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Matthias Kaldorf Florian Wicknig

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Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements *

MATTHIAS KALDORF† FLORIAN WICKNIG‡

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Abstract

How does the eligibility of corporate sector assets as collateral affect collateral supply and risk-taking by the corporate sector? Since banks are willing to pay collateral premia on eligible assets, this makes debt financing cheaper for all firms satisfying eligibility requirements, which are best thought of minimum ratings. We provide a novel analytical characterization of heterogeneous firm responses to collateral easing, i.e., relaxing eligibility requirements. While high-quality firms respond by increasing their debt issuance, some low-quality firms are incentivized to reduce their debt outstanding to benefit from collateral premia. If risk-taking effects are sufficiently large, firm responses increase the resources losses from corporate default. Applying the model to the ECB's collateral easing policy during the 2008 financial crisis, our results suggest that firm responses introduce a central bank trade-off between collateral supply and resource losses of default. Our analysis suggests that a *covenant* conditioning eligibility on debt outstanding *and* current default risk is a powerful instrument to mitigate the adverse impact of collateral premia on default risk while at the same time maintaining a high level of collateral supply.

Keywords: Collateral Premia, Eligibility Requirements, Firm Heterogeneity, Corporate Default Risk, Collateral Policy

JEL Classification: E44, E58, G12, G32

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†University of Cologne, Center for Macroeconomic Research, Albertus-Magnus-Platz, 50923 Cologne, Germany. Email: kaldorf@wiso.uni-koeln.de (Corresponding Author).

‡University of Cologne, Center for Macroeconomic Research. Email: fwicknig@wiso.uni-koeln.de.

1 Introduction

Central banks implement monetary policy by lending to banks against collateral, which makes a sufficiently high supply of collateral essential to the functioning of the financial system. During the financial crisis of 2008, this restriction required many central banks to expand the pool of assets they accept as collateral to facilitate the conduct of expansionary monetary policy. For example, the European Central Bank (ECB) engaged in collateral easing when switching towards a full allotment regime in its Main Refinancing Operations and prior to introducing Long-Term Refinancing Operations. To expand the pool of collateral, the ECB added corporate sector assets of intermediate quality, such as BBB-rated corporate bonds and securitized bank loans, to the list of eligible assets.[1] The inclusion of corporate sector assets is quantitatively relevant: corporate bonds and credit claims make up 27% of collateral in ECB operations.[2]

While collateral easing facilitates the smooth conduct of monetary policy, a thorough assessment of this policy must also account for endogenous responses of the corporate sector. Firm responses arise, since banks increase demand for assets if they become eligible as collateral and firms cater to this demand by increasing their debt issuance and indebtedness (Mésonnier et al., 2022; Pelizzon et al., 2020; Mota, 2021). The positive response of debt issuance and indebtedness is particularly pronounced for high-rated borrowers (Grosse-Rueschkamp et al., 2019). While debt supply effects are desirable for monetary policy implementation, higher indebtedness of firms also increases their risk of default, which can also limit the efficacy of collateral easing, if higher indebtedness is associated with rating downgrades in future periods. This paper presents a novel theoretical framework to study endogenous firm responses to eligibility requirements in the presence of default risk. While this framework can be applied to many situations where eligibility is specified in a discontinuous way through minimum rating requirements, we present an application to the ECB's collateral easing policy of 2008.

We study endogenous firm responses to eligibility requirements through the lens of a model with heterogeneous firms that issue risky debt securities (corporate bonds) to banks.[3] Firms are subject to idiosyncratic revenue shocks and have an incentive to issue bonds, because they are more impatient than their creditors. Firms default on their bonds if revenues fall short of current repayment obligations, in which case all current revenues are wasted. Thus, bond issuance is

[1] See Wolff (2014), Heider et al. (2015), Nyborg (2017), and Blot and Hubert (2018) for a discussion on the collateral eligibility of risky private sector assets and the monetary policy implementation by the ECB. We show the collateral treatment of corporate sector assets by different central banks in Appendix A.

[2] As of 2020Q4, corporate bonds are the second largest asset class accepted as collateral by the ECB with a market value of EUR 1871 billion. This is only exceeded by government bonds (see European Central Bank, 2022).

