, the offering side is chosen at random. Taylor characterizes equilibria without delay and shows that the price is determined by the market composition when traders are impatient, and by the long-run population dynamics when traders are patient. We show these results also hold in a decentralized market when frictions are small and arrival is fixed. Endogenous arrival and evolving market conditions may nevertheless reverse these results, leading to price dispersion and trade delay. Our novel characterization of price dynamics in terms of the future discounted time with excess demand also permits us to obtain new comparative statics results regarding price dynamics and the effects of changes in the interest rate on the distribution of prices.[2]

[2]Other, less closely related papers in this literature are those of Coles and Muthoo (1998) and Loertscher et al. (2022), who consider a model similar to that of Taylor (1995) in which heterogeneous sellers and buyers arrive in pairs, and Said

Our paper is also related to the extensive literature on decentralized large markets, which is reviewed in Osborne and Rubinstein (1990) and Gale (2000).[3] The models in this literature feature a continuum of traders and non-stochastic population dynamics, which are often assumed to be in a stationary state (an exception is the model of Manea, 2017a, which represents a non-stationary market with a continuum of traders). For the frictionless market, Gale (1987) shows that the outcome is competitive without delay or price dispersion. By contrast, we study small markets in which the market composition undergoes permanent, endogenous stochastic evolution. The equilibrium interrelation between arrival and pricing is key to generating delay and price dispersion.

Our work is also related to recent work on bargaining in networks. For example, Talamàs (2019) and Elliott and Nava (2019) look at bargaining in networks with traders who trade goods and leave.[4] In these papers, either there is no arrival, or traders are immediately replaced by copies of themselves upon transaction; furthermore, traders are heterogeneous because of their different valuations and/or different positions in the network. Both Talamàs and Elliott and Nava show that the network incompleteness and trader heterogeneity generate inefficiency, price dispersion, and delay. Here, we instead study an endogenously evolving unstructured market (i.e., with a complete network) with homogeneous traders and ask how the endogenous arrival determines—and is determined by—the bargaining outcome. We show that, although our network is simple and traders are homogeneous, price dispersion and delay may still arise.

Finally, our paper is also related to the literature on dynamic thin financial markets, which feature few but large traders trading multiple shares over time. Vayanos (1999) and Rostek and Weretka (2015) study efficiency in dynamic Kyle (1989) models, where heterogeneous traders possess private information and gauge the impact of their actions on public information and prices. Our focus, on the other hand, is on understanding how arrival affects bargaining outcomes in markets where each trader trades only once, and therefore does not consider the price impact of her transactions. In this context, inefficiencies arise not because of private information but because current trades affect future arrivals.

(2011), who studies a dynamic market in which buyers compete in a sequence of private-value second-price auctions.

[3] Important contributions are those of Rubinstein and Wolinsky (1985), Gale (1987), Burdett and Coles (1997), Shimer and Smith (2000), Atakan (2006), Satterthwaite and Shneyerov (2007), Manea (2011), and Lauermann (2012). In Section 4.3 we consider the limit where traders arrive increasingly frequently, which is interpreted as the market growing by replication, and we compare our convergence results to the competitive outcome of this literature.

[4] Manea (2018) and Condorelli et al. (2016) study models where a single good is transacted along a network, and where some of the traders endogenously become intermediaries.

2 The model

In this section, we introduce a model similar to those of Rubinstein and Wolinsky (1985) and Gale (1987).

Our setting has two key distinguishing features. The first is that the market is assumed to be "small"; that is, the number of traders in the market at any given moment of time is a non-negative integer (instead of a mass) that changes stochastically over time. Second, we place few assumptions on the arrival and matching processes, allowing them to depend on the market composition and an endogenously evolving variable that encodes the economic conditions. Sections 3.3 and 5.1 discuss the motivations for our assumptions and the robustness of the results if they are changed.

State of the market. Time is continuous with an infinite horizon, $t \in \mathbb{R}_+$. There are infinitely many potential sellers and buyers. At any given moment t, there are $N_{s,t} \in \{0, ..., \bar{N}_s\}$ sellers and $N_{b,t} \in \{0, ..., \bar{N}_b\}$ buyers in the market, for some large $\bar{N}_s, \bar{N}_b \in \mathbb{N}$. There is also a variable ω_t denoting the *economic conditions*, which belongs to a finite set $\Omega \subset \mathbb{R}^n$. We refer to $(N_{s,t}, N_{b,t}, \omega_t)$ as the *state of the market* at time t, which is observed by all traders (sellers and buyers) in the market.

Arrival process. Sellers arrive in the market at a Poisson rate $\gamma_s(N_{s,t}, N_{b,t}, \omega_t) \in \mathbb{R}_+$, and buyers arrive at a Poisson rate $\gamma_b(N_{s,t}, N_{b,t}, \omega_t) \in \mathbb{R}_+$. Note that $\gamma_s(\bar{N}_s, \cdot, \cdot) \equiv \gamma_b(\cdot, \bar{N}_b, \cdot) \equiv 0$.

Bargaining. For ease of exposition, our base model uses a simplistic (yet canonical) bargaining protocol. If, at time t, there are sellers and buyers in the market (i.e., $N_{s,t}, N_{b,t} > 0$), then a match occurs at a Poisson rate $\lambda(N_{s,t}, N_{b,t}, \omega_t) > 0$. When a match occurs, nature randomly selects one of the sellers and one of the buyers in the market and also chooses which trader makes a price offer. The probability that the seller makes the offer is $\xi \in (0,1)$. The trading counter-party decides whether to accept the offer or not. If the offer is accepted, the good is transacted, and the traders leave the market, whereas if the offer is rejected, they remain in the market.

Dynamics of the economic conditions. The economic conditions ω_t change through shocks arriving at a Poisson rate $\gamma_c(N_{s,t}, N_{b,t}, \omega_t) \geq 0$. The new conditions are drawn from a distribution that depends on the time-t state of the market.[5]

Payoffs. All sellers value the good at 0, and all buyers value it at 1. Both sellers and buyers discount the future at a rate $r > 0$. If a seller and a buyer trade at time t at price p, then they obtain, respectively, $e^{-rt} p$ and $e^{-rt}(1-p)$. If they never trade, they both obtain 0. Both sellers and buyers are risk-neutral and are expected-utility maximizers.

[5] The economic conditions capture a broad set of economic variables relevant to the entry of traders into the market. Some components of ω, such as shocks to the national or world economy, may evolve independently of the rest. Other components, such as market hotness or local government policies, may be endogenous.

Strategies. To simplify the model setting, we focus directly on Markov strategies using the state of the market as the state variable. Thus, a trader's strategy maps each state (N_s, N_b, ω) with $N_s, N_b > 0$ both to a price offer distribution in $\Delta(\mathbb{R})$ and to a probability of acceptance for each possible offer received. These are interpreted to be the trader's strategy in the bargaining game if she is matched and the market state is (N_s, N_b, ω).

Equilibrium concept. We focus on *symmetric Markov perfect equilibria*, in which all traders on each side of the market use the same strategy (see Appendix A.1 for the formal definition). We will refer to symmetric Markov perfect equilibria simply as "equilibria."

3 Equilibrium analysis

3.1 Equilibrium continuation values and existence

We begin this section by presenting the equations satisfied by the continuation value of a trader in equilibrium, then state that an equilibrium exists. See Appendix A.1 for formal derivations.

Fix an equilibrium. We use $V_s(N_s, N_b, \omega)$ to denote a seller's continuation value in state (N_s, N_b, ω) with $N_s > 0$, and $V_b(N_s, N_b, \omega)$ to denote a buyer's continuation value in state (N_s, N_b, ω) with $N_b > 0$; both of these quantities are formally defined in Appendix A.1. We will sometimes refer to sellers and buyers as, respectively, s-traders and b-traders.

Fix some type $\theta \in \{s, b\}$ and some state (N_s, N_b, ω). The continuation value of a θ-trader in state (N_s, N_b, ω) is divided into three components, corresponding to the possible events that can happen next in the market, each weighted by the relative rate at which it occurs:

$$V_\theta = \overbrace{\frac{\frac{1}{N_\theta}\lambda}{\lambda+\gamma+r} V_\theta^{m}}^{\text{own match}} + \overbrace{\frac{\frac{N_\theta - 1}{N_\theta}\lambda}{\lambda+\gamma+r} V_\theta^{o}}^{\text{others' match}} + \overbrace{\frac{\gamma}{\lambda+\gamma+r} V_\theta^{e}}^{\text{exog. change}} . \tag{3.1}$$

Here and in the rest of the paper, when there is no risk of confusion, we omit the dependence of V_θ, V_θ^m, V_θ^o, V_θ^e, λ, and $\gamma \equiv \gamma_s + \gamma_b + \gamma_c$ on the state of the market. The quantities V_θ^m, V_θ^o, and V_θ^e are defined and described below.

1. **Own match:** The first possibility is that the θ-trader is matched with a trader from the other side of the market, which occurs at rate $\frac{1}{N_\theta}\lambda$. Assume first $\theta = s$ (i.e., the θ-trader is a seller). With probability ξ, she makes the offer. She can decide to offer an unacceptable price (say above 1) that the buyer rejects, in which case she obtains her continuation value of V_s. Alternatively, she can make an offer intended to be acceptable to the buyer. Since the buyer's continuation value from rejecting the offer is V_b, he accepts for sure offers strictly below $1 - V_b$ and rejects offers strictly above $1 - V_b$. The standard take-it-or-leave-it logic implies that all equilibrium

offers from the seller that are accepted with positive probability are equal to $1-V_{\mathrm{b}}$; hence the seller's continuation payoff when she makes an offer is $\max\{V_{\mathrm{s}}, 1-V_{\mathrm{b}}\}$. With complementary probability, $1-\xi$, the buyer is chosen to make the offer. In this case, the seller receives a payoff of V_{s}: in equilibrium, either the offer is not acceptable, or the offer is acceptable and she is indifferent whether to accept it or not. Hence, we have

$$V_{\mathrm{s}}^{\mathrm{m}} = \xi \max\{V_{\mathrm{s}}, 1-V_{\mathrm{b}}\} + (1-\xi)\, V_{\mathrm{s}} \,. \tag{3.2}$$

The analogous equation for a buyer is given by

$$V_{\mathrm{b}}^{\mathrm{m}} = \xi\, V_{\mathrm{b}} + (1-\xi) \max\{V_{\mathrm{b}}, 1-V_{\mathrm{s}}\} \,. \tag{3.3}$$

2. **Others' match:** The second possibility is that another θ-trader is matched, which occurs at rate $\frac{N_\theta-1}{N_\theta}\,\lambda$. The continuation value of a θ-trader if other traders match depends on the probability that equilibrium offers will be accepted. This value can be written as

$$V_\theta^{\mathrm{o}} = \alpha\, V_\theta(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \omega) + (1-\alpha)\, V_\theta \;, \tag{3.4}$$

where $\alpha \equiv \alpha(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$ is the equilibrium probability that a seller and a buyer trade when they are matched in state $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$. It is important to notice that, if the state $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$ is such that the net surplus from trade is positive (i.e., $1-V_{\mathrm{s}}-V_{\mathrm{b}} > 0$), then the equilibrium offer is accepted for sure in any match in this state (hence, $\alpha = 1$), whereas if the net surplus from trade is negative (i.e., $1-V_{\mathrm{s}}-V_{\mathrm{b}} < 0$), then the equilibrium offer is rejected for sure (hence, $\alpha = 0$).

3. **Exogenous change:** The third possibility is that the state of the market changes exogenously, which occurs at rate $\gamma \equiv \gamma_{\mathrm{s}}+\gamma_{\mathrm{b}}+\gamma_{\mathrm{c}}$.[6] Conditional on the occurrence of an exogenous change, it consists of the arrival of a seller with probability $\frac{\gamma_{\mathrm{s}}}{\gamma}$, the arrival of a buyer with probability $\frac{\gamma_{\mathrm{b}}}{\gamma}$, and a change in the economic conditions with probability $\frac{\gamma_{\mathrm{c}}}{\gamma}$. The continuation value of a θ-trader conditional on an exogenous change in the state of the market can then be written as

$$V_\theta^{\mathrm{e}} = \tfrac{\gamma_{\mathrm{s}}}{\gamma}\, V_\theta(N_{\mathrm{s}}+1, N_{\mathrm{b}}, \omega) + \tfrac{\gamma_{\mathrm{b}}}{\gamma}\, V_\theta(N_{\mathrm{s}}, N_{\mathrm{b}}+1, \omega) + \tfrac{\gamma_{\mathrm{c}}}{\gamma}\, \mathbb{E}^{\mathrm{c}}[V_\theta(N_{\mathrm{s}}, N_{\mathrm{b}}, \tilde{\omega})] \tag{3.5}$$

for both $\theta \in \{\mathrm{s}, \mathrm{b}\}$, where $\mathbb{E}^{\mathrm{c}}$ is the expectation over the new economic conditions.

An important observation is that even though our small dynamic market evolves endogenously through multiple events (arrivals, transactions, changes in market conditions), we can uniquely determine an equilibrium from the corresponding probability of agreement in each state. Indeed, α

[6] In each state, the market conditions change when a transaction occurs (the rate of this event is endogenous because it depends on the equilibrium) and when a trader arrives or there is an exogenous change (the rates of these events are exogenous, because they do not depend on the equilibrium behavior in this state).

uniquely pins down the continuation payoffs through equations (3.1)–(3.5), while the transaction price is either V_s (if the buyer makes the offer) or $1-V_\mathrm{b}$ (if the seller makes the offer). Also, as we argued before, α is part of an equilibrium if and only if $\alpha = 1$ when $V_\mathrm{s}+V_\mathrm{b} < 1$ and $\alpha = 0$ when $V_\mathrm{s}+V_\mathrm{b} > 1$. These observations can be used to apply standard fixed-point theorems to prove the existence of equilibria.

Proposition 3.1. *An equilibrium exists. The continuation values in an equilibrium are uniquely determined by the probability of agreement α, and satisfy equations* (3.1)–(3.5).

3.2 Equilibrium properties

We continue our analysis with a result that characterizes some important features of equilibrium behavior, in particular the possibility of delay. In the statement, and in the rest of the paper, we use $\bar{\theta}$ to denote the type complementary to a trader's type $\theta \in \{\mathrm{s},\mathrm{b}\}$, so $\{\theta,\bar{\theta}\}=\{\mathrm{s},\mathrm{b}\}$.

Proposition 3.2. *In any equilibrium, for any state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ with $N_\mathrm{s}, N_\mathrm{b} > 0$, the following hold:*

1. *There is a strictly positive probability of trade in every match; that is, $\alpha > 0$.*
2. *There is trade for sure when the market is balanced; that is, $\alpha = 1$ when $N_\mathrm{s} = N_\mathrm{b}$.*
3. *If $\alpha < 1$, then the traders who are on the long (short) side of the market gain (lose) from other trades; that is, for $\theta \in \{\mathrm{s},\mathrm{b}\}$ such that $N_\theta > N_{\bar{\theta}}$, we have $V^\mathrm{o}_\theta > V_\theta$ and $V^\mathrm{o}_{\bar{\theta}} < V_{\bar{\theta}}$.*

The results in Proposition 3.2 are either new or assumed in the literature on small centralized dynamic markets, because delay is ruled out by construction. Conversely, in the network literature, the heterogeneity of traders (with respect to values or position in the network) implies that some trades never happen, while there is no notion of market balancedness in general. These results will be important in our analysis as they will help determine the bargaining dynamics in the market.

The first part of Proposition 3.2 establishes that there is no state of the market where equilibrium offers are rejected for sure. Hence, even though equilibrium offers may be rejected with strictly positive probability, there is never a "market breakdown." To see why, let $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ be a state with maximal V across all states, and assume for the sake of contradiction that $\alpha = 0$. We then have that

$$V = \tfrac{\gamma}{\gamma+r} V^\mathrm{e} \leq \tfrac{\gamma}{\gamma+r} V < V \ ,$$

which is a contradiction. Intuitively, under no trade, the joint present value from agreeing in the future is lower than V because of discounting; hence there should be trade in the state with the highest V.

The second part of Proposition 3.2 establishes that when there is a match and the market is balanced (i.e., $N_\mathrm{s} = N_\mathrm{b}$), there is trade with probability one. Indeed, because the joint surplus of the seller and the buyer is never larger than 1 (by part 1 of the proposition), we have

$$V = \frac{\frac{1}{N_s}\lambda}{\lambda+\gamma+r} \underbrace{V^{m}}_{=1} + \frac{\frac{N_s-1}{N_s}\lambda}{\lambda+\gamma+r} \underbrace{V^{o}}_{\leq 1} + \frac{\gamma}{\lambda+\gamma+r} \underbrace{V^{e}}_{\leq 1} \leq \frac{\lambda+\gamma}{\lambda+\gamma+r} < 1 \ .$$

As we can see, the joint surplus of the seller and the buyer from not agreeing is strictly lower than 1, because they discount the time when the next event occurs. Intuitively, when the market is balanced, the seller and the buyer "agree" on the relative likelihoods of the three types of events that could change the state (their own matching, others' matching, and exogenous changes).