[3] We refer to these securities as corporate bonds, even though they can be interpreted as securitized bank loans or other marketable corporate sector assets, which are also eligible in many central bank collateral frameworks.

determined by a trade-off between relative impatience and expected default costs. Bonds are held and priced by banks. We assume that banks value these bonds if they can be used to collateralize borrowing from the central bank. Consistent with actual central bank practice, only sufficiently safe bonds are eligible as collateral and the central bank sets the minimum quality (rating) requirement as a policy instrument. The dual role of bonds as investment object and collateral implies that spreads on *eligible* bonds contain a fundamental component and a collateral premium that, ceteris paribus, shifts the pricing schedule outwards in a discontinuous way.

As our first contribution, we provide a characterization of firm responses to collateral eligibility in a model with discontinuous demand for corporate bonds. We obtain analytical solutions in a setting with one period bonds, i.i.d. revenue shocks, and permanent differences in firm productivity. Making corporate bonds eligible affects the firm's borrowing decision in a discontinuous way. The discussion of firm responses is organized around a key firm characteristic, the *eligible debt capacity*, defined as the maximum amount of bonds a firm can issue without losing eligibility.

Compared to the case of no collateral premia, firms' debt choices differ in sign above and below the discontinuity in bond demand induced by eligibility requirements. High-quality firms (with a large eligible debt capacity) take advantage of banks' high valuation of corporate bonds and issue more bonds to front-load dividend payouts: firms increase their *risk-taking*.[4] In contrast, medium-quality firms (which issue bonds at or near their eligible debt capacity) may find it worthwhile to reduce their debt issuance, if this makes their bonds eligible: a *disciplining* effect.[5] Both firm level effects imply that bond prices and debt choices are not solely determined by firm fundamentals. The latter case will be referred to as *market discipline*, while risk-taking and disciplining effects constitute violations thereof.[6]

While both firm level effects increase collateral supply, they have an opposing effect on default risk. This makes a heterogeneous firm model essential to study aggregate implications,

[4]Front-loading dividends can be thought of as an intertemporal substitution effect. On the other hand, those firms can sustain the same dividend-payout by issuing a smaller face value of bonds: an income effect. Under a standard assumptions on firm's revenue distribution, the former effect dominates.

[5]Disciplining effects associated with rating up- and downgrades are well documented in the literature. Kisgen (2006) and Kisgen (2009) show that firms near rating thresholds reduce their debt issuance, either due to the threat of losing a high rating or due to the opportunity of being upgraded into a higher category.

[6]There are two potential confounding factors. First, firms may substitute from loan to bond financing and leave total debt constant. Pelizzon et al. (2020) provide evidence for this substitution, but still find a sizable positive effect on leverage. A second potential concern arises if firms simultaneously increase investment and keep leverage constant. However, Grosse-Rueschkamp et al., 2019 identify sizable leverage responses of firms rated A or higher after the ECB's corporate sector purchase program (CSPP). Moreover Todorov (2020) and Santis and Zaghini (2021) find that QE-eligible firms primarily issued bonds to increase dividend payouts.

because the relative strength of both effects depends on the firm distribution. To illustrate the aggregate effect, consider collateral easing, which increases the eligible debt capacity for all firms. The change of aggregate collateral supply contains a *mechanical component*, the change caused by a lower rating threshold all else equal, and *endogenous firm responses*, which depend on the relative size of risk-taking and disciplining effects and the mass of firms subject to each effect. Within this framework, we show that endogenous firm responses unambiguously amplify the positive mechanical effect of collateral easing. Risk-taking and disciplining effects positively contribute to the increase. However, they have an ambiguous impact on aggregate default cost. Note that we obtain these results under the assumptions of one-period bonds and i.i.d. revenue shocks, which we relax in the following.

As our second contribution, we illustrate the relevance of firm responses in the context of the ECB's collateral easing policy in a setting with long-term debt and persistent revenue shocks. We solve the model using global methods and calibrate the firm cross-section to euro area data by employing a merged dataset of corporate bonds from *IHS Markit* and corporate balance sheet data from *Compustat Global*. The calibrated model can replicate several features of firm debt issuance, corporate bond spreads, and collateral premia, which are crucial to evaluate the impact of eligibility requirements.

We study two different policies: our benchmark scenario are tight eligibility requirements, which only accept bonds rated A or higher, corresponding to the ECB collateral framework before the 2008 crisis. Second, we consider lenient eligibility requirements, under which all bonds rated BBB or higher are accepted, in line with ECB practice after 2008. In the setting with long-term bonds and persistent revenues, firm responses dampen the positive mechanical effect of collateral easing on collateral supply and increase aggregate cost from default. These effects are sizable: instead of mechanically expanding by 71%, collateral supply only increases by 62% if firm responses are taken into account. This is reflected by the relative importance of risk-taking and disciplining effects. While 51% of firms engage in risk-taking and 19% are subject to disciplining under a tight policy, these numbers shift to 79% and 3% under a lenient policy. Aggregate default costs increase by 8%.

The dampening effect of firm responses on collateral supply is associated with the large debt issuance of high-revenue firms. While the risk-taking response at the firm level is increasing the *current* market value of bonds outstanding, it exposes firms to rollover risk: when hit by a series of adverse revenue shocks, previously issued bonds experience a drop in their market value. If firms also lose their eligibility status, this rollover problem becomes more severe, since bonds also lose the collateral premium. As a result, indebtedness increases, default becomes more likely, and collateral supply contracts. Notably, increasing bond issuance is still optimal for firms that experience high revenue draws due to the relative impatience of firm managers: the adverse effects of rollover risk are discounted sufficiently heavily. Key to this phenomenon is the combination of persistent shocks and long-term debt. It is important to note that this mechanism is also present under tight collateral policy but becomes stronger after collateral easing. Similar effects have been described in the macro-finance (Gomes et al., 2016 or Jungherr and Schott, 2022) and sovereign default literature (Hatchondo et al., 2016).

We investigate an *eligibility covenant* as a potential instrument to tackle the dampening effect of endogenous firm responses. Our focus is on covenants depending on *current* debt outstanding, which effectively is public information for firms that are large enough to issue corporate bonds. A covenant limits the eligible debt capacity of firms with high levels of debt outstanding and provides deleveraging incentives. This reduces the rollover burden of these firms once they experience a sequence of adverse revenue draws. Conceptually, most collateral frameworks condition the eligibility of bonds only on ratings, which is a one-dimensional measure of default risk. Conditioning eligibility also on debt outstanding allows the central bank to dis-incentivize 'unsustainable' debt issuance of high-revenue firms, while at the same time allowing low-revenue firms with low debt outstanding to roll over their bonds.

The policy problem in choosing a covenant is to set a sufficiently tight covenant to provide deleveraging incentives for high-revenue firms without shutting down the issuance of bonds altogether. Restricting our attention to a simple parametric class for the covenant, we demonstrate the existence of a *collateral Laffer curve* for any given minimum rating requirement. Our numerical results suggest that conditioning eligibility not only on default risk but also on current debt outstanding, has a positive effect on collateral supply. For example, under a BBB minimum rating requirement, covenants can expand collateral supply by up to 22% compared to the case without covenant. Finally, we investigate the potential of adding covenants to the central bank toolkit in addition to the minimum rating requirement. Since the representation of banks and liquidity risk is too simplistic to allow for a fully-fledged welfare analysis, we build on the literature on optimal collateral policy (Koulischer and Struyven, 2014 and Choi et al., 2021 among others) and embed our analysis in a central bank trade-off between maintaining high collateral supply and limiting the additional default cost from violating market discipline. Our model predicts that using both instruments increases collateral supply, lowers cost from corporate default, and, thereby, shifts the *collateral policy frontier* outward.

While we propose a model that is suited to study discontinuous collateral eligibility, our framework can also be applied in other situations where firms respond endogenously to a discontinuous demand schedule for their debt. This includes the eligibility for asset purchase programs, where the anticipation of substantial demand increases for targeted assets induces a willingness to pay eligibility premia on them. Furthermore, pension funds are typically restricted to investment grade bonds for regulatory reasons, such that bond demand exhibits a jump from BB+ to BBB-ratings (Boot et al., 2005). Similarly, Kisgen (2006) argues that speculative grade firms (CCC+ or lower) are subject to more stringent disclosure requirements, while only firms rated at least AA- are able to issue commercial paper in the US (Hahn, 1993).

Related Literature. Our paper builds on a large strand of literature providing empirical results on the bond market impact of collateral policy and eligibility for QE programs. Ashcraft et al. (2011) find a sizable impact of haircuts on bond prices using an event study around announcement and implementation of the Term Asset-Backed Securities Loan Facility in the US. Exploiting an unexpected policy change regarding eligibility of Chinese corporate bonds, Chen et al. (2019) and Fang et al. (2020) identify large pledgeability premia. Mésonnier et al. (2022) use an extension of eligibility criteria as part of the Additional Credit Claims program and find a premium of 8bp on bank loans relative to a non-eligible control group. Firm indebtedness are presented in Grosse-Rueschkamp et al. (2019), who identify heterogeneous responses of firms in different rating brackets.