The last part of Proposition 3.2 establishes that if equilibrium offers are rejected with positive probability in some state (N_s, N_b, ω), then a trader on the long side of the market must benefit from other traders' transactions in such a state. To see this, consider the case where sellers are on the long side of the market (i.e., $N_s > N_b$). The rate at which there is a match involving other traders is, from a seller's perspective, $\frac{N_s-1}{N_s}\lambda$, while this rate is lower from a buyer's perspective, $\frac{N_b-1}{N_b}\lambda$. Thus, the weight of the event that other traders match is larger in determining the sellers' continuation value than in determining the buyers'. The joint continuation value of a seller and a buyer can then be written as

$$V = \frac{\lambda}{\lambda+\gamma+r}\Big(\overbrace{\tfrac{1}{N_s} V_s^{m} + \tfrac{N_s-1}{N_s} V_s^{o} + \tfrac{1}{N_b} V_b^{m} + \tfrac{N_b-1}{N_b} V_b^{o}}^{(*)}\Big) + \frac{\gamma}{\lambda+\gamma+r} V^{e} \ . \tag{3.6}$$

If offers are rejected with positive probability, $V = V^{m} = 1$. By the first part of Proposition 3.2, V^{o} and V^{e} are weakly lower than 1. The greater weight that a seller assigns to the event that two other traders match makes the term $(*)$ in the previous expression strictly greater than 1 (which is necessary for V to be equal to 1) only if $V_s^{o} > V_s$ and $V_b^{o} < V_b$. In fact, this term can be written as

$$(*) = \underbrace{\frac{N_s-N_b}{N_s N_b}\left(V_s^{o} - V_s\right)}_{=(**)} + \underbrace{\frac{1}{N_b} V + \frac{N_b-1}{N_b} V^{o}}_{\leq 1} = \underbrace{\frac{N_s-N_b}{N_s N_b}\left(V_b - V_b^{o}\right)}_{=(***)} + \underbrace{\frac{1}{N_s} V + \frac{N_s-1}{N_s} V^{o}}_{\leq 1} \ .$$

It is then clear that $(*) > 1$ only if $(**) > 0$ and $(***) > 0$,

Trade delay

Delay in our model arises because of a novel effect: the event that other traders match (and trade) occurs more often from the perspective of a trader on the long side of the market than from the perspective of a trader on the short side. If a trader waits and lets others trade, this affects the endogenous arrival and market dynamics, but the effect depends on which side of the market the trader is on (see Appendix B for an illustrative example where all equilibria exhibit trade delay). Hence, even though the traders have homogeneous beliefs about the future evolution of the market, the evolution of the market conditional on a given trader's deviating (and not trading)—which provides the trader's outside option when bargaining—depends on the side of the market to which she belongs.

Inefficient delay is often found in bargaining models with private information and adverse selection. Examples of delay in large markets with adverse selection are presented in Moreno and Wooders (2002) and Camargo and Lester (2014).[7] In small markets, trade delay may also occur for other reasons; here we mention three that are closely related to our results. Jehiel and Moldovanu (1995) analyze a bargaining setting with one seller and many asymmetric buyers, where buyers' transactions impose externalities on other buyers. They find that if the externalities are negative, there are equilibria with "cyclical delay." In our model, delaying trade imposes an endogenous externality on other traders by not changing the state of the market, hence affecting its future evolution. Such an externality is positive for traders on the long side of the market and negative for traders on the short side. Yildiz (2004) shows that trade delay can occur when "optimistic" players with heterogeneous beliefs bargain. Yildiz's logic is similar to ours because, in our model, a trader's anticipated dynamics upon deviating not to trade depend on whether she is on the long side of the market or not. In our model, nevertheless, players are rational and have correct (and endogenous) beliefs about the market evolution. In Elliott and Nava (2019), bargaining delay occurs when traders expect their bargaining position to improve because of their bargaining counterpart' disappearing trading options (exit of possible trade partners). Such an effect is not present in a complete network with homogeneous traders, in which we show that delay only arises when arrival is endogenous (see Proposition 4.2 below).

3.3 Discussion of assumptions

Information: We assume that traders observe the state of the market, which is a common assumption in the bargaining literature. Given our focus on Markov perfect equilibria, all the information a player has other than the state of the market is irrelevant (for concreteness, we can assume that the game is of perfect information). In practice, knowledge about active markets may come from public listings (e.g., apps, websites), word of mouth, applications for interviews, etc. Relaxing the assumption would lead to price discovery effects, where different traders would have different beliefs about the state of the market (see Lauermann et al., 2017, for a treatment of this situation in large decentralized markets).

Homogeneity: We study the benchmark case of homogeneous sellers and buyers, which facilitates comparison with models of large markets (e.g., Gale, 1987). Homogeneity makes for tractability and

[7]In bilateral trade models with private information, the Coase (1972) conjecture asserts that trade occurs with no delay (see Gul et al., 1986). Nevertheless, a number of papers have identified reasons that the Coase conjecture fails in certain settings, such as endogenous timing (Admati and Perry, 1987), adverse selection (Deneckere and Liang, 2006), capacity choice (McAfee and Wiseman, 2008), arrival of traders (Fuchs and Skrzypacz, 2010), deadlines (Fuchs and Skrzypacz, 2013), and stochastic costs (Ortner, 2017).

is a reasonable assumption for some markets. Intuitively, some small markets are defined by the narrow set of characteristics that their participants share. For example, to a firm offering a technical apprenticeship, all newly graduated candidates with a given degree may be indistinguishable; to a family choosing accommodations for a short vacation, all two-bedroom apartments in a given location may be equivalent.

Allowing sellers and buyers to have heterogeneous qualities or valuations for goods would make the analysis less tractable. It would not only increase the dimensions of the state and strategy spaces, but also make the outcome of bargaining stochastic. Indeed, Abreu and Manea (2012a,b) and Elliott and Nava (2019) show that, in a bargaining model in networks, sometimes transactions with low gains from trade are realized in the presence of more beneficial trade opportunities, even in the limit where matching frictions vanish.

Stationarity of gains from trade: The gains from trade in our model are constant and normalized to 1. This assumption allows us to focus on how the arrival process affects outcomes in a small dynamic market. Still, most of our analysis can be straightforwardly generalized to the case where the gains from trade depend on the economic conditions and do not increase too quickly in expectation.[8] For example, we could assume that in each state (N_s, N_b, ω), the gains from trade are equal to $g(\omega)$ instead of 1, where $g : \Omega \to \mathbb{R}_+$ satisfies

$$g(\omega) > \tfrac{\gamma_c}{\gamma_c + r} \max_{\omega' \in \Omega^c(\omega)} g(\omega') \tag{3.7}$$

for all states (N_s, N_b, ω) (note that the equation holds if $g(\cdot) \equiv 1$), with $\Omega^c \equiv \Omega^c(N_s, N_b, \omega)$ denoting the support of the new market conditions when an exogenous change of the economic conditions occurs. If (3.7) holds, then Proposition 3.2 holds as well, and the results below can be generalized. If (3.7) does not hold, then there may be states exhibiting market breakdown, where players do not trade and wait for the surplus to increase.

Endogenous arrival: We consider a general arrival process, where arrival rates depend arbitrarily on the state of the market. Hence, our analysis accommodates a broad range of arrival processes and provides predictions independent of the arrival process (see Section 5.1 for a discussion on costly entry). Allowing such generality in the study of small markets is important, as arrival processes may differ across markets and may be difficult to discern in practice.

Relatedly, we assume that the number of traders in the market is bounded. This technical assumption, which requires that the arrival rates depend on the state of the market, simplifies the intuition

[8] The trade surplus may depend on economic conditions affecting taxes or determining the future profitability of the match. For example, in response to a shortage of specialized labor, a local government may offer free housing to newly-hired workers; such a subsidy may increase the joint gains from trade for an employer and employee.

and the proofs; however, it can easily be dropped.[9] In a model without bounds on the number of traders, an arrival process independent of the composition of the market leads to a market with a growing number of traders; hence the market is not small in the long run.[10]

Endogenous matching rates: Most of the literature on large decentralized markets has focused, mostly for analytical convenience, on quadratic matching technologies, where $\lambda(N_s, N_b, \omega) = \overline{\lambda} N_s N_b$ for some $\overline{\lambda} \in \mathbb{R}_{++}$. We do not make assumptions on the functional form of the matching rate, but we will mostly focus on the limit of vanishing matching frictions. In Section 5.1 we discuss the interpretation of matching frictions as attention frictions.

4 Small matching frictions

We now turn to the case where matching frictions are small, that is, where traders in the market are matched frequently. This may be a plausible assumption in some small dynamic markets, such as localized housing markets or job markets for specific occupations, where the rate at which traders (can) meet once they are in the market is much higher than the rate of arrivals in the market. As in the large markets literature, studying the case where frictions are small allows us to provide a sharper characterization of the equilibrium outcome.

4.1 Notation and a preliminary result

To analyze the case where matching frictions are small, we separate each state's matching rate $\lambda = \lambda(N_s, N_b, \omega)$ into two parts. The first is a state-independent common factor $k > 0$. The second is a function $\ell(N_s, N_b, \omega)$, which measures the relative rate at which traders match in each state. Thus, from now on, we use the expressions $\lambda(N_s, N_b, \omega)$ and $k\,\ell(N_s, N_b, \omega)$ interchangeably.

The difficulty in characterizing how equilibrium outcomes change "when matching frictions are small" is that our model may have multiple equilibria. We therefore introduce the following notation to obtain the properties of equilibrium outcomes as k increases. The notation "$\simeq$" indicates that the difference between the terms on the two sides tends to 0 in any equilibrium as k goes to $+\infty$ under any equilibrium sequence (see footnote 11). The following result establishes that when the matching

[9] See the proof of Proposition 3.1 for a demonstration of the existence of an equilibrium for $\bar{N}_b = \bar{N}_s = +\infty$. The results in Proposition 3.2 extend to the situation where γ is bounded.

[10] Indeed, since sellers and buyers leave the market in pairs, $N_t \equiv N_{s,t} - N_{b,t}$ evolves independently of the equilibrium behavior when γ is independent of $(N_{s,t}, N_{b,t})$ (which requires $\bar{N}_s = \bar{N}_b = +\infty$). The variance of $N_{t+\Delta} - N_t$ is then independent of N_t, implying that, from the time-t perspective, the variance of $N_{\hat{t}}$ grows to infinity as $\hat{t} \to \infty$. If instead, for example, buyers are more likely to arrive when they are on the short side of the market than when they are on the long side, the market remains small in the long run (see Section IV in Taylor, 1995, for a simple specification).

frictions are small, the joint continuation value of a seller and a buyer is close to the joint surplus they obtain from trade.

Lemma 4.1. *$V \simeq 1$ for all states $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ with $N_\mathrm{s}, N_\mathrm{b} > 0$.*[11]

To obtain intuition for Lemma 4.1, assume that matches occur very frequently; that is, λ (or k) is large. Note that, for a fixed equilibrium, there are three kinds of states. The first kind comprises all states where $N_\mathrm{s}, N_\mathrm{b} > 0$ and equilibrium offers are rejected with positive probability. For these states, $V = 1$. The second kind comprises all states where either $N_\mathrm{s} = 1$ or $N_\mathrm{b} = 1$ (or both). For example, if there is only one buyer, he can wait almost costlessly until he is able to make an offer and then obtain $1-V_\mathrm{s} \geq V_\mathrm{b}$; hence $V_\mathrm{s}+V_\mathrm{b}$ is close to 1 for these states. Finally, the third kind of state comprises those where $N_\mathrm{s}, N_\mathrm{b} > 1$ and there is immediate trade. In a state in this set, a θ-trader has the option to wait for a transaction between two other traders to take place (which occurs soon, because there is immediate trade and matches are frequent), so $V_\theta(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)$ is an approximate lower bound on V_θ. This implies that $V(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)$ is an approximate lower bound on V, which is itself bounded above by 1. If $(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)$ belongs to the first or second kind of state, we have that $V(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)$ is approximately equal to 1. We can use the same argument again if it belongs to the third kind. It is clear that after at most $\min\{N_\mathrm{s}, N_\mathrm{b}\}-1$ times, a state of one of the first two kinds is reached. Because the expected time for this to happen shrinks to 0 as $k \to \infty$, we have that V is approximately equal to 1 for states of the third kind as well.

An immediate and important consequence of Lemma 4.1 is that, when the matching frictions are small, a seller is approximately indifferent between trading and not trading when $N_\mathrm{s} > N_\mathrm{b} \geq 1$. This is obviously true if $\alpha < 1$ (i.e., in the first kind of state defined above). When, instead, $\alpha = 1$, the payoff of a seller is

$$V_\mathrm{s} \simeq \tfrac{1}{N_\mathrm{s}} V_\mathrm{s} + \tfrac{N_\mathrm{s}-1}{N_\mathrm{s}} V_\mathrm{s}(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)\ . \tag{4.1}$$

Thus, from the previous equation, it follows that $V_\mathrm{s} \simeq V_\mathrm{s}(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)$; that is, the seller obtains approximately the same payoff if she follows the equilibrium strategy and if she decides not to trade in state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$.

Another implication of Lemma 4.1 is that within-state price dispersion vanishes when the matching frictions are small. Indeed, the transaction price in state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ is either V_s (if the buyer makes the offer) or $1-V_\mathrm{b}$ (if the seller makes the offer). Because $V \simeq 1$, we have that $V_\mathrm{s} \simeq 1-V_\mathrm{b}$. Therefore, we call $V_\mathrm{s} = V_\mathrm{s}(N_\mathrm{s}, N_\mathrm{b}, \omega)$ the *market price* (in state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$); this is the approximate price at which

[11] The notation "$V \simeq 1$" should be read as follows: "For any sequence $(k_n)_n$ tending to $+\infty$ and for any corresponding sequence of equilibria, we have that $\limsup_{n\to\infty} |V_n - 1| = 0$." That is, for any $\varepsilon > 0$, $|V_n - 1| < \varepsilon$ in any equilibrium for n big enough (note that V_n can be different in different equilibria).

transactions take place when the state is $(N_{\rm s}, N_{\rm b}, \omega)$. The following section shows that there is market price dispersion across states.

4.2 Characterization of the market price

We now provide a characterization of the market price. We do so by modifying the probability measure that determines the evolution of the state of the market.

Each equilibrium induces a probability measure determining the evolution of the state of the market. Under such a measure, transactions occur at rate $\alpha(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)\,\lambda(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)$, and exogenous changes in the state of the market occur at rate $\gamma(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)$. While the equilibrium measure characterizes the market dynamics, it cannot be directly used to obtain each trader's outside option when bargaining. The reason is that to obtain a trader's outside option when bargaining, we have to compute her value from deviating to not trading, which changes the market evolution.

As we saw earlier, traders on the long side of the market are indifferent between trading and not trading. To compute the value from not trading, we introduce a new measure, under which exogenous changes in the state of the market occur in exactly the same way as under the equilibrium measure, but transactions occur at rate

$$\begin{cases} \frac{N_{{\rm b},t}-1}{N_{{\rm b},t}}\,\alpha(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)\,\lambda(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t) & \text{if } N_{{\rm s},t} \leq N_{{\rm b},t}, \\ \frac{N_{{\rm s},t}-1}{N_{{\rm s},t}}\,\alpha(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)\,\lambda(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t) & \text{if } N_{{\rm s},t} > N_{{\rm b},t}, \end{cases} \tag{4.2}$$

instead of at rate $\alpha(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)\,\lambda(N_{{\rm s},t}, N_{{\rm b},t}, \omega_t)$. If sellers are on the long side of the market (i.e., $N_{{\rm s},t} > N_{{\rm b},t}$) and a seller decides to deviate and refrain from trading until the market is balanced, the market evolves according to the modified measure. Note that the dynamics of the state of the market under the modified measure are determined from the equilibrium dynamics of the state of the market (and can be uniquely pinned down by an external observer who observes only the evolution of the state).

Our technique of changing the measure to compute equilibrium prices resembles the use of risk-neutral measures in studying financial markets. The risk-neutral measure makes an asset's current price equal to the value that a competitive, risk-neutral seller derives by trading later instead of now. The current value of a financial asset in a complete market is then equal to its payoffs in the future, discounted at the risk-free rate, averaged using the risk-neutral measure. In a slight abuse of language, we call our modified measure the *risk-neutral measure* (of the fixed equilibrium). Because the competitive side of the market (i.e., the one with more traders) stochastically changes over time in our model, our risk-neutral measure makes a trader on the long side of the market indifferent between trading now and trading later. The following proposition shows that our risk-neutral measure can be used to characterize the market price.

Proposition 4.1. *For any state* $(N_{s,0}, N_{b,0}, \omega_0)$*, we have that*

$$V_s(N_{s,0}, N_{b,0}, \omega_0) \simeq \tilde{\mathbb{E}}\left[\int_0^{\infty} e^{-r\,t} \left(\mathbb{I}_{N_{s,t}<N_{b,t}} + \xi\, \mathbb{I}_{N_{s,t}=N_{b,t}} \right) r\, \mathrm{d}t \right], \tag{4.3}$$

where $\tilde{\mathbb{E}}$ *is the expectation using the risk-neutral measure.*

Proposition 4.1 gives an approximation of the transaction (and market) price in state $(N_{s,0}, N_{b,0}, \omega_0)$ in terms of the equilibrium dynamics of the state of the market and the probability that a seller makes an offer. It is a discounted average (under the risk-neutral measure) of the amount of future time for which the market exhibits excess demand, adjusted by the amount of time for which it is balanced.

Our characterization of prices in terms of the future evolution of the market composition is novel in the literature. Since the price only depends on the bargaining protocol in proportion to the discounted amount of future time for which the market is balanced, Proposition 4.1 can be used in applied work without much knowledge of the primitives of the model. From observing the evolution of the market composition, one can test whether prices were accurate predictors of the future market imbalance. As we will see, the simplicity of equation (4.3) will permit us to obtain new comparative statics results.

To obtain some intuition for Proposition 4.1, fix a state (N_s, N_b, ω) where the market is imbalanced. If there are more sellers than buyers (i.e., if $N_s > N_b$), sellers are approximately indifferent about whether to trade or not, and this implies that

$$V_s \simeq \frac{\frac{N_s-1}{N_s}\alpha\lambda}{\frac{N_s-1}{N_s}\alpha\lambda+\gamma+r} V_s(N_s-1, N_b-1, \omega) + \frac{\gamma}{\frac{N_s-1}{N_s}\alpha\lambda+\gamma+r} V_s^e\,. \tag{4.4}$$

A similar equation can be obtained when there are more buyers than sellers in the market (replacing s by b and N_s by N_b). Using Lemma 4.1 we can write, when $N_s < N_b$,

$$\overbrace{1-V_b}^{\simeq V_s} \simeq \frac{r}{\frac{N_b-1}{N_b}\alpha\lambda+\gamma+r} + \frac{\frac{N_b-1}{N_b}\alpha\lambda}{\frac{N_b-1}{N_b}\alpha\lambda+\gamma+r} \overbrace{\left(1-V_b(N_s-1, N_b-1, \omega)\right)}^{\simeq V_s(N_s-1,N_b-1,\omega)} + \frac{\gamma}{\frac{N_b-1}{N_b}\alpha\lambda+\gamma+r} \overbrace{\left(1-V_b^e\right)}^{\simeq V_s^e}\,. \tag{4.5}$$

When the market is imbalanced, the outcome resembles Bertrand competition between agents on the long side of the market (e.g., Taylor, 1995). Indeed, in a match, the payoff a trader on the long side of the market obtains from trading is very close to her continuation value if she does not trade and instead waits until the state of the market changes. Importantly, in a small dynamic market, the continuation value is endogenous and is driven by the expectation of future trade opportunities.

Consider now a state (N_s, N_b, ω) where the market is balanced (i.e., $N_s = N_b$). Proposition 3.2 establishes that there is trade in every match. Consequently, when $N_s > 1$, equation (4.1) holds, so $V_s \simeq V_s(N_s-1, N_b-1, \omega)$. Each seller is approximately indifferent between trading and letting other traders trade until she is alone in the market with a single buyer. Consider then the case where only one seller and one buyer are in the market (i.e., $N_s = N_b = 1$). The disagreement value of the seller (i.e., her value from deciding not to trade) is $\frac{\gamma}{\gamma+r} V_{\mathrm{s}}^{\mathrm{e}}$. Similarly, the reservation value of the buyer is $\frac{\gamma}{\gamma+r} V_{\mathrm{b}}^{\mathrm{e}}$. As the frequency with which offers are made increases, the transaction price is determined by the limit outcome of a two-player bargaining game à la Rubinstein (1982). The "size of the pie" over which the traders bargain is not 1, but rather the trade surplus net of the sum of the outside options; that is,

$$1-\frac{\gamma}{\gamma+r}\left(V_{\mathrm{s}}^{\mathrm{e}}+V_{\mathrm{b}}^{\mathrm{e}}\right) \simeq \frac{r}{\gamma+r} \ .$$

The seller obtains her disagreement value, $\frac{r}{\gamma+r} V_{\mathrm{s}}^{\mathrm{e}}$, plus a fraction of the pie equal to the probability with which she makes offers, $\xi \frac{r}{\gamma+r}$; so[12]

$$V_{\mathrm{s}} \simeq \frac{r}{\gamma+r}\, \xi+\frac{\gamma}{\gamma+r} V_{\mathrm{s}}^{\mathrm{e}} \ . \tag{4.6}$$

Equations (4.4)–(4.6) indicate that under the risk-neutral measure, V_{s} approximately satisfies the equations satisfied by the continuation payoff of a fictitious agent who receives a flow payoff equal to 0 when there is excess supply (equation (4.4)), a flow payoff equal to 1 when there is excess demand (equation (4.5)), and a flow payoff equal to ξ when the market is balanced (equation (4.6)). The right-hand side of equation (4.3) gives an expression for this value.

Changes in continuation values

As we argued before, when the matching frictions are small and $N_{\mathrm{s},0} \geq N_{\mathrm{b},0}$, a close-to-optimal strategy for a seller at time 0 is to refrain from trading until she is the only seller in the market. The same applies to a buyer if, instead, $N_{\mathrm{s},0}<N_{\mathrm{b},0}$. It then follows that

$$V_{\mathrm{s}}(N_{\mathrm{s},0}, N_{\mathrm{b},0}, \omega_0) \simeq \begin{cases} \tilde{\mathbb{E}}[e^{-r\,\tau_0} V_{\mathrm{s}}(1,1,\omega_{\tau_0})] & \text{if } N_{\mathrm{s},0} \geq N_{\mathrm{b},0} \ , \\ 1-\tilde{\mathbb{E}}[e^{-r\,\tau_0}(1-V_{\mathrm{s}}(1,1,\omega_{\tau_0}))] & \text{if } N_{\mathrm{s},0}<N_{\mathrm{b},0} \ , \end{cases} \tag{4.7}$$

where τ_0 is the first time at which the market contains only one seller and one buyer. This observation can be used to prove the following result.

Corollary 4.1. *Fix an imbalanced state $(N_{\mathrm{s},0}, N_{\mathrm{b},0}, \omega_0)$ where θ-traders are on the long side of the market. If $V_{\theta,0}>0$ and $t>0$ is small, then $\mathbb{E}[V_{\theta,t}] \succeq \tilde{\mathbb{E}}[V_{\theta,t}] \succ V_{\theta,0}$, where "$\succeq$" means that the inferior limit of the left-hand side minus the right-hand side is non-negative under any equilibrium sequence as $k \to \infty$.*

An implication of Corollary 4.1 is that when the market is imbalanced, the market price increases in expectation when there is excess supply and decreases when there is excess demand. Remarkably, this result is independent of whether the state of the market tends toward being balanced or not.

[12]In the proof of Proposition 4.1, we obtain equation (4.6) by first computing $V_{\mathrm{s}}(1,1,\omega)$ and $V_{\mathrm{b}}(1,1,\omega)$ from equation (3.1) and then taking the limit as $\lambda \to +\infty$ (see equations (A.5) and (A.6)).

To obtain some intuition for Corollary 4.1, assume that there is excess supply at time t. Because sellers are approximately indifferent between trading and not trading, the expected price must increase over time to compensate for the cost of waiting to trade. The rate at which transactions happen in this state under the equilibrium strategy is equal to $\alpha\lambda$, which is higher than the transaction rate under the risk-neutral measure, $\frac{N_{s,t}-1}{N_{s,t}}\alpha\lambda$. Hence, given that a seller's continuation payoff increases when other traders trade (see Proposition 3.2), the expected increase in the sellers' continuation payoff is larger under the equilibrium measure than under the risk-neutral measure.

4.3 No delay

We now study the case where trade delay vanishes when the matching frictions become small. To this end, we first present a condition on the primitives, which will turn out to be sufficient for our analysis.

Condition 1. $\frac{\gamma_\theta(N_s-1,N_b-1,\omega)}{\gamma(N_s-1,N_b-1,\omega)+r} \le \frac{\gamma_\theta}{\gamma+r} + \frac{r}{\gamma+r}\frac{1}{3}$ for all $\theta \in \{s,b,c\}$ and (N_s, N_b, ω) with $N_s, N_b > 1$.

Condition 1 requires that transactions have little effect on the arrival of traders and on economic conditions, limiting the possibility that traders on the long side of the market benefit significantly from the transactions of other traders.

Proposition 4.2. *Assume Condition 1 holds. Then trade delay vanishes as matching frictions disappear; that is,* $\frac{\alpha\lambda}{\alpha\lambda+r} \simeq 1$ *for any state* (N_s, N_b, ω) *with* $N_s, N_b > 0$. *Furthermore, there exists a function* $p : \{-\bar{N}_b, \ldots, \bar{N}_s\} \times \Omega \to [0,1]$ *such that* $V_s(N_s, N_b, \omega) \simeq p(N_s - N_b, \omega)$ *for all states* (N_s, N_b, ω).

Proposition 4.2 establishes that Condition 1 is sufficient to ensure that trade delay disappears when the matching frictions are small. Intuitively, by Proposition 3.2, trade delay occurs in a given state only if the gain that traders on the long side of the market obtain from other traders' transactions is big enough to compensate for discounting. The proof of Proposition 4.2 shows that, because (by Condition 1) the arrival of buyers cannot increase much after a transaction, this gain is small compared to the cost of waiting, so there is no equilibrium with trade delay.

Note that Condition 1 holds trivially in the large markets studied by Rubinstein and Wolinsky (1985) and Gale (1987), which exhibit no trade delay. Indeed, the equilibrium rate of arrival of traders—which, in their models, is a discrete-time flow—is independent of the state of the market. Hence, in these models, delaying trade does not change the joint continuation value of the traders. This argument cannot be applied to a small dynamic market when Condition 1 does not hold: because transactions affect the state of the market, traders may have the incentive to let others trade, delaying their own transactions until their bargaining power is higher.

Under Condition 1, the time it takes for the short side of the market to clear vanishes as the matching frictions disappear, so, for most of the time, one side of the market is empty. That is, when

k is large, it is likely that either $N_{s,t}=0$ or $N_{b,t}=0$ at any given time $t>0$. The limit dynamics of the state of the market (under either the equilibrium or the risk-neutral measure) can then be described by the evolution of the net supply, $N_t \equiv N_{s,t} - N_{b,t}$, and the economic conditions, ω_t.

The limit equilibrium and risk-neutral measures of the net supply coincide when the market is imbalanced. Hence, for example, equation (4.7) also holds for the equilibrium measure. If $N_t > 0$, the net supply N_t increases by 1 at rate $\gamma_s(N_t, 0, \omega_t)$ and decreases by 1 at rate $\gamma_b(N_t, 0, \omega_t)$, while the economic conditions ω_t change at rate $\gamma_c(N_t, 0, \omega_t)$. When the market is balanced, the limit measures may differ: under the equilibrium measure, a seller and a buyer immediately agree, and the market state changes at rate $\gamma(0, 0, \omega_t)$, whereas, under the risk-neutral measure, the seller and buyer remain in the market, and so the market state changes at rate $\gamma(1, 1, \omega_t)$.

Changes in r

We now consider the effect that changing the discount rate has on the distribution of market prices.

In this section, we focus on a *stationary market*, that is, the case where the economic conditions do not change over time (i.e., $\Omega = \{\omega\}$). We also require the following condition on the arrival rates.

Condition 2. For any state (N_s, N_b, ω) we have $\gamma_s > 0$ if $N_s \leq N_b$ and $\gamma_b > 0$ if $N_s \geq N_b$.

Condition 2 is mild, because it requires only that sellers arrive with positive probability when buyers are on the long side of the market, and vice versa. It is not difficult to see that if the market is stationary and Conditions 1 and 2 hold, then each equilibrium generates an ergodic distribution of the state of the market; that is, there is a unique probability measure F on $\mathbb{Z}$ such that, for each value of the net supply N, $\lim_{t\to\infty} \Pr(N_t = N) \simeq F(\{N\})$ independently of the initial state. Because by Proposition 4.2 there is no trade delay when k is large under Condition 1, this distribution of states does not depend on the discount rate when the matching frictions are small. Nevertheless, the corresponding ergodic distribution of market prices does.

Proposition 4.3. *If Condition 1 holds and the market is stationary, then p defined in Proposition 4.2 is decreasing in N. If Condition 2 also holds, then increases in r generate spreads of the ergodic distribution of market prices.*[13] *If, additionally, $\gamma_\theta(0, 0, \omega) = \gamma_\theta(1, 1, \omega)$ for both $\theta \in \{s,b\}$, then such spreads are mean-preserving.*

Proposition 4.3 establishes three important features of the market price. The first feature is intuitive, and appears in different forms in different settings (e.g., Gale, 1987, and Taylor, 1995): the

[13] Recall that a cumulative distribution function (CDF) F_1 on $\mathbb{R}$ is said to be a *spread* of another CDF F_2 if there exists a value $\bar{x} \in \mathbb{R}$ such that $F_1(x) \geq F_2(x)$ for all $x < \bar{x}$ and $F_1(x) \leq F_2(x)$ for all $x > \bar{x}$.

market price depends negatively on the net market supply. Our result is general in that it is independent of the process governing the evolution of the market composition. It follows from the competition between agents on the long side of the market (recall equation (4.7)) and the fact that the time it takes for the market to become balanced is increasing in the degree of imbalance of the market when the market is stationary, as well as the fact that there is no delay.

The second feature of the market price is new: increasing r raises the ergodic market price dispersion. This follows again from (4.7), and because an increase in the discount rate r lowers the expected discount factor of the time it takes for the market to become balanced from a given net supply. Hence, the market price for a given N tends to become more extreme when r increases.[14] For instance, in the limit where $r \to \infty$, $p(N,\omega) \to 0$ for all $N > 0$, and $p(N,\omega) \to 1$ for all $N < 0$.

The third feature of the market price is also new: the spread generated by an increase in r is mean-preserving when $\gamma_\theta(0,0,\omega) = \gamma_\theta(1,1,\omega)$ for both $\theta \in \{\mathrm{s},\mathrm{b}\}$. The reason is that, because trade delay vanishes when the matching frictions disappear, the limit equilibrium and risk-neutral measures coincide under the additional assumption of Condition 2, and the ergodic probability that the market has excess demand is independent of r. Also, from equation (4.3), the ergodic mean of the market price can be approximated by the ergodic probability with which the market exhibits excess demand plus the ergodic probability that the market is balanced multiplied by ξ. Hence it is independent of r. In summary, although changes in r change the ergodic distribution of market prices, they do not change its mean.[15]

4.4 Large arrival rates

In this section we analyze the trade outcome when the rate of arrival of traders into the market is large. Our analysis sheds light on the role of the friction that remains in the market when the matching frequency is high, that is, the time that a trader must wait to trade when she is on the long side of the market. We then answer, from a small-market perspective, one of the salient questions in the literature on decentralized bargaining in large markets: that of whether lowering frictions leads to a competitive outcome. As in the previous models of large markets, our answer sheds light on whether and how frictions may be magnified or mitigated by the traders' equilibrium behavior in the

[14]The negative direct effect that an increase in r has on $p(N,\omega)$ when $N > 0$ may sometimes be compensated for by an increase in $p(0,\omega)$ (recall that $p(N_0,\omega_0) \simeq \tilde{\mathbb{E}}[e^{-r\,\tau_0}\,p(0,\omega)]$ from equation (4.7)). Nevertheless, as the proof of Proposition 4.3 shows, the direct effect dominates if N is large enough.

[15]More generally, if the arrival rates in states $(0,0,\omega)$ and $(1,1,\omega)$ are close (but not equal), or if the ergodic likelihood that the market is balanced is low, Proposition 4.3 establishes that increases in the interest rate will increase the spread of prices while keeping their mean approximately unchanged.

market.[16]

In practice, increases in the arrival rate occur when similar markets are unified into bigger ones. Such market unification may arise, for example, from the launch of websites providing information on job offerings, rental prices, or housing prices in nearby locations, which make it easier for sellers and buyers to be active in different small markets. Market unification may also result from improvements in transportation infrastructure, such as new metro stations or roads, that reduce commute time.

We begin with some notation. Fix two functions, $\tilde{\gamma}_{\mathrm{s}}$ and $\tilde{\gamma}_{\mathrm{b}}$, mapping states to non-negative numbers, and two non-negative sequences, $(k_n)_n$ and $(M_n)_n$, tending to $+\infty$. For each n, we consider the model with arrival rates $\gamma_{\mathrm{s}} = M_n \, \tilde{\gamma}_{\mathrm{s}}$, $\gamma_{\mathrm{b}} = M_n \, \tilde{\gamma}_{\mathrm{b}}$ and, as before, $\lambda = k_n \, \ell$. In this section, the notation "$\simeq$" indicates that, for any sequences $(k_n)_n$ and $(M_n)_n$ tending to $+\infty$ and any sequence of corresponding equilibria, the difference between the terms on the two sides tends to zero (see footnote 11). The following result characterizes the limit trade outcome as the market grows and matching frictions disappear.

Proposition 4.4. *For all states $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$ with $N_{\mathrm{s}}, N_{\mathrm{b}} > 0$ we have $\frac{\alpha \lambda}{\alpha \lambda + r} \simeq 1$. If Conditions 1 and 2 hold, then for each $\omega \in \Omega$ there is some $p^*(\omega) \in (0,1)$ such that $V_{\mathrm{s}}(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega) \simeq p^*(\omega)$ for all $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$.*

Proposition 4.4 provides two results regarding the limit trade outcome. The first establishes that the discount factor until a transaction happens, which is equal to $\frac{\alpha \lambda}{\alpha \lambda + r}$, tends to 1; that is, trade delay disappears. The second states that price dispersion is only a function of the market conditions; that is, for each ω the market price tends to a "competitive price" $p^*(\omega)$ independent of the market composition. Hence, like large decentralized markets, small dynamic markets with high entry of traders and low matching frictions feature both low trade delay and a single price, even if the number of traders at each point in time is small.