While the previous group of papers uses surprise policy changes to identify causal effects, there are two complementary approaches leading to similar findings. First, Pelizzon et al. (2020) document collateral eligibility premia and bond supply effects using security-level data from the euro area. Their identification relies on ECB-discretion over when formally eligible bonds are put on the list of eligible assets. They identify collateral eligibility premia of 11-24bp. Second, building an identification strategy around the US treasury safety premium, Mota (2021) uses US corporate bond data and finds that non-financial corporate bonds carry a premium, which can be related to collateral service. The premium decreases in the bond default risk and depends on idiosyncratic firm characteristics as well as an aggregate component encompassing economy-wide collateral supply and demand factors. Mota (2021) finds that firm debt issuance and dividend payout responses rise in the size of the premium. We complement this literature by proposing a model that captures the empirically documented heterogeneity of firm responses regarding debt issuance and indebtedness.

The results of our paper can be related to a group of papers studying the collateral eligibility of risky assets and implications for central bank policy. Chapman et al. (2011) propose a model where the central bank faces a collateral policy trade-off between relaxing banks' liquidity constraints and incentivizing them to invest into illiquid and risky assets. Koulischer and Struyven (2014) argue that relaxing eligibility requirements can alleviate credit crunches if collateral supply or collateral quality fall below specific levels, as banks' ability to extend credit depends on both. Cassola and Koulischer (2019) quantify a collateral policy trade-off between liquidity provision and risk-taking by the central bank. In Choi et al. (2021) banks prefer to use high-quality collateral on the interbank market so that central banks negatively affect access to liquidity when accepting only high-quality assets. At the same time, lending against low-quality collateral exposes the central bank to counterparty default risk. In contrast to these papers, we make collateral supply and its riskiness endogenous but abstract from further frictions on money markets. Combining these approaches might deliver interesting interactions, which we leave to future research.

Outline. The paper is structured as follows. Section 2 introduces collateral premia and eligibility requirements into a corporate capital structure model and presents our main conceptual results. We extend and apply the model to the ECB's collateral easing policy in Section 3. In Section 4, we conduct policy experiments regarding eligibility covenants. Section 5 concludes.

2 A Model of Eligibility Requirements

This section introduces a model of endogenous collateral supply and firm default risk to analyze the impact of eligibility requirements on firms. Time is discrete and there are two groups of agents: a non-financial sector (*firms*) and financial intermediaries (*banks*). The *central bank* sets an eligibility requirement, which we treat as an exogenous parameter.

2.1 Environment

Firms are endowed with a technology that generates stochastic revenues, which can be interpreted as earnings before interest and taxes (EBIT). To maintain tractability, we do not endogenize investment. Revenue shocks realize at the beginning of each period t and are i.i.d. across firms and over time. In addition to being subject to idiosyncratic revenue shocks, firms are ex-ante heterogeneous with respect to the probability distribution over revenue shocks: some firms are permanently more productive than others and we denote this heterogeneity by the parameter s. We will use this parameter to index bonds issued by each firm as well.

Each period, firms issue debt instruments to banks. These debt instruments are referred to as corporate bonds but reflect all marketable debt instruments including securitized bank loans. Bonds are real one-period discount bonds, i.e., they promise to pay one unit of the all-purpose good in period $t+1$. In our model, firms are the natural borrowers, because they are more impatient than banks. Given their shock realization and bonds outstanding, firms either default or repay. Bonds have a dual role in the economy, since banks can pledge eligible bonds with the central bank to obtain funding. The demand for central bank funding can be motivated by liquidity deficits that require immediate settlement, such as net deposit outflows (see Bianchi and Bigio, 2022, and De Fiore et al., 2019). We follow Mota (2021) and assume a constant willingness to pay collateral premia. We present a robustness check where the size of collateral premia depends on collateral supply in Appendix D.