Intuition for Proposition 4.4 can be obtained as follows: as the rate of arrival of traders increases, the current state of the market becomes less relevant in determining the current price, because each trader in the market can wait for the market composition to change without incurring much of a delay cost. For some initial state $(N_{\mathrm{s},0}, N_{\mathrm{b},0}, \omega_0)$, the delay cost of not trading until the state reaches $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega_0)$ in the support of the ergodic distribution of the state of the market tends to 0 as $n \to \infty$. It may then seem that the option of waiting to trade becomes increasingly attractive to all traders

[16] For instance, Gale (1987) characterizes the limit trade outcome in the large-market version of our model as the discount rate tends to 0, and obtains that it converges to that of a competitive market. The limit price is either 0 or 1, depending on whether there are more sellers or buyers in the market. Other papers have identified settings where the trade outcome fails to converge to the competitive one. Such failure of convergence may be due, for example, to asymmetric information between traders (Satterthwaite and Shneyerov, 2007; Lauermann and Wolinsky, 2016), heterogeneity on each side of the market (Lauermann, 2012), or a lack of knowledge about the state of the market (Lauermann et al., 2017). See Lauermann (2013) for a review of other reasons for the failure of convergence.

in the market. However, this is not possible when the matching frictions are small: the sum of the continuation values of a seller and a buyer is always close to 1, independently of the value of the arrival rates. Hence, even though waiting is increasingly cheap, it is also increasingly worthless, because the price variation across states becomes increasingly small.

To provide further intuition for Proposition 4.4, let $N^{\Sigma}_{s,t}$ and $N^{\Sigma}_{b,t}$ denote, respectively, the number of sellers and the number of buyers who arrived in the market between time 0 and time t, including the ones who "arrived" (or were present) at time 0. Then $N_t = N_{s,t} - N_{b,t} = N^{\Sigma}_{s,t} - N^{\Sigma}_{b,t}$; thus, equation (4.3) holds if we replace $N_{s,t}$ and $N_{b,t}$ by $N^{\Sigma}_{s,t}$ and $N^{\Sigma}_{b,t}$, respectively. As traders arrive more frequently, the price (e.g., at time 0) approximates the (ergodic) probability that more sellers than buyers will arrive in the future. Hence, the effective market to which a trader (at time 0) has access grows intertemporally. Given that the endogenous arrival process keeps a small market balanced, the price is interior. In contrast, the competitive price in large markets with small matching frictions is typically equal to either 0 (when there is excess supply) or 1 (when there is excess demand). The competitive price in a small dynamic market is a convex combination of the two extremes, each weighted according to the probability that the market features excess supply or demand.

The price can be obtained by solving a system of equations (one equation for each value $\omega \in \Omega$), which is given by

$$p^*(\omega) = \tfrac{r}{\gamma_c + r}\, \bar{p}(\omega) + \tfrac{\gamma_c}{\gamma_c + r}\, \mathbb{E}^c[p^*(\tilde{\omega})|\omega] \ , \tag{4.8}$$

where $\bar{p}(\omega) \equiv \mathbb{E}[\mathbb{I}_{N_s < N_b} + \xi\, \mathbb{I}_{N_s = N_b}|\omega]$ is the expected market price under the ergodic distribution of market compositions for ω, and where $\mathbb{E}^c[\cdot|\omega]$ is the expectation over the new market conditions given the ergodic distribution of market compositions for ω. Hence, current and future economic conditions affect the current price in the limit where the market grows by replication, but the price is independent of the current market composition.

5 Extensions and conclusions

5.1 Extensions

This section discusses some generalizations of our model, which illustrate how our results can be extended beyond some of the assumptions we have made. They also give a sense of the robustness of our findings, by indicating that different specifications of a small dynamic market yield similar sets of results.

Costly entry. Entering some markets requires a sunk investment. For example, people selling or renting out houses have to condition their housing units and design advertisements; workers entering a job market may have to update their market-specific knowledge and prepare documentation (CVs, cover letters, reference letters, etc.). Potential traders may therefore perform a cost–benefit analysis, comparing the cost of entering a market with the expected gains, before deciding whether to enter. To accommodate such a possibility, we could extend our model in the following way.[17]

Consider an extended model where, instead of directly entering the market, sellers and buyers become active at some respective state-independent rates $\bar{\gamma}_{\rm s}$ and $\bar{\gamma}_{\rm b}$. Assume, for concreteness, that $\bar{\gamma}_{\rm s} > \bar{\gamma}_{\rm b}$, and that sellers have entry cost $c \in (0,1)$, while buyers' entry cost is 0. In an equilibrium of this model, if a seller becomes active when the state is $(N_{\rm s}, N_{\rm b}, \omega)$, she enters the market if the net payoff from doing so, $V_{\rm s}(N_{\rm s}+1, N_{\rm b}, \omega) - c$, is above the payoff from some fixed outside option normalized to be 0 (which may come from selling the good in another market or keeping it for herself). This implies that the arrival of sellers into the market is equal to 0 if $V_{\rm s}(N_{\rm s}+1, N_{\rm b}, \omega) < c$ and to $\bar{\gamma}_{\rm s}$ if $V_{\rm s}(N_{\rm s}+1, N_{\rm b}, \omega) > c$; otherwise it belongs to $[0, \bar{\gamma}_{\rm s}]$ (the entry rate can be smoothed out assuming the entry cost is drawn from some continuous random variable).

Any equilibrium outcome in such a model corresponds to an equilibrium outcome of our model under some specification of the primitives. Hence, our characterization of the market price in Proposition 4.1 holds with costly entry, as does the tendency of prices to revert to the price in a balanced market (Corollary 4.1). Under costly entry, equilibrium prices would fluctuate closely around the competitive price c with frequent entry, despite the potential for a hold-up problem after the sunk cost has been paid. Costly entry would naturally keep the market frequently balanced, since excess supply would decrease the net entry of sellers, while excess demand would decrease the net entry of buyers.

Exogenous and endogenous exit. A common assumption in the large-market literature is that θ-traders leave the market at some (typically state-independent) Poisson rate $\rho_\theta > 0$, for each $\theta \in \{{\rm s}, {\rm b}\}$. This assumption is often made to keep the size of the market stationary when the arrival rates are constant, incorporating the observation that traders sometimes exit the market for exogenous reasons. Making such an assumption in our model adds an extra term equal to $N_{\rm s}\,\rho_{\rm s} + N_{\rm b}\,\rho_{\rm b}$ to each denominator in equation (3.1), as well as an extra term

$$\tfrac{1}{\lambda+\gamma+r+N_{\rm s}\,\rho_{\rm s}+N_{\rm b}\,\rho_{\rm b}}\Big((N_{\rm s}-1)\,\rho_{\rm s}\,V_{\rm s}(N_{\rm s}-1, N_{\rm b}, \omega) + N_{\rm b}\,\rho_{\rm b}\,V_{\rm s}(N_{\rm s}, N_{\rm b}-1, \omega)\Big)$$

on the right-hand side of the equation when $\theta = {\rm s}$ (and a similar term when $\theta = {\rm b}$). The additional term plays a role similar to that of the term corresponding to the exogenous change of the state of the market in equation (3.1): it also corresponds to an exogenous (i.e., equilibrium-independent) change in the state of the market.

[17] In the large markets literature, Manea (2017b) shows the existence of steady states in a similar specification.

More generally, our model can be adapted to accommodate endogenous exit. In a job market, endogenous exit may correspond to workers moving to other commuting zones when wages are low in their current zones. Similarly, house-seekers may end up looking for houses with different characteristics when the prices of houses in their desired sub-market are high (e.g., a family may end up buying an apartment instead of a house with a garden). To incorporate endogenous exit, we can add a stochastic process determining the decision times (arriving at some Poisson rate) at which a trader can decide whether to leave the market or not. Leaving the market gives a θ-trader an exogenous continuation value equal to $\underline{V}_\theta \geq 0$ (possibly depending on ω and satisfying $\underline{V}_\mathrm{s}+\underline{V}_\mathrm{b} < 1$). Equilibria of this extended model would feature some states (typically highly imbalanced) where traders on the long side of the market would choose to exit with some probability, and other states with no exit. Additionally, there would be some highly imbalanced states such that, after the arrival of a trader on the long side of the market, another trader on the long side of the market would almost immediately leave. As in the previous case with costly entry, the state of the market would tend to stay approximately balanced in a model with either exogenous or endogenous exit.

Attention frictions. The matching rate of our model can be interpreted in terms of attention frictions faced by traders. Consider, for example, a model where traders draw "attention times" instead of "meeting" other traders; a θ-trader in the market draws attention times at a (possibly state-independent) Poisson rate $\lambda_\theta > 0$, for $\theta \in \{\mathrm{s}, \mathrm{b}\}$. When a trader draws an attention time, she chooses a trader on the other side of the market (if any) and makes him an offer. This model with attention frictions generates the same (symmetric Markov perfect) equilibria as our model with[18]

$$\lambda \equiv N_\mathrm{s}\,\lambda_\mathrm{s} + N_\mathrm{b}\,\lambda_\mathrm{b} \quad \text{and} \quad \xi \equiv \tfrac{N_\mathrm{s}\,\lambda_\mathrm{s}}{N_\mathrm{s}\,\lambda_\mathrm{s}+N_\mathrm{b}\,\lambda_\mathrm{b}}\ .$$

The limit where "matching frictions vanish" considered in Section 4 corresponds, in the model with attention frictions, to the limit where "attention frictions vanish."

More general bargaining protocol. In our base model, the bargaining protocol in each match consists of a take-it-or-leave-it offer by a randomly chosen trader. This bargaining protocol is often used in the literature, which helps us compare our results to previous work. We now argue that our results apply to a broader set of bargaining protocols satisfying individual rationality.

We can black-box the bargaining protocol into the outcome of a match between a seller and a buyer, summarized as a probability of agreement α and some (potentially stochastic) transfers (dependent

[18]Our results can be straightforwardly generalized to the case where the bargaining protocol (captured by ξ) depends on the state of the market. In this case, ξ in equation (4.3) should be replaced by $\xi(1,1,\omega_t)$. Our results also apply when r, interpreted as the interest rate, depends on the economic conditions.

on whether agreement occurs). Thus, we can write equations (3.2) and (3.3) as

$$V_{\mathrm{s}}^{\mathrm{m}} = \alpha\,\mathbb{E}[\tilde{p}|\text{agreement}] + (1-\alpha)\,(V_{\mathrm{s}} + \mathbb{E}[\tilde{p}|\text{disagreement}])\,,$$
$$V_{\mathrm{b}}^{\mathrm{m}} = \alpha\,\mathbb{E}[1-\tilde{p}|\text{agreement}] + (1-\alpha)\,(V_{\mathrm{b}} - \mathbb{E}[\tilde{p}|\text{disagreement}])\,,$$

while equations (3.4) and (3.5) remain the same. If the bargaining protocol satisfies individual rationality, we have that $V_\theta^{\mathrm{m}} \geq V_\theta$ for both $\theta \in \{\mathrm{s},\mathrm{b}\}$ (i.e., $V_\theta^{\mathrm{m}} \in [V_\theta, 1-V_{\bar{\theta}}]$), which implies $\alpha = 0$ and $\mathbb{E}[\tilde{p}|\text{disagree}] = 0$ whenever $V_{\mathrm{s}}+V_{\mathrm{b}} > 1$. If one additionally requires that agreement occur whenever the gains from agreement are strictly positive, then $\alpha = 1$ and $\mathbb{E}[\tilde{p}|\text{agree}] \in [V_{\mathrm{s}}, 1-V_{\mathrm{b}}]$ whenever $V_{\mathrm{s}}+V_{\mathrm{b}} < 1$. It is then not difficult to see that if the bargaining protocol satisfies individual rationality and there is some $\xi = \xi(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega) \in (0,1)$ such that $\mathbb{E}[\tilde{p}|\text{agree}] = \xi\,V_{\mathrm{s}} + \xi\,(1-V_{\mathrm{b}})$, then our results continue to hold.[19]

5.2 Conclusions

We have studied decentralized bargaining in small dynamic markets. These are markets where traders arrive and leave over time, and there are few active traders at any given date.

The model is tractable while keeping the endogenous arrival process general. Our explicit characterization of the price in terms of the evolution of the market may serve as a guide for future empirical work and has allowed us to obtain novel implications for the price process that are amenable to testing. First, the higher competitiveness of traders on the long side of the market implies that the primitives affect prices only through the discounted length of future time during which the market has excess demand. It also implies that prices tend to converge to the price in a balanced market, even if the market composition fails to become more balanced. Second, the ergodic distribution of market compositions tends not to depend on the interest rate, which implies that increases in the interest rate tend to result in mean-preserving spreads of the distribution of market prices. Finally, since the market composition changes faster when multiple markets are unified, prices tend to depend less on the number of traders currently in the market and more on the evolution of the economic conditions.

Our setting is flexible and can be generalized in several directions. We have already indicated some interesting avenues for research, such as allowing for trader heterogeneity or studying price discovery by limiting the information about the state of the market. Another possible extension is to explicitly model the costly relocation of traders between different sub-markets. This could lead to new insights on endogenous gentrification (see Guerrieri et al., 2013, for a centralized large-market

[19] If $\xi = 0$ for all states or $\xi = 1$ for all states, then prices degenerate to 0 or 1, respectively, by the usual Diamond paradox. If the bargaining protocol is such that there is a probability $1-\bar{\alpha}$ of exogenous breakdown, then the match frequency λ can be normalized to $\lambda/\bar{\alpha}$; the results for our model then apply to the normalized model (where $\bar{\alpha} = 1$).

approach) or the sectorial mobility of workers (see Artuç and McLaren, 2015, for evidence), as well as on the effects that idiosyncratic and common shocks have on mobility across markets. The analysis of these and other extensions is left to future research.

A Omitted expressions and proofs of the results

A.1 Payoffs and equilibria

In the Appendix we use $\mathscr{S}\equiv\{0,...,\bar{N}_\mathrm{s}\}$, $\mathscr{B}\equiv\{0,...,\bar{N}_\mathrm{b}\}$, $\mathscr{S}^*\equiv\mathscr{S}\setminus\{0\}$, and $\mathscr{B}^*\equiv\mathscr{B}\setminus\{0\}$. Fix a strategy for the sellers $(\pi_\mathrm{s},\alpha_\mathrm{s})$ and a strategy for the buyers $(\pi_\mathrm{b},\alpha_\mathrm{b})$. For each type $\theta\in\{\mathrm{s},\mathrm{b}\}$ and state $(N_\mathrm{s},N_\mathrm{b},\omega)\in\mathscr{S}^*\times\mathscr{B}^*\times\Omega$, $\pi_\theta(N_\mathrm{s},N_\mathrm{b},\omega)\in\Delta(\mathbb{R})$ is the distribution of price offers that θ-traders make if they are matched and chosen to make the offer in state $(N_\mathrm{s},N_\mathrm{b},\omega)$, and $\alpha_\theta(\,\cdot\,;N_\mathrm{s},N_\mathrm{b},\omega):\mathbb{R}\to[0,1]$ maps each price offer received to a probability of acceptance.

Payoffs: Fix a strategy profile $\{(\pi_\theta,\alpha_\theta)\}_{\theta\in\{\mathrm{s},\mathrm{b}\}}$ and state $(N_\mathrm{s},N_\mathrm{b},\omega)$. We compute the continuation value the strategy profile gives to a seller (denoted $V_\mathrm{s}(N_\mathrm{s},N_\mathrm{b},\omega)$) and to a buyer (denoted $V_\mathrm{b}(N_\mathrm{s},N_\mathrm{b},\omega)$) using standard recursive analysis. They satisfy equation (3.1) (for both $\theta\in\{\mathrm{s},\mathrm{b}\}$), where now the expected continuation values conditional on being selected in the match given by

$$\begin{aligned} V_\mathrm{s}^\mathrm{m}(N_\mathrm{s},N_\mathrm{b},\omega)\equiv{}&\xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{b}(\tilde{p})\,\tilde{p}+(1-\alpha_\mathrm{b}(\tilde{p}))\,V_\mathrm{s}(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{s}\,\big]\\ &+(1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{s}(\tilde{p})\,\tilde{p}+(1-\alpha_\mathrm{s}(\tilde{p}))\,V_\mathrm{s}(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{b}\,\big]\ \text{ and} \qquad (\mathrm{A}.1)\\ V_\mathrm{b}^\mathrm{m}(N_\mathrm{s},N_\mathrm{b},\omega)\equiv{}&\xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{b}(\tilde{p})\,(1-\tilde{p})+(1-\alpha_\mathrm{b}(\tilde{p}))\,V_\mathrm{b}(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{s}\,\big]\\ &+(1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{s}(\tilde{p})\,(1-\tilde{p})+(1-\alpha_\mathrm{s}(\tilde{p}))\,V_\mathrm{b}(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{b}\,\big] \qquad (\mathrm{A}.2)\end{aligned}$$

instead of equations (3.2) and (3.3), where the continuation value of the type-θ trader conditional on some other traders being selected in the match is given by

$$\begin{aligned} V_\theta^\mathrm{o}(N_\mathrm{s},N_\mathrm{b},\omega)\equiv{}&\xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{b}(\tilde{p})\,V_\theta(N_\mathrm{s}-1,N_\mathrm{b}-1,\omega)+(1-\alpha_\mathrm{b}(\tilde{p}))\,V_\theta(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{s}\,\big]\\ &+(1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_\mathrm{s}(\tilde{p})V_\theta(N_\mathrm{s}-1,N_\mathrm{b}-1,\omega)+(1-\alpha_\mathrm{s}(\tilde{p}))\,V_\theta(N_\mathrm{s},N_\mathrm{b},\omega)\,\big|\,\pi_\mathrm{b}\,\big] \qquad (\mathrm{A}.3)\end{aligned}$$

instead of by equation (3.4), and where V_θ^e satisfies equation (3.5).[20] It is convenient to set $V_\mathrm{s}(0,N_\mathrm{b},\omega)=V_\mathrm{b}(N_\mathrm{s},0,\omega)=0$ for all $N_\mathrm{s}\in\mathscr{S}$ and $N_\mathrm{b}\in\mathscr{B}$, so the domain of V_s and V_b is $\mathscr{S}\times\mathscr{B}\times\Omega$.

For each fixed strategy profile, the system of equations $\langle(3.1),(\mathrm{A}.1),(\mathrm{A}.2),(\mathrm{A}.3),(3.5)\rangle$ has a unique solution for $(V_\mathrm{s},V_\mathrm{b})$ by the standard fixed-point argument. Indeed, we can replace V_b with $W_\mathrm{s}\equiv 1-V_\mathrm{b}$ and use the system of equations to define an operator mapping each pair of functions

[20] As in the main text we keep the notation short by omitting the explicit dependence of λ and γ on the state $(N_\mathrm{s},N_\mathrm{b},\omega)$, and we use $\alpha_\theta(\tilde{p})$ to denote $\alpha_\theta(\tilde{p};N_\mathrm{s},N_\mathrm{b},\omega)$.

$(V_s, W_s) : \mathscr{S} \times \mathscr{B} \times \Omega \to \mathbb{R}^2$ into another pair of similar functions. It is then easy to verify that such an operator satisfies the sufficient Blackwell conditions for a contraction.

Equilibrium concept: We use the principle of optimality to define our equilibrium concept. More concretely, we say that $\{(\pi_\theta, \alpha_\theta)\}_{\theta \in \{s,b\}}$ is a *symmetric Markov perfect equilibrium* if the corresponding continuation values—solving the system of equations $\langle(3.1), (A.1), (A.2), (A.3), (3.5)\rangle$—are such that, for each (N_s, N_b, ω), $\theta \in \{s, b\}$ and $\pi_{\overline{\theta}}$, the pair $(\pi_\theta(N_s, N_b, \omega), \alpha_\theta(\cdot; N_s, N_b, \omega))$ maximizes the right-hand side of equation (A.1) if $\theta = s$ and right-hand side of (A.2) if $\theta = b$.

A.2 Proofs of the results

Proof of Proposition 3.1

Proof. Fix an equilibrium, assuming it exists, given by $(\pi_\theta, \alpha_\theta, V_\theta)$, hence solving the system of equations $\langle(3.1), (A.1), (A.2), (A.3), (3.5)\rangle$. Standard arguments imply that if there is a positive probability that offers made by a seller are accepted in state (N_s, N_b, ω), then the equilibrium probability that such offers are equal to $1 - V_b(N_s, N_b, \omega)$ is one. Similarly, an equilibrium offer by a buyer in state (N_s, N_b, ω) is accepted with positive probability in equilibrium if and only if it is equal to $V_s(N_s, N_b, \omega)$. Since these offers make the receiver of the offer indifferent between accepting them or not, it is without loss of generality (to prove the existence of equilibria) to focus on equilibria where, in each state (N_s, N_b, ω) and for all $\theta \in \{s, b\}$, sellers offer $1 - V_b(N_s, N_b, \omega)$ and buyers offer $V_s(N_s, N_b, \omega)$ for sure, and a θ-trader accepts the equilibrium offer with some probability

$$\hat{\alpha}_s(N_s, N_b, \omega) = \alpha_s(V_s; N_s, N_b, \omega) \text{ and } \hat{\alpha}_b(N_s, N_b, \omega) = \alpha_b(1 - V_b; N_s, N_b, \omega) .$$

Thus, equations (A.1) and (A.2) can be replaced by equations (3.2) and (3.3). Note that the continuation values of a seller and a buyer only depend on $\hat{\alpha}_b$ and $\hat{\alpha}_s$ through

$$\alpha \equiv (1 - \xi)\, \hat{\alpha}_b + \xi\, \hat{\alpha}_s \tag{A.4}$$

(see equation (A.3)), with the convention that $\alpha(N_s, N_b, \omega) = 0$ whenever $N_s = 0$ or $N_b = 0$, and so equation (A.3) can be replaced by equation (3.4). Hence, equations (3.1)–(3.5) determine the continuation payoffs in an equilibrium.

Fix some $\alpha \in [0, 1]^{\mathscr{S}^* \times \mathscr{B}^* \times \Omega}$, interpreted as a putative equilibrium probability of trade. We can compute the equilibrium continuation values in each state by solving equations in (3.1)–(3.5), and let $V_s(\cdot; \alpha)$ and $V_b(\cdot; \alpha)$ denote the corresponding solutions. Note also that a seller and a buyer are indifferent on making an acceptable offer at state (N_s, N_b, ω) if and only if $V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) = 1$. Hence, there is no $\theta \in \{s, b\}$ such that the θ-trader has a profitable deviation at a given state $(N_s, N_b, \omega) \in \mathscr{S}^* \times \mathscr{B}^* \times \Omega$ if and only if

$$\alpha(N_s, N_b, \omega) \in \begin{cases} \{0\} & \text{if } V_b(N_s, N_b, \omega; \alpha) + V_s(N_s, N_b, \omega; \alpha) > 1, \\ [0,1] & \text{if } V_b(N_s, N_b, \omega; \alpha) + V_s(N_s, N_b, \omega; \alpha) = 1, \\ \{1\} & \text{if } V_b(N_s, N_b, \omega; \alpha) + V_s(N_s, N_b, \omega; \alpha) < 1. \end{cases}$$

The argument is standard. Intuitively, for example, consider the case $V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) > 1$ and assume, for the sake of contradiction, that $\alpha_s(N_s, N_b, \omega) > 0$. If a buyer makes the equilibrium offer (equal to $V_s(N_s, N_b, \omega; \alpha)$) at state (N_s, N_b, ω) he obtains

$$\begin{aligned} &\alpha_s\,(1 - V_s(N_s, N_b, \omega; \alpha)) + (1 - \alpha_s)\, V_b(N_s, N_b, \omega; \alpha) \\ &\quad = V_b(N_s, N_b, \omega; \alpha) - \alpha_s \left(V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) - 1\right) \\ &\quad < V_b(N_s, N_b, \omega; \alpha)\ . \end{aligned}$$

If, instead, he offers $V_s(N_s, N_b, \omega; \alpha) - \varepsilon$, for some $\varepsilon > 0$, the seller rejects the offer for sure, and so the buyer obtains $V_b(N_s, N_b, \omega; \alpha)$. The buyer is then better off deviating.

To conclude the proof of existence of equilibria, we define $A : [0,1]^{\mathscr{S}^* \times \mathscr{B}^* \times \Omega} \rightrightarrows [0,1]^{\mathscr{S}^* \times \mathscr{B}^* \times \Omega}$ as follows:

$$A(\alpha)(N_s, N_b, \omega) = \begin{cases} \{0\} & \text{if } V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) > 1, \\ [0,1] & \text{if } V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) = 1, \\ \{1\} & \text{if } V_s(N_s, N_b, \omega; \alpha) + V_b(N_s, N_b, \omega; \alpha) < 1. \end{cases}$$

Standard arguments apply to show that $A(\cdot)$ has a closed graph, and that $A(\alpha)$ is, for all $\alpha \in [0,1]$, non-empty and convex. Hence, the existence of equilibria follows from Kakutani's fixed point theorem.

Existence when $\bar{N}_s = \bar{N}_b = +\infty$. Fix some functions $\lambda, \gamma_s, \gamma_b : \mathbb{Z}_+^2 \times \Omega \to \mathbb{R}_{++} \times \mathbb{R}_+ \times \mathbb{R}_+$, where γ_s and γ_b are bounded. Consider a sequence $(\bar{N}_{sn}, \bar{N}_{bn})_n$ strictly increasing in both arguments. For each n, we can construct a model with a finite state space as follows ("the n-th model"). In the n-th model, λ^n coincides with λ, now with domain $\mathscr{S}^n \times \mathscr{B}^n \times \Omega \equiv \{0, ..., \bar{N}_{sn}\} \times \{0, ..., \bar{N}_{bn}\} \times \Omega$. The arrival rates in the n-th model is, for each $(N_s, N_b, \omega) \in \mathscr{S}^n \times \mathscr{B}^n \times \Omega$, are

$$\gamma_s^n(N_s, N_b, \omega) = \begin{cases} \gamma_s(N_s, N_b, \omega) & \text{if } N_s < \bar{N}_s{}^n, \\ 0 & \text{if } N_s = \bar{N}_s{}^n, \end{cases} \quad \text{and}$$

$$\gamma_b^n(N_s, N_b, \omega) = \begin{cases} \gamma_s(N_s, N_b, \omega) & \text{if } N_b < \bar{N}_b{}^n, \\ 0 & \text{if } N_b = \bar{N}_b{}^n. \end{cases}$$

For each n, let α^n characterize an equilibrium of the n-th model. Fix some $\omega \in \Omega$ (the argument can be made for each value of the economic conditions). Let $\mu : \mathbb{N} \to \mathbb{N}^2$ be a bijective ordering of $\mathbb{N}^2$. Initialize $(\alpha_0^n)_n = (\alpha^n)_n$. Then, for each $m \in \mathbb{N}$, we use $(\alpha_{m-1}^n)_n$ to recursively construct $(\alpha_m^n)_n$ as follows. Let $\underline{\alpha}_m$ be the minimum cluster point of $(\alpha_{m-1}^n(\mu(m), \omega))_n$ (recall that the set of cluster points of a sequence is closed). If there is an increasing subsequence of $(\alpha_{m-1}^n(\mu(m), \omega))_n$ converging to $\underline{\alpha}_m$, then we let $(\alpha_m^n)_n$ be the biggest subsequence of $(\alpha_{m-1}^n)_n$ such that $(\alpha_{m-1}^n(\mu(m), \omega))_n$ is increasing and converging to $\underline{\alpha}_m$. If no increasing subsequence of $(\alpha_{m-1}^n(\mu(m)))_n$ converging to $\underline{\alpha}_m$ exists, then there must exist a decreasing subsequence of $(\alpha_{m-1}^n(\mu(m), \omega))_n$ converging to $\underline{\alpha}_m$. In this case, $(\alpha_m^n)_n$ be the largest decreasing subsequence of $(\alpha_m^n)_n$ such that $(\alpha_{m-1}^n(\mu(m), \omega))_n$ is decreasing and converges to $\underline{\alpha}_m$.

Note that, as explained above, for each m there exists some $\underline{\alpha}_m \in [0,1]$ and a subsequence $(\alpha_m^n)_n$ of $(\alpha^n)_n$ such that $(\alpha_m^n(\mu(m'), \omega))_n$ converges to $\underline{\alpha}_{m'}$ for all $m' \in \{1, ..., m\}$. It is then easy to prove that, in fact, $\alpha : \mathbb{N}^2 \times \Omega \to [0,1]$ given by $\alpha(N_s, N_b, \omega) = \underline{\alpha}_{\mu^{-1}(N_s, N_b, \omega)}(N_s, N_b, \omega)$ characterizes an equilibrium in the infinite model. □

Proof of Proposition 3.2

Proof. The proof follows from the arguments in the main text. □

Proof of Lemma 4.1

Proof. Throughout the proof, we fix a sequence $(k_n)_n$ tending to $+\infty$ and, for each n, an equilibrium for the model in which the matching rate is $\lambda = k_n \ell$. For each n and fixed state (N_s, N_b, ω), we let $V_{\theta,n} \equiv V_{\theta,n}(N_s, N_b, \omega)$ denote the continuation value of a θ-trader in the n-th equilibrium in state (N_s, N_b, ω), for $\theta \in \{s, b\}$, and $\alpha_n \equiv \alpha_n(N_s, N_b, \omega)$ denote the probability of trade in a match in this equilibrium.

Let $\mathscr{M}^* \equiv \mathscr{S}^* \times \mathscr{B}^* \times \Omega$ denote the set of all states of the market with at least one seller and one buyer. Let $\mathscr{M}_1^n$, $\mathscr{M}_2^n$, and $\mathscr{M}_3^n$, denote, respectively, the first, the second, and the third, kinds of states described in the main text after the statement of Lemma 4.1 in the n-th equilibrium (so $\{\mathscr{M}_i^n\}_{i=1,2,3}$ is a partition of $\mathscr{M}^*$ for all n). Taking a subsequence if necessary, assume that for all n and for each state (N_s, N_b, ω) there is some $i = i(N_s, N_b, \omega) \in \{1, 2, 3\}$ such that $(N_s, N_b, \omega) \in \mathscr{M}_{i(N_s, N_b, \omega)}^n$ for all n in the subsequence. Assume, to ease notation and without loss of generality for the argument, that the subsequence above is equal to the original sequence. There are then three cases:

1. It is clear that for all $(N_s, N_b, \omega) \in \mathscr{M}_1^n$ for all n we have $V_n = 1$ for all n, so $V_n \to 1$.

2. Assume $(N_s, N_b, \omega) \in \mathscr{M}_2^n$ for all n and, without loss of generality, that $N_b = 1$ (the case $N_s = 1$

is analogous). Then, as indicated in the main text, a strategy available to the buyer is to wait until he makes the offer and offer $V_{\mathrm{s},n}$ (if the state of the market has not changed before). This implies

$$V_{\mathrm{b},n} \geq \underbrace{\frac{(1-\xi)\,k_n\,\ell}{(1-\xi)\,k_n\,\ell+\gamma+r}}_{\to 1}\,(1-V_{\mathrm{s},n}) + \underbrace{\frac{\gamma}{(1-\xi)\,k_n\,\ell+\gamma+r}}_{\to 0}\,V^{\mathrm{e}}_{\mathrm{b},n}\,.$$

Hence, since $V_{\mathrm{s},n}+V_{\mathrm{b},n} \leq 1$ for all n (by Proposition 3.2), we have $V_{\mathrm{s}}+V_{\mathrm{b}} \simeq 1$.

3. Assume finally that $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega) \in \mathscr{M}_3^n$ for all n and, without loss of generality, that $N_{\mathrm{s}} \geq N_{\mathrm{b}} > 1$ (the case $1 < N_{\mathrm{s}} < N_{\mathrm{b}}$ is analogous). Here we proceed by induction. Assume that $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega) \in \mathscr{M}_3^n$ is such that N_{s} is minimal among all states $(N'_{\mathrm{s}}, N'_{\mathrm{b}}, \omega') \in \mathscr{M}_3^n$ with $N'_{\mathrm{s}} \geq N'_{\mathrm{b}} > 1$. A strategy available to a seller is not trading until the state changes and then continuing with the equilibrium strategy, which implies

$$V_{\mathrm{s},n} \geq \underbrace{\frac{\frac{N_{\mathrm{s}}-1}{N_{\mathrm{s}}}\,k_n\,\ell}{\frac{N_{\mathrm{s}}-1}{N_{\mathrm{s}}}\,k_n\,\ell+\gamma+r}}_{\to 1}\,V_{\mathrm{s},n}(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \tilde{\omega}) + \underbrace{\frac{\gamma}{\frac{N_{\mathrm{s}}-1}{N_{\mathrm{s}}}\,k_n\,\ell+\gamma+r}}_{\to 0}\,V^{\mathrm{e}}_{\mathrm{s},n}\,.$$

A buyer who follows the same strategy obtains

$$V_{\mathrm{b},n} \geq \underbrace{\frac{\frac{N_{\mathrm{b}}-1}{N_{\mathrm{b}}}\,k_n\,\ell}{\frac{N_{\mathrm{b}}-1}{N_{\mathrm{b}}}\,k_n\,\ell+\gamma+r}}_{\to 1}\,V_{\mathrm{b},n}(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \omega) + \underbrace{\frac{\gamma}{\frac{N_{\mathrm{b}}-1}{N_{\mathrm{b}}}\,k_n\,\ell+\gamma+r}}_{\to 0}\,V^{\mathrm{e}}_{\mathrm{s},n}\,.$$

Hence, we have that $V \succeq V(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \omega)$. By the assumption that N_{s} is minimal we have $(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \omega) \in \mathscr{M}_1^n \cup \mathscr{M}_2^n$ for all n, and therefore $V(N_{\mathrm{s}}-1, N_{\mathrm{b}}-1, \omega) \simeq 1$. This implies that $V \simeq 1$. One can proceed recursively to show that the result holds for all N_{s}.

□

Proof Proposition 4.1

Proof. As in the proof of Lemma 4.1, throughout the proof, we fix a sequence $(k_n)_n$ tending to $+\infty$ and, for each n, an equilibrium for the model in which the matching rate is $\lambda = k_n\,\ell$. For each n and fixed state $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$, we let $V_{\theta,n} \equiv V_{\theta,n}(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$ denote the continuation value of a θ-trader in the n-th equilibrium in state $(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$, for $\theta \in \{\mathrm{s}, \mathrm{b}\}$, and $\alpha_n \equiv \alpha_n(N_{\mathrm{s}}, N_{\mathrm{b}}, \omega)$ denote the probability of trade in a match in this equilibrium.

Assume, without loss of generality for our arguments and taking a subsequence if necessary, that for each state $(N'_{\mathrm{s}}, N'_{\mathrm{b}}, \omega') \in \mathscr{S}^* \times \mathscr{B}^* \times \Omega$ the agreement rate at such a state, equal to

$$\alpha_n(N'_{\mathrm{s}}, N'_{\mathrm{b}}, \omega')\,k_n\,\ell(N'_{\mathrm{s}}, N'_{\mathrm{b}}, \omega')\,,$$

tends to some value $\delta(N'_{\rm s},N'_{\rm b},\omega')\in[0,+\infty]$ as n increases (with the convention that $\alpha_n(N'_{\rm s},N'_{\rm b},\omega')=0$ when $N'_{\rm s}=0$ or $N'_{\rm b}=0$).

Preliminary result: We first note that using the standard analysis in Rubinstein (1982), we have that equation (4.6) holds for $V_{{\rm s},n}$. Indeed, when $(N_{\rm s},N_{\rm b},\omega)$ is such that $N_{\rm s}=N_{\rm b}=1$ we have

$$V_{\theta,n}=\tfrac{k_n\,\ell}{k_n\,\ell+\gamma+r}\left(\xi_\theta\,(1-V_{\bar\theta,n})+(1-\xi_\theta)\,V_{\theta,n}\right)+\tfrac{\gamma}{k_n\,\ell+\gamma+r}\,V^{\rm e}_{\theta,n} \tag{A.5}$$

for all $\theta\in\{{\rm s},{\rm b}\}$, where $\xi_{\rm s}\equiv\xi$ and $\xi_{\rm b}\equiv 1-\xi$. The previous equation coincides with the equation for the continuation payoff in a two-player Rubinstein bargaining where the "threat point" (i.e., value from not trading) for the θ-trader is $\frac{r}{\gamma+r}\,V^{\rm e}_{\theta,n}$. Solving for $V_{{\rm b},n}$ and $V_{{\rm s},n}$, we have

$$V_{\theta,n}=\tfrac{k_n\,\ell}{k_n\,\ell+\gamma+r}\left(\tfrac{r}{\gamma+r}+\gamma\,(1-V^{\rm e}_n)\right)\xi_\theta+\tfrac{\gamma}{\gamma+r}\,V^{\rm e}_{\theta,n}\ .$$

Hence, since $V^{\rm e}_n\simeq 1$ by Lemma 4.1, we have

$$\lim_{n\to\infty}\left|\tfrac{r}{\gamma+r}\,\xi_\theta+\tfrac{\gamma}{\gamma+r}\,V^{\rm e}_{\theta,n}-V_{\theta,n}\right|=0\ , \tag{A.6}$$

that is, $V_{\theta,n}\simeq\frac{r}{\gamma+r}\,\xi_\theta+\frac{\gamma}{\gamma+r}\,V^{\rm e}_{\theta,n}$.

Definitions: For each n, define a function $\tilde V_{\theta,n}:\mathscr{S}\times\mathscr{B}\times\Omega\to[0,1]$, interpreted as the payoff of a θ-trader when she decides to trade only when the market is balanced, as follows. It is obtained solving equations (3.1), (3.4) and (3.5) (adding tildes to all V_θ's), and instead of equations (3.2) and (3.3), we now require that $\tilde V^{\rm m}_{\theta,n}=\tilde V_{\theta,n}$ when $N_{\rm s}\neq N_{\rm b}$ (no trade when the market is imbalanced) and $\tilde V^{\rm m}_{\theta,n}=V^{\rm m}_{\theta,n}$ when $N_{\rm s}=N_{\rm b}$ (trade for sure when the market is balanced). Note that for $\theta={\rm s}$ and $N_{\rm s}\neq N_{\rm b}$, equation (3.1) can be rewritten as equation (4.4) replacing "$\simeq$" by "$=$", adding tildes to all V's, and replacing α by α_n. We can obtain an analogous equation for $\tilde V_{{\rm b},n}$.

Proof that $\tilde V_\theta\simeq V_\theta$ whenever $N_\theta\geq N_{\bar\theta}$: Our first goal is to show that, independently of the choice of the the sequence $(k_n)_n$ and a corresponding sequence equilibria,

$$\lim_{n\to\infty}|\tilde V_{s,n}-V_{s,n}|=0 \text{ for all } N_{\rm s}\geq N_{\rm b}\ \text{ and }\ \lim_{n\to\infty}|\tilde V_{{\rm b},n}-V_{{\rm b},n}|=0 \text{ for all } N_{\rm s}\leq N_{\rm b}\ ;$$

or, in our notation, $\tilde V_{\rm s}\simeq V_{\rm s}$ for all $N_{\rm s}\geq N_{\rm b}$ and $\tilde V_{\rm b}\simeq V_b$ for all $N_{\rm s}\leq N_{\rm b}$. (As in the main text, "$\simeq$" means equal except for terms that go to 0 as n increases.) We then define $W_n\equiv W_n(N_{\rm s},N_{\rm b},\omega)$ to be equal to $\tilde V_{{\rm s},n}$ when $N_{\rm s}\geq N_{\rm b}$, and to be equal to $1-\tilde V_{{\rm b},n}$ when $N_{\rm s}<N_{\rm b}$.

For each n and state $(N_{\rm s},N_{\rm b},\omega)$, let $D_{{\rm s},n}\equiv D_{{\rm s},n}(N_{\rm s},N_{\rm b},\omega)$ denote $|V_{{\rm s},n}-\tilde V_{{\rm s},n}|$, and let $D_{{\rm b},n}\equiv D_{{\rm b},n}(N_{\rm s},N_{\rm b},\omega)$ denote $|V_{{\rm b},n}-\tilde V_{{\rm b},n}|$. Let also D_n denote $D_{{\rm b},n}$ when $N_{\rm s}\geq N_{\rm b}$ and $D_{{\rm s},n}$ when $N_{\rm s}<N_{\rm b}$. Assume, without loss of generality and for simplicity (considering a subsequence if necessary), assume that $D\equiv\lim_{n\to\infty}D_n$ is well defined for all states.

We want to prove that $D(N_\mathrm{s}, N_\mathrm{b}, \omega) = 0$ in any state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$. To do so we assume, for the sake of contradiction, that $D(N'_\mathrm{s}, N'_\mathrm{b}, \omega') > 0$ for some state $(N'_\mathrm{s}, N'_\mathrm{b}, \omega')$. We let $\bar{D}$ denote the maximal value of D among all states, and we assume without loss of generality that there is at least one state $(N'_\mathrm{s}, N'_\mathrm{b}, \omega')$ with $N'_\mathrm{s} \geq N'_\mathrm{b} > 0$ where $D(N'_\mathrm{s}, N'_\mathrm{b}, \omega') = \bar{D}$. We let $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ be a state with satisfying that $D(N_\mathrm{s}, N_\mathrm{b}, \omega) = \bar{D}$ and N_s is minimal among all states $(N'_\mathrm{s}, N'_\mathrm{b}, \omega')$ such that both $N'_\mathrm{s} \geq N'_\mathrm{b} > 0$ and $D(N'_\mathrm{s}, N'_\mathrm{b}, \omega') = \bar{D}$. We rule out the possibility that $\bar{D} > 0$ by reaching a contradiction in four separate cases (which cover all possible states):

1. **Assume first that** $N_\mathrm{s} = N_\mathrm{b}$**.** We aim at proving that for all $\varepsilon > 0$ there is some n such that $\left|\tilde{V}_{\mathrm{s},n} - V_{\mathrm{s},n}\right| < \varepsilon$. To see this recall that, by Proposition 3.2, there is immediate trade when the market is balanced and, by Lemma 4.1, $V_\mathrm{b} \simeq 1 - V_\mathrm{s}$. If $N_\mathrm{s} = N_\mathrm{b} = 1$ it is clear, by the definition of $\tilde{V}_s$, that $V_s \simeq V_s^\mathrm{m} \simeq \tilde{V}_s$. Proceeding similarly, we have that it is also the case that $W = \tilde{V}_\mathrm{s} \simeq V_\mathrm{s}$ because, as n increases, it is increasingly unlikely that an arrival happens before the market clears. If instead $N_\mathrm{s} = N_\mathrm{b} > 1$ we have

$$V_\mathrm{s} \simeq \tfrac{1}{N_\mathrm{s}} V_\mathrm{s} + \tfrac{N_\mathrm{s}-1}{N_\mathrm{s}} V_\mathrm{s}(N_\mathrm{s}-1, N_\mathrm{b}-1, \tilde{\omega}) \;\;\Rightarrow\;\; V_\mathrm{s} \simeq V_\mathrm{s}(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)\ . \tag{A.7}$$

and similarly

$$\tilde{V}_\mathrm{s} \simeq \tfrac{1}{N_\mathrm{s}} V_\mathrm{s} + \tfrac{N_\mathrm{s}-1}{N_\mathrm{s}} \tilde{V}_\mathrm{s}(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega)\ .$$

Hence, if $N_\mathrm{s} = N_\mathrm{b} = 2$ it is clear that we also have $V_s \simeq V_s^\mathrm{m} \simeq \tilde{V}_s$. Proceeding recursively, we have that $V_s \simeq V_s^\mathrm{m} \simeq \tilde{V}_s$ holds for any $N_\mathrm{s} = N_\mathrm{b} > 0$. Hence, $V_s \simeq \tilde{V}_s = W_s$ (i.e., $\bar{D} = 0$) whenever $N_\mathrm{s} = N_\mathrm{b}$, which is a contradiction.

2. **Assume now that** $N_\mathrm{s} > N_\mathrm{b}$ **and** $\delta = \delta(N_\mathrm{s}, N_\mathrm{b}, \omega) < +\infty$**.** This is the case where the rate at which transactions happen remains finite as the matching frictions disappear. This implies that, for n large enough, we have $V_n = 1$. Also, since a seller is indifferent between trading or not, we have

$$D_n = \frac{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \alpha_n k_n \ell}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \alpha_n k_n \ell + \gamma + r} D_n(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega) + \frac{\gamma}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \alpha_n k_n \ell + \gamma + r} D_n^\mathrm{e}\ .$$

While the left-hand side tends to $\bar{D}$, the right-hand side tends to

$$\frac{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \delta}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \delta + \gamma + r} D(N_\mathrm{s}-1, N_\mathrm{b}-1, \omega) + \frac{\gamma}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \delta\gamma + r} D^\mathrm{e} \leq \frac{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \delta + \gamma}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}} \delta + \gamma + r} \bar{D} < \bar{D}\ .$$

This is a contradiction.

3. **Assume next that** $N_\mathrm{s} > N_\mathrm{b} = 0$ **(and so** $\ell(N_\mathrm{s}, N_\mathrm{b}, \omega) = 0$**).** In this case, we have

$$D_n = \tfrac{\gamma}{\gamma + r} D_n^\mathrm{e} \;\;\Rightarrow\;\; \bar{D} \leq \tfrac{\gamma}{\gamma + r} \bar{D}\ .$$

This is, again, a contradiction.

4. **Assume finally that $N_s > N_b > 0$ and $\delta = \infty$.** Note that this implies that equation (A.7) holds and, additionally,

$$\tilde{V}_s \simeq \tfrac{1}{N_s}\,\tilde{V}_s + \tfrac{N_s-1}{N_s}\,\tilde{V}_s(N_s-1, N_b-1, \omega) \quad\Rightarrow\quad \tilde{V}_s \simeq \tilde{V}_s(N_s-1, N_b-1, \omega)\ .$$

This implies that

$$\bar{D} \le D(N_s-1, N_b-1, \omega)\ .$$

(The inequality is owed to the fact that the absolute value is a convex function.) This contradicts the assumption that N_s is minimal among all states (N_s', N_b', ω') with $N_s' \ge N_b' > 0$ where $D(N_s', N_b', \omega') = \bar{D}$.

Conclusion of the proof: We have proven that $D(N_s, N_b, \omega) = 0$ for any state (N_s, N_b, ω). Hence, we have $W \simeq 1 - V_b$ for all $N_s \ge N_b$ and $W \simeq 1 - V_b$ for all $N_s < N_b$. Furthermore, when $N_s = N_b = 1$, we can rewrite equation (A.6) as

$$W \simeq \tfrac{r}{\gamma+r}\,\xi + \tfrac{\gamma}{\gamma+r}\,W^{e}\ .$$

As a result, W satisfies equations (4.4)–(4.6) replacing all V_s's by the corresponding W's. As argued in the main text, this implies that W satisfies equation (4.3). □

Proof of Corollary 4.1

Proof. As in the proof of Proposition 4.1, we fix a sequence $(k_n)_n$ tending to $+\infty$ and, for each n, an equilibrium for the model with $\lambda = k_n\,\ell$. Also as in the proof of Proposition 4.1, we assume without loss of generality that, for each state (N_s, N_b, ω), the values $V_{s,n}$, $V_{b,n}$, and $\alpha_n\,k_n\,\ell$ tend to some (state-dependent) values V_s, V_b, and $\delta \in [0, +\infty]$ as n increases, respectively.

Fix time $t = 0$ and state (N_s, N_b, ω). Assume, without loss of generality for our arguments, that $N_s > N_b$. For each $\Delta > 0$, we use $V_{s,\Delta} \equiv V_{s,\Delta}(N_s, N_b, \omega)$ denote the continuation value of the seller at time Δ if $(N_{s,0}, N_{b,0}, \omega_0) = (N_s, N_b, \omega)$ (which is a stochastic random variable). Then, $\mathbb{E}[V_{s,\Delta}] \equiv \mathbb{E}[V_{s,\Delta}(N_s, N_b, \omega)]$ indicates the expected continuation value of the seller at time Δ if $(N_{s,0}, N_{b,0}, \omega_0) = (N_s, N_b, \omega)$. (Note that since the agreement rate converges to δ, the limit distribution of $(N_{s,\Delta}, N_{b,\Delta}, \omega_\Delta)$ as $n \to \infty$ is well defined for any $\Delta > 0$.)

We will now prove that, for any imbalanced state $(N_{s,0}, N_{b,0}, \omega_0) \equiv (N_s, N_b, \omega)$ where θ-traders are on the long side of the market,[21]

$$r\,V_{\theta,0} \simeq \frac{\tilde{\mathbb{E}}[V_{\theta,\Delta}]-V_{\theta,0}}{\Delta}+O(\Delta) \preceq \frac{\mathbb{E}[V_{\theta,\Delta}]-V_{\theta,0}}{\Delta}+O(\Delta)\;, \tag{A.8}$$

where $V_{\theta,\Delta} \equiv V_\theta(N_{s,\Delta}, N_{b,\Delta}, \omega_\Delta)$. It is clear that (A.8) implies Corollary 4.1, since it implies

$$V_{\theta,0} \prec V_{\theta,0}+r\,V_{\theta,0}\,\Delta \simeq \tilde{\mathbb{E}}[V_{\theta,\Delta}]+O(\Delta^2) \preceq \mathbb{E}[V_{\theta,\Delta}]+O(\Delta^2)\;.$$

To prove that equation (A.8) holds, we divide the analysis into three cases (which cover all states) for the case $\theta=\mathrm{s}$ (the case $\theta=\mathrm{b}$ is analogous):

1. Assume first that the state (N_s, N_b, ω) is such that $N_b=0$. It is clear that, in this case,[22]

   $$\mathbb{E}[V_{s,\Delta}] = (1-\gamma\,\Delta)\,V_s+\gamma\,\Delta\,V_s^e+O(\Delta^2) = \tilde{\mathbb{E}}[V_{s,\Delta}]+O(\Delta^2)\;.$$

   Since, from equation (3.1), we have $r\,V_s=\gamma\,(V_s^e-V_s)$, we have that

   $$r\,V_s = \frac{\mathbb{E}[V_{s,\Delta}]-V_s}{\Delta}+O(\Delta) = \frac{\tilde{\mathbb{E}}[V_{s,\Delta}]-V_s}{\Delta}+O(\Delta)\;.$$

2. Assume now that the state (N_s, N_b, ω) is such that $\delta<+\infty$. In this case, we have that

   $$\mathbb{E}[V_{s,\Delta}] =(1-(\delta+\gamma)\,\Delta)\,V_s+\delta\,\Delta\,V_s(N_s-1,N_b-1,\tilde{\omega})+\gamma\,\Delta\,V_s^e+O(\Delta^2)$$

   as $\Delta\to 0$. The same equation holds if $\mathbb{E}$ is replaced by $\tilde{\mathbb{E}}$ on the left-hand side and δ is replaced by $\frac{N_s-1}{N_s}\,\delta$ on the right-hand side. Also, we can use equation (3.1) to obtain

   $$r\,V_s = \tfrac{N_s-1}{N_s}\,\delta\,(V_s(N_s-1,N_b-1,\omega)-V_s)+\gamma\,(V_s^e-V_s)\;.$$

   It is then clear that $\frac{\tilde{\mathbb{E}}[V_{s,\Delta}]-V_s}{\Delta} \simeq r\,V_s+O(\Delta)$ as $\Delta\to 0$. Furthermore, it is easy to show (see the proof of Proposition 4.2) that $V_s(N_s-1,N_b-1,\omega)-V_s = \frac{N_b\,N_s}{N_s-N_b}\,\frac{r}{\delta} > 0$, and so we have $\frac{\mathbb{E}[V_{s+\Delta}]-V_s}{\Delta} \succeq r\,V_s+O(\Delta)$ as $\Delta\to 0$.

3. Assume finally that the state (N_s, N_b, ω) is such that $\delta=+\infty$. In this case, we have

   $$V_s = V_s(N_s-1, N_b-1, \tilde{\omega})\;.$$

   We consider two subcases:

[21] Consistently with footnote 11, the equality "$\simeq$" in equation (A.8) should be read as "For any sequence $(k_n)_n$ and corresponding sequence of equilibria, and for any state $(N_{s,t}, N_{b,t}, \omega_t)$, the function $\Delta \mapsto R(\Delta) \equiv \limsup_{n\to\infty}\big(\frac{\tilde{\mathbb{E}}_t[V_{\theta,n,t+\Delta}]-V_{\theta,n,t}}{\Delta}-r\,V_{\theta,n,t}\big)$ is such that $\lim_{\Delta\searrow 0} R(\Delta)=0$."

[22] Note that, since $(N_{s,0}, N_{b,0}, \omega_0)=(N_s, N_b, \omega)$, we have $V_s=V_{s,0}$.

(a) Assume first that $N_\mathrm{b} = 1$. This implies that, after a transaction, the new state has zero buyers, and then the analysis of part 1 above applies. We then have

$$r\,\overbrace{V_\mathrm{s}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)}^{=V_\mathrm{s}} = \frac{\overbrace{\mathbb{E}[V_{\mathrm{s},\Delta}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)]}^{=\mathbb{E}[V_{\mathrm{s},\Delta}]} - \overbrace{V_\mathrm{s}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)}^{=V_\mathrm{s}}}{\Delta} + O(\Delta)\,.$$

The same argument applies by replacing $\mathbb{E}$ with $\tilde{\mathbb{E}}$.

(b) Now, we proceed by induction. For each $N_\mathrm{b} > 1$, we assume that equation (A.8) holds for all states with lower number of buyers. Hence, either $\delta(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega) = +\infty$ (and the result applies then to state $(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)$ by part 2 of this proof), or $\delta(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \tilde{\omega}) < +\infty$, so the induction hypothesis dictates that the result holds. Then, we have

$$r\,V_\mathrm{s}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega) \preceq \frac{\mathbb{E}[V_{\mathrm{s},\Delta}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)] - V_\mathrm{s}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)}{\Delta} + O(\Delta)\,.$$

The previous inequality holds with equality "$\simeq$" if the expectation operator $\mathbb{E}$ is replaced by $\tilde{\mathbb{E}}$. □

Proof of Proposition 4.2

Proof. We assume that Condition 1 holds. For each state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$, we use $V_{\mathrm{s},n}$ and $V_{\mathrm{b},n}$ to denote the continuation values of sellers and buyers in the n-th equilibrium, respectively, and α_n to denote the probability of acceptance of an equilibrium offer.

As in the proof Proposition 4.1 we assume, taking a subsequence if necessary, that for each state $(N_\mathrm{s}, N_\mathrm{b}, \omega)$, the continuation values $V_{\mathrm{b},n}$ and $V_{\mathrm{s},n}$ converge to some values $V_\mathrm{b} = V_\mathrm{b}(N_\mathrm{s}, N_\mathrm{b}, \omega)$ and $V_\mathrm{s} = V_\mathrm{s}(N_\mathrm{s}, N_\mathrm{b}, \omega)$, and the transaction rate $\delta_n \equiv \alpha_n\, k_n\, \ell$ converges to some $\delta = \delta(N_\mathrm{s}, N_\mathrm{b}, \omega) \in [0, +\infty]$.

Equations for V_s: We first focus on characterizing the limit continuation value of seller, V_s for states $(N_\mathrm{s}, N_\mathrm{b}, \omega)$ such that $N_\mathrm{s} > N_\mathrm{b} \geq 1$. The equations are given by:

1. Consider first the case where $N_\mathrm{s} > N_\mathrm{b} \geq 1$ and $\delta < +\infty$. Using equation (3.1) for both $\theta = \mathrm{s}, \mathrm{b}$, and using the fact that $V_{\mathrm{s},n} + V_{\mathrm{b},n} = 1$ if n is big enough, we can write

$$\begin{aligned} V_{\mathrm{s},n} &= \frac{\frac{N_\mathrm{s}-1}{N_\mathrm{s}}\,\delta_n}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}}\,\delta_n + \gamma + r}\,\big(V_{\mathrm{s},n}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)\big) + \frac{\gamma}{\frac{N_\mathrm{s}-1}{N_\mathrm{s}}\,\delta_n + \gamma + r}\,V^\mathrm{e}_{\mathrm{s},n} \\ &= 1 - \frac{r}{\frac{N_\mathrm{b}-1}{N_\mathrm{b}}\,\delta_n + \gamma + r} + \frac{\frac{N_\mathrm{b}-1}{N_\mathrm{b}}\,\delta_n}{\frac{N_\mathrm{b}-1}{N_\mathrm{b}}\,\delta_n + \gamma + r}\,\big(1 - V_{\mathrm{b},n}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega)\big) + \frac{\gamma}{\frac{N_\mathrm{b}-1}{N_\mathrm{b}}\,\delta_n + \gamma + r}\,\big(1 - V^\mathrm{e}_{\mathrm{s},n}\big)\,. \end{aligned}$$

Using Lemma 4.1 we have that $V_\mathrm{b} = 1 - V_\mathrm{s}$ for all states (given that V_s and V_b are the limits of $V_{\mathrm{s},n}$ and $V_{\mathrm{b},n}$ as $n \to \infty$, respectively, we can replace "$\simeq$" by "$=$"). Therefore, the previous two equalities can be rewritten in the limit, as

$$V_\mathrm{s} = V_\mathrm{s}(N_\mathrm{s}{-}1, N_\mathrm{b}{-}1, \omega) - \frac{N_\mathrm{b}\,N_\mathrm{s}}{N_\mathrm{s}-N_\mathrm{b}}\,\frac{r}{\delta} = \frac{\gamma}{\gamma+r}\,V^\mathrm{e}_\mathrm{s} + \frac{r}{\gamma+r}\,\frac{(N_\mathrm{s}-1)\,N_\mathrm{b}}{N_\mathrm{s}-N_\mathrm{b}}\,. \tag{A.9}$$

It is then clear that there is no state where $\delta = 0$; that is, there is no state where trade occurs at a rate that becomes arbitrarily small as k increases (the logic for this result is analogous to part 1 of Proposition 3.2). Note that the second equality of equation (A.9) implies that, as indicated in part 3 of Proposition 3.2, traders on the long side of the market gain from other's transactions in states where is trade delay; i.e., when $V_s(N_s{-}1, N_b{-}1, \omega) > V_s$ in our case (since $N_s > N_b$).

2. Consider now the case where $N_s > N_b \geq 1$ and $\delta = +\infty$. In this case, equation (4.1) implies that

$$V_s = V_s(N_s{-}1, N_b{-}1, \omega) \; . \tag{A.10}$$

Proof that of $V_s = V_s(N_s{-}1, N_b{-}1, \omega)$ whenever $N_s > N_b \geq 1$: For each state (N_s, N_b, ω) with $N_s > N_b \geq 1$, we define $\Delta \equiv V_s(N_s{-}1, N_b{-}1, \omega) - V_s$, and $\Delta = 0$ for each state (N_s, N_b, ω) with $N_s = N_b > 0$. Since, from equations (A.9) and (A.10) it follows that $V_s(N_s{-}1, N_b{-}1, \omega) \geq V_s$, it is the case that $\Delta \geq 0$ for all states with $N_s \geq N_b$. Let (N_s, N_b, ω) be a state which maximizes $\Delta(N'_s, N'_b, \omega')$ among all states with $N'_s \geq N'_b \geq 1$ and assume, for the sake of contradiction, that $\Delta > 0$ (so necessarily $N_s > N_b > 0$). If there are multiple states with this property, assume that (N_s, N_b, ω) is such that N_s is minimal among all of them.

Assume first that (N_s, N_b, ω) is such that $\delta = +\infty$. In this case, $V_s(N_s{-}1, N_b{-}1, \omega) = V_s$ (from equation (A.10)), hence $\Delta = 0$, a contradiction. Then, it is necessarily the case that (N_s, N_b, ω) is such that $\delta < +\infty$. In this case, using equation (A.9), we have that

$$\begin{aligned}\Delta = {} & \tfrac{\hat{\gamma}_s}{\hat{\gamma}+r} V_s(N_s, N_b{-}1, \omega) + \tfrac{\hat{\gamma}_b}{\hat{\gamma}+r} V_s(N_s{-}1, N_b, \omega) + \tfrac{\hat{\gamma}_c}{\hat{\gamma}+r} \mathbb{E}^c[V_s(N_s{-}1, N_b{-}1, \tilde{\omega})] \\ & - \Big(\tfrac{\gamma_s}{\gamma+r} V_s(N_s{+}1, N_b, \omega) + \tfrac{\gamma_b}{\gamma+r} V_s(N_s, N_b{+}1, \omega) + \tfrac{\gamma_c}{\gamma+r} \mathbb{E}^c[V_s(N_s, N_b, \tilde{\omega})] + \tfrac{r}{\gamma+r} \tfrac{N_b\,(N_s-1)}{N_s-N_b} \Big) \; ,\end{aligned}$$

where the variables with a hat are evaluated at the state $(N_s{-}1, N_b{-}1, \omega)$. After some algebra, we obtain that the right-hand side of the previous equation is weakly smaller than

$$\begin{aligned}& \tfrac{\gamma}{\gamma+r} \Delta \overbrace{- \tfrac{r}{\gamma+r} \tfrac{N_b\,(N_s-1)}{N_s-N_b}}^{\equiv(*)} \\ & + \underbrace{\Big(\tfrac{\hat{\gamma}_s}{\hat{\gamma}+r} - \tfrac{\gamma_s}{\gamma+r}\Big) V_s(N_s{+}1, N_b, \omega) + \Big(\tfrac{\hat{\gamma}_b}{\hat{\gamma}+r} - \tfrac{\gamma_b}{\gamma+r}\Big) V_s(N_s, N_b{+}1, \omega) + \Big(\tfrac{\hat{\gamma}_c}{\hat{\gamma}+r} - \tfrac{\gamma_c}{\gamma+r}\Big) \mathbb{E}^c[V_s(N_s, N_b, \tilde{\omega})]}_{\equiv(**)} \; ,\end{aligned}$$

We will reach a contradiction (and then rule out that $\Delta > 0$) if the summation of the terms $(*)$ and $(**)$ in the previous equation is non-positive. (Indeed, in this case, $\Delta \leq \frac{\gamma}{\gamma+r} \Delta$, which is not possible if $\Delta > 0$.) Note first that

$$(**) \leq \max\Big\{\tfrac{\hat{\gamma}_s}{\hat{\gamma}+r} - \tfrac{\gamma_s}{\gamma+r}, 0\Big\} + \max\Big\{\tfrac{\hat{\gamma}_b}{\hat{\gamma}+r} - \tfrac{\gamma_b}{\gamma+r}, 0\Big\} + \max\Big\{\tfrac{\hat{\gamma}_c}{\hat{\gamma}+r} - \tfrac{\gamma_c}{\gamma+r}, 0\Big\} \leq \tfrac{r}{\gamma+r} \; ,$$

where the last inequality holds because of Condition 1(b). Therefore,

$$(*) + (**) \leq -\tfrac{r}{\gamma+r} \tfrac{N_b\,(N_s-1)}{N_s-N_b} + \tfrac{r}{\gamma+r} \leq 0 \; ,$$

and so the result holds.

Proof of no delay: We have shown that $V_s = V_s(N_s - 1, N_b - 1, \omega)$ for all (N_s, N_b, ω) satisfying $N_s > N_b \geq 1$. Equation (A.9) cannot be satisfied for $\delta < +\infty$, and therefore it must be that $\delta = +\infty$. In words, there is no asymptotic delay in all states (N_s, N_b, ω) with $N_s > N_b \geq 1$. Symmetric arguments can be used for states where $N_b > N_s \geq 1$. Since, by Proposition 3.2, there is no delay when the market is balanced, we conclude that there is no asymptotic delay in any state.

Properties of p: We now prove that p satisfying the conditions in the statement exists. We use $p(\cdot,\cdot)$ to denote the solution of equations

$$p(N,\omega) = \tfrac{\gamma_s}{\gamma+r}\, p(N+1,\omega) + \tfrac{\gamma_b}{\gamma+r}\, p(N-1,\omega) + \tfrac{\gamma_c}{\gamma+r}\, \mathbb{E}^c[p(N,\tilde{\omega})] \qquad \text{if } N>0\,, \tag{A.11}$$

$$p(N,\omega) = \tfrac{r}{\gamma+r} + \tfrac{\gamma_s}{\gamma+r}\, p(N+1,\omega) + \tfrac{\gamma_b}{\gamma+r}\, p(N-1,\omega) + \tfrac{\gamma_c}{\gamma+r}\, \mathbb{E}^c[p(N,\tilde{\omega})] \qquad \text{if } N<0\,, \tag{A.12}$$

$$p(0,\omega) = \tfrac{r}{\gamma(1,1,\omega)+r}\, \xi + \tfrac{\gamma_s(1,1,\omega)}{\gamma(1,1,\omega)+r}\, p(1,\omega) + \tfrac{\gamma_b(1,1,\omega)}{\gamma(1,1,\omega)+r}\, p(-1,\omega) + \tfrac{\gamma_c(1,1,\omega)}{\gamma(1,1,\omega)+r}\, \mathbb{E}^c[p(0,\tilde{\omega})]\,, \tag{A.13}$$

where the arrival rates are evaluated at state $(N,0,\omega)$ when $N > 0$ and at state $(0,-N,\omega)$ when $N < 0$. The function $p(\cdot,\cdot)$ can be proven to be unique using standard fixed-point arguments similar to those in Section A.1. These equations are equivalent to equations (4.4)–(4.6) (as $n \to \infty$) replacing $V_s(N_s, N_b, \omega)$ by $p(N_s - N_b, \omega)$, so it is clear that $V_s(N_s, N_b, \omega) \simeq p(N_s - N_b, \omega)$ for all states (N_s, N_b, ω).

□

Proof of Proposition 4.3

Proof. Assume Conditions 1 and 2 hold and the market is stationary. Since there is only one value ω for the economic conditions, it is convenient to use $p(\cdot)$ to denote $p(\cdot,\omega)$ defined in Proposition 4.2.

Proof of the first result: For each $\bar{N} \geq 0$, one can rewrite equation (4.7) (recall that, under Condition 1, equation (4.7) holds also for the equilibrium measure) for all $N \geq \bar{N}$ as

$$p(N) = \mathbb{E}[e^{-r\,\bar{\tau}}\, p(\bar{N}) | N_0 = N]\,,$$

where $\bar{\tau}$ is the stochastic time it takes the net supply to reach $\bar{N}$ for the first time. Intuitively, for a fixed net supply $\bar{N} \geq 0$ and a state with more excess supply $N \geq 0$, a seller is indifferent to waiting to trade until the excess supply reaches $\bar{N}$.[23] It is then clear, using for example $\bar{N} = 0$, that $p(\cdot)$ is decreasing on $\{-\bar{N}_b, \ldots, \bar{N}_s\}$.

[23]In fact, as $n \to \infty$, a seller is indifferent between waiting or trading at any state with excess supply $N > 0$. Since the excess supply's changes are of size one, a seller's payoff when the excess supply is $N \geq \bar{N} \geq 0$ can be computed as her payoff if she does not trade until the excess supply is $\bar{N}$ and then she trades immediately.

Proof of the second result: Consider now an increase on the discount rate from r_1 to r_2, with $r_1 < r_2$, and let $p^{r_i}(\cdot)$ denote the market price function for each r_i, $i = 1,2$. Assume that $p^{r_1}(0) \geq p^{r_2}(0)$ (the reverse case is analogous). In this case, for all $N > 0$ we have $p^{r_1}(N) > p^{r_2}(N)$. Indeed, using τ_0 to denote the (stochastic) time it takes for the market to become balanced (which is independent of r) and using equation (4.7) (recall again that, under Condition 1, equation (4.7) holds also for the equilibrium measure), we can write

$$p^{r_1}(N) = \mathbb{E}\big[\, e^{-r_1\,\tau_0} \,\big|\, N_0 = N \big]\, p^{r_1}(0) > \mathbb{E}\big[\, e^{-r_2\,\tau_0} \,\big|\, N_0 = N \big]\, p^{r_2}(0) = p^{r_2}(N) \ . \tag{A.14}$$

Let $\bar{N}$ be the maximum value satisfying $p^{r_1}(\bar{N}) < p^{r_2}(\bar{N})$. Notice that equation (4.7) can be rewritten, for any $N \leq \bar{N} < 0$ and $i \in \{1,2\}$, as

$$p^{r_i}(N) = 1 - \mathbb{E}\big[\, e^{-r_i\,\bar{\tau}} \,\big|\, N_0 = N \big]\,(1 - p^{r_i}(\bar{N})) \ ,$$

where $\bar{\tau}$ is the first time where $N_t = \bar{N}$. It is then clear, using equation (A.14) and $p^{r_1}(\bar{N}) < p^{r_2}(\bar{N})$, that for all $N \leq \bar{N}$ we have $p^{r_1}(N) < p^{r_2}(N)$. Thus, in fact, $\bar{N}$ is such that

$$p^{r_1}(N) \geq p^{r_2}(N) \text{ for all } N > \bar{N} \text{ and } p^{r_1}(N) < p^{r_2}(N) \text{ for all } N \leq \bar{N} \ .$$

This property (and the fact that the ergodic distribution of N is independent of the discount rate) ensures that the ergodic distribution of $p^{r_2}(N)$ is a spread of $p^{r_1}(N)$.

Proof of the third result: Assume finally that $\gamma_{\mathrm{b}}(0,0,\omega) = \gamma_{\mathrm{b}}(1,1,\omega)$ and $\gamma_{\mathrm{s}}(0,0,\omega) = \gamma_{\mathrm{s}}(1,1,\omega)$. In this case, the limit ergodic distributions of N under both the equilibrium and the risk-neutral measures coincide. Let F be such a distribution. Then, the expected price under such a distribution is

$$\mathbb{E}[p(\tilde{N})|F] = \sum_{\tilde{N} \in \mathbb{Z}} F(\{\tilde{N}\})\, p(N) \ .$$

It is also the case that

$$\begin{aligned} \lim_{t\to\infty} \mathbb{E}[p_t] = \mathbb{E}[p(\tilde{N})|F] &= \lim_{t\to\infty} \mathbb{E}\Big[\int_t^{\infty} e^{-r\,(s-t)}\, (\mathbb{I}_{N_s<0} + \xi\, \mathbb{I}_{N_s=0})\, r\, \mathrm{d}s \,\Big] \\ &= \mathbb{E}\big[I_{\tilde{N}<0} + \xi\, \mathbb{I}_{\tilde{N}=0} \big| F \big] = F(-\mathbb{N}) + \xi\, F(\{0\}) \ , \end{aligned}$$

where $-\mathbb{N}$ is the set of strictly negative integers. This proves that the ergodic mean of the market price is independent of r. □

Proof of Proposition 4.4

Proof. **Proof of the first result.** Similar to the proof of Proposition 4.1, now fix a sequence $(k_n, M_n)_n$ tending to $(+\infty, +\infty)$ and, for each n, an equilibrium of the model where $\lambda = k_n\,\ell$ and $(\gamma_{\mathrm{b},n}, \gamma_{\mathrm{s},n}) =$

$M_n(\tilde{\gamma}_\text{b}, \tilde{\gamma}_\text{s})$. For each state $(N_\text{s}, N_\text{b}, \omega)$ with $N_\text{s}, N_\text{b} > 0$, we use $\tilde{\delta}_n(N_\text{s}, N_\text{b}, \omega)$ the trade rate at such state in the n-th equilibrium under the risk-neutral measure as in equation (4.2). Taking a subsequence if necessary, assume that $(\tilde{\delta}_n(N_\text{s}, N_\text{b}, \omega))_n$ tends to some limit $\tilde{\delta}(N_\text{s}, N_\text{b}, \omega) \in \mathbb{R}_+ \cup \{+\infty\}$ for all states $(N_\text{s}, N_\text{b}, \omega)$.

As in the main text, we use τ_0 to denote the stopping time until the market is balanced. Furthermore, for each state $(N_\text{s}, N_\text{b}, \omega)$ and n, we define[24]

$$\phi_n(N_\text{s}, N_\text{b}, \omega) \equiv 1 - \tilde{\mathbb{E}}_n\left[e^{-r\,\tau_0} \middle| (N_{\text{s},0}, N_{\text{b},0}, \omega_0) = (N_\text{s}, N_\text{b}, \omega)\right] \in [0,1] , \tag{A.15}$$

where the expectation is computed using the risk-neutral measure of the n-th equilibrium. Note that if $N_\text{s} = N_\text{b}$ then $\phi_n(N_\text{s}, N_\text{b}, \omega) = 0$. Also, it satisfies the equation

$$\begin{aligned} \phi_n(N_\text{s}, N_\text{b}, \omega) = {} & \tfrac{r}{\tilde{\delta}_n+\gamma+r} + \tfrac{\tilde{\delta}_n}{\tilde{\delta}_n+\gamma_n+r}\, \phi_n(N_\text{s}-1, N_\text{b}-1, \omega) \\ & + \tfrac{\gamma_{\text{s},n}}{\tilde{\delta}_n+\gamma_n+r}\, \phi_n(N_\text{s}+1, N_\text{b}, \omega) + \tfrac{\gamma_{\text{b},n}}{\tilde{\delta}_n+\gamma_n+r}\, \phi_n(N_\text{s}, N_\text{b}+1, \omega) + \tfrac{\gamma_\omega}{\tilde{\delta}_n+\gamma_n+r}\, \mathbb{E}^\text{c}[\phi_n(N_\text{s}, N_\text{b}, \tilde{\omega})] , \end{aligned} \tag{A.16}$$

where $\tilde{\delta}_n$ should be replaced with 0 when $N_\text{s} = 0$ or $N_\text{b} = 0$.

Take a subsequence of our original sequence satisfying that, for all states of the world $(N'_\text{s}, N'_\text{b}, \omega')$, $(\phi_n(N'_\text{s}, N'_\text{b}, \omega'))_n$ converges to some $\phi(N'_\text{s}, N'_\text{b}, \omega')$. Assume, for the sake of contradiction, that $\bar{\phi} \equiv \max_{(N'_\text{s}, N'_\text{b}, \omega')} \phi(N'_\text{s}, N'_\text{b}, \omega') > 0$. Assume $\phi(N'_\text{s}, N'_\text{b}, \omega') = \bar{\phi}$ for some state $(N'_\text{s}, N'_\text{b}, \omega')$ with $N'_\text{s} > N'_\text{b}$ (the other case is analogous). Let $(N_\text{s}, N_\text{b}, \omega)$ be such that $N_\text{s} > N_\text{b}$ and satisfy that, for all other states $(N'_\text{s}, N'_\text{b}, \omega')$ with $N'_\text{s} > N'_\text{b}$ and $\phi(N'_\text{s}, N'_\text{b}, \omega') = \bar{\phi}$, (i) $N'_\text{s} - N'_\text{b} \geq N_\text{s} - N_\text{b}$, and that (ii) if $N'_\text{s} - N'_\text{b} = N_\text{s} - N_\text{b}$ then $N'_\text{s} > N_\text{s}$. Thus, $(N_\text{s}, N_\text{b}, \omega)$ has a minimal excess supply among all states with maximal ϕ and, among those with the lowest excess supply, has a minimal number of sellers. If $N_\text{b} = 0$ then we can write (A.16) as

$$\begin{aligned} \overbrace{\phi_n(N_\text{s}, N_\text{b}, \omega)}^{\to\bar{\phi}} = {} & \overbrace{\tfrac{r}{\gamma_n}\,(1-\phi_n(N_\text{s}, N_\text{b}, \omega))}^{\to 0} \\ & + \tfrac{\gamma_{\text{s},n}}{\gamma_n}\, \underbrace{\phi_n(N_\text{s}+1, N_\text{b}, \tilde{\omega})}_{\to\leq\bar{\phi}} + \underbrace{\tfrac{\gamma_{\text{b},n}}{\gamma}}_{>0}\, \underbrace{\phi_n(N_\text{s}, N_\text{b}+1, \tilde{\omega})}_{\to<\bar{\phi}} + \tfrac{\gamma_\text{c}}{\gamma_n}\, \underbrace{\mathbb{E}^\text{c}[\phi_n(N_\text{s}+1, N_\text{b}, \tilde{\omega})]}_{\to\leq\bar{\phi}} , \end{aligned}$$

where we used that $\tilde{\gamma}_\text{b} > 0$ because $N_\text{b} < N_\text{s}$ (by Condition 2) and that $N_\text{s} - (N_\text{b}+1) < N_\text{s} - N_\text{b}$. This is a clear contradiction; hence, it must be that $N_\text{b} > 0$. Assume, then, that $N_\text{b} > 0$. In this case, since

[24] In equation (A.15), the expectation operator on the right-hand side averages the discounting of all times it takes for the market to become balanced if the initial state $(N_{\text{s},0}, N_{\text{b},0}, \omega_0)$ is equal to $(N_\text{s}, N_\text{b}, \omega)$.

$(N_s - 1) - (N_b - 1) = N_s - N_b$ we have $\phi(N_s - 1, N_b - 1, \omega) < \bar{\phi}$. We can write (A.16) as

$$\overbrace{\phi(N_s, N_b, \omega)}^{\to \bar{\phi}} = \overbrace{\tfrac{r}{\tilde{\delta}_n + \gamma_n} (1 - \phi(N_s, N_b, \omega))}^{\to 0} + \tfrac{\tilde{\delta}_n}{\tilde{\delta}_n + \gamma_n} \overbrace{\phi_n(N_s - 1, N_b - 1, \omega)}^{\to < \bar{\phi}}$$
$$+ \tfrac{\gamma_{s,n}}{\tilde{\delta}_n + \gamma_n} \underbrace{\phi_n(N_s + 1, N_b, \omega)}_{\to \le \bar{\phi}} + \tfrac{\gamma_{b,n}}{\tilde{\delta}_n + \gamma_n} \underbrace{\phi_n(N_s, N_b + 1, \omega)}_{\to < \bar{\phi}} + \tfrac{\gamma_c}{\tilde{\delta}_n + \gamma_n} \underbrace{\mathbb{E}^c[\phi_n(N_s, N_b, \tilde{\omega})]}_{\to < \bar{\phi}} .$$

Given that the $\tilde{\delta}_n$ cannot tend to 0 (see the proof of Corollary 4.1), the limit of the right-hand side of the previous expression as $n \to \infty$ is strictly smaller than $\bar{\phi}$. It is then clear that we reach, again, a contradiction.

The previous argument implies that $\phi(N_s, N_b, \omega) = 0$ for all states (N_s, N_b, ω); that is, the discounting until the market balances (under the risk-neutral measure) is 1. Using equation (4.7), we have then that $\lim_{n\to\infty} V_{s,n}(N_s, N_b, \omega) = \lim_{n\to\infty} V_{s,n}(1, 1, \omega)$ for all states (N_s, N_b, ω). Hence, we have $\lim_{n\to\infty} \frac{r}{\tilde{\delta}_n} = 0$ for all states. This implies that delay vanishes as $n \to 0$ also under the equilibrium measure.

Proof of the second result. We want to prove that there is a well-defined $p^*(\omega)$ for each $\omega \in \Omega$. By the first part of this proof, the trade rate under the risk-neutral measure $\tilde{\delta}_n$ is such that $\lim_{n\to\infty} \frac{r}{\tilde{\delta}_n} = 0$ for all states. Hence, the discounted time it takes for the distribution of the market composition to approximate the ergodic distribution of the market composition for a given initial ω_0 shrinks to 0. It is then clear that equation (4.8) follows from equation (4.3). Note that when the market is stationary, we have that, by Proposition 4.1, $p^*(\omega_0)$ is equal to the probability that $N_s < N_b$ under the ergodic distribution plus ξ multiplied by the probability that $N_s = N_b$ under the ergodic distribution. □

B An example with trade delay

This section sheds light on how to trade delay may arise in equilibrium.

Instead of describing the whole model, we focus on a particular state and presume some continuation values in neighboring states. It is not difficult to construct a full model where the same arguments we use here apply. We focus on one state of the world $(N_s, N_b, \omega) = (2, 1, \omega)$, that is, a state with two sellers and one buyer in the market, and where the economic conditions are equal to some $\omega \in \Omega$. We set, for simplicity, $\gamma_s > \gamma_b = \gamma_c = 0$ (all rates are evaluated at $(2, 1, \omega)$). If a seller arrives, the buyer has a high continuation payoff, assumed to be 1, and the sellers obtain 0. If instead, a transaction occurs before the arrival of a seller, the remaining seller obtains a high continuation payoff, which is assumed to be equal to 1.[25]

[25]The continuation values after arrivals are assumed to be 0 or 1 to simplify the analysis, and can be approximated in a

We first compute the continuation values of the sellers and the buyer under the assumption that, in each match, the price offer is equal to the continuation value of the trader receiving the offer, and that such an offer is accepted for sure (i.e., equations (3.1)–(3.5) hold with $\alpha = 1$). The continuation values in state $(2,1,\omega)$ solve the following system of equations:

$$\begin{aligned} V_s &= \overbrace{\tfrac{\lambda/2}{\lambda+\gamma+r}\big(\xi(1-V_b)+(1-\xi)V_s\big)}^{\text{own match}} + \overbrace{\tfrac{\lambda/2}{\lambda+\gamma+r}\,1}^{\text{others' match}} + \overbrace{\tfrac{\gamma}{\lambda+\gamma+r}\,0}^{\text{exog. change}} ,\\ V_b &= \underbrace{\tfrac{\lambda}{\lambda+\gamma+r}\big(\xi V_b+(1-\xi)(1-V_s)\big)}_{\text{own match}} + \underbrace{\tfrac{0}{\lambda+\gamma+r}\,0}_{\text{others' match}} + \underbrace{\tfrac{\gamma}{\lambda+\gamma+r}\,1}_{\text{exog. change}} . \end{aligned}$$

Solving the previous system of equations, we obtain

$$V_s + V_b = 1 + \tfrac{\gamma\,(\lambda-2\,r)-2\,r^2}{(\gamma+\lambda+r)\,(2\,\gamma+(1-\xi)\,\lambda+2\,r)} ,$$

which is strictly higher than 1 if λ is large or r is small. As we see, a seller's value if she decides not to trade at state $(2,1,\omega)$ is $\frac{\lambda/2}{\lambda/2+\gamma+r}$, so she is not willing to trade at a price lower than this value. Also, the buyer obtains a continuation value equal to $\frac{\gamma}{\gamma+r}$ from not trading, so he is not willing to trade at a price higher than $\frac{r}{\gamma+r} = 1-\frac{\gamma}{\gamma+r}$. As a result, if either λ is large or r is small, any equilibrium in this reduced version of our model has the property that offers are rejected with positive probability. Using α to denote the probability of agreement in a match in state $(2,1,\omega)$ (which ensures that $V_s+V_b=1$), we have

$$\alpha = \min\left\{1, \tfrac{2\,r\,(\gamma+r)}{\gamma\,\lambda}\right\} .$$

Notice that the rate at which an agreement occurs in state $(2,1,\omega)$ (which equals $\alpha\,\lambda$) converges to $\frac{2\,r\,(\gamma+r)}{\gamma}$ as λ becomes big; that is, a significant trade delay remains even in the limit where matching frictions disappear.

To obtain further intuition, observe that the weight of a (own or other's) match in both the sellers' and the buyer's payoffs is the same, equal to $\frac{\lambda}{\lambda+\gamma+r}$. Nevertheless, conditional on a match occurring, each seller is chosen with probability $\frac{1}{2}$, and therefore she obtains $\frac{1}{2}\,V_s^m + \frac{1}{2}\,V_s^o$; while the buyer is chosen for sure and he obtains V_b^m. Hence, even if $V^m \leq 1$, the sum of the payoffs of a seller and a buyer conditional on a match may be bigger than 1 if sellers benefit from other's transactions (note that $\frac{1}{2}\,V_s^o + \frac{1}{2}\,V_b^m$ may be bigger than $\frac{1}{2}$). Then, equilibrium trade delay occurs when traders on the long side of the market benefit when other traders trade, as they assign a larger probability that other traders trade than the traders on the short side of the market.

full version of our model. For instance, a high continuation payoff for the remaining seller after a transaction is obtained if the arrival of buyers is high after the transaction. Analogously, a high continuation value of the buyers when a seller arrives follows from the fact that there are more sellers than buyers in the market even if the other buyer arrives. Such changes in the market can be encoded in changes in the market conditions ω.

In the example, equilibrium offers are rejected with a positive probability. Trade delay occurs, even when matching frictions are small, despite the fact that the sellers are identical and there is no information asymmetry. The equilibrium behavior of the sellers in the market resembles a war of attrition: each one trades at the rate that makes the other seller (and buyer) indifferent regarding whether to trade or not. From each seller's perspective, this delay lowers the value of making unacceptable offers because doing so comes with the risk of another seller arriving. As time passes, either one of the sellers trades (and the remaining seller obtains a high payoff), or another seller arrives (and all sellers obtain a low continuation payoff).

Our example illustrates that trade delay may occur when transactions affect the future arrival rates of traders into the market. This may be the case when a house sold to a celebrity in an unfashionable neighborhood attracts other celebrities, leading to gentrification. Similarly, in a local labor market, the absence of specialized workers (teachers, doctors, etc.) may lead the local government to initiate a subsidy program (free housing, lower taxes, etc.) to increase the arrival of workers into the market. Employers may then have the incentive to let other employers hire the current workers in the market and wait to hire until the subsidy program is in place. More generally, transactions may make some markets more visible to otherwise inattentive traders, increase the value of the goods being sold in the market (see the discussion on time-varying gains from trade in Section 5.1), or may trigger actions by third parties (regulators, market makers, etc.).

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