Banks. There is a unit mass of perfectly competitive banks, which price bonds risk-neutrally without discounting. They purchase bonds $b_{t+1}(s)$ issued by firm s at price $q(b_{t+1}|s)$, which reflects its value as an investment object, i.e., the repayment probability (described below), and the collateral benefit they provide. The collateral premium on an eligible bond will be denoted by L. Consistent with actual central bank policy, we assume that bonds are only eligible as collateral if their default probability $F(b_{t+1}|s)$ does not exceed an eligibility threshold $\overline{F}$ set by the central bank. The eligibility indicator Ψ is given by

$$\Psi(b_{t+1}|s) = \begin{cases} 1 & \text{if } F(b_{t+1}|s) \leq \overline{F} \\ 0 & \text{else} \end{cases} .$$

We model collateral policy in terms of bond eligibility thresholds, i.e., bonds either receive a 100% or a 0% haircut.[7]

Firms. Firm managers/owners are risk neutral and their discount factor is denoted by $\beta < 1$. They operate a technology generating random revenues $\mu_t \in [\underline{\mu}, \overline{\mu}]$ with $\underline{\mu} < 0$ and $\overline{\mu} > 0$.[8] We assume that μ_t is independent across firms and over time, and denote its pdf and cdf by $f(\mu_t|s)$ and $F(\mu_t|s)$, respectively. Firms are *ex-ante* heterogeneous with respect to their probability distribution over revenues, which allows us to analytically disentangle how individual firms react to eligibility requirements and how firm heterogeneity affects aggregate collateral supply responses to collateral easing. The ex-ante heterogeneity is governed by a productivity parameter s, which characterizes the revenue distribution in a first order stochastic dominance sense. Firms with a high s are more productive on average. In particular, s shifts the probability mass according to $F(\mu_t|s) = F(\mu_t - s)$. We assume that s follows some continuous distribution $G(s)$ over the open interval $S \equiv [s^-, \infty]$, to which we refer as the *firm type space*. Furthermore, we

[7] In practice, eligible bonds have collateral values in between due to other risk factors, like market illiquidity, which are not present in our setup. Nevertheless, collateral frameworks exhibit large discontinuities at the eligibility thresholds, as we show in Appendix A.

[8] Allowing for negative realizations of the revenue shock is consistent with the interpretation of μ_t as EBIT.

assume that s^- is sufficiently low such that at least one firm is not eligible even when it chooses not to issue any bonds, i.e., $F(0|s^-) = F(s^-) > \overline{F}$.

Firm managers maximize the present value of dividends. Dividends can become negative, which we interpret as equity issuance. Firms issue bonds $b_{t+1}(s)$ to banks. These bonds are subject to default risk: if firm revenues μ_t fall short of the repayment obligation b_t, the firm is unable to raise funds by issuing additional equity and defaults. The default and repayment probabilities implied by the debt choice b_{t+1} are, therefore, given by $F(b_{t+1}|s)$ and $1-F(b_{t+1}|s)$, respectively. In case of default, all firm revenues are lost and there is no recovery for banks.[9]

Bond Pricing. Expressing the expected payoff from investing into bonds of firm s in terms of the revenue distribution, the bond pricing condition can be written as

$$q(b_{t+1}|s) = \big(1 - F(b_{t+1}|s)\big)\left(1 + \Psi(b_{t+1}|s)\cdot L\right) . \tag{1}$$

It depends on the expected payoff, determined by the firm default decision in $t+1$, and the collateral premium L if bond s is eligible, which, in turn, depends on firm default risk. In the absence of a collateral premium, bond prices merely reflect the expected payoffs.

2.2 Debt Choices at the Firm Level

In this section, we analyze how firms' debt choice is affected by the eligibility of their corporate bonds as collateral. We assume there are no delays in the restructuring of defaulted bonds and no exclusion from the corporate bond market after a default. The maximization problem of firm s in period t can be written as

$$V(b_{t+1}|s) = q(b_{t+1}|s)b_{t+1} + \beta \int_{b_{t+1}}^{\overline{\mu}} (\mu_t - b_{t+1})\, dF(\mu_t|s) . \tag{2}$$

Maximizing (2) over b_{t+1} yields the first order condition

$$\beta(1 - F(b_{t+1}|s)) = q(b_{t+1}|s) + \frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}} b_{t+1} ,$$

[9] Our approach is motivated by Lian and Ma (2021), who show that most corporate borrowing is tied to the going-concern value of the firm. Allowing for a positive recovery rate would not change our qualitative results.

which we express using the bond price derivative $\frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}} = -f(b_{t+1}|s)(1+\Psi(F(b_{t+1}|s))\cdot L)$ as

$$\beta\left(1-F(b_{t+1}|s)\right) = \left(1-F(b_{t+1}|s)\right) - f(b_{t+1}|s)\cdot b_{t+1} \qquad \text{if } F(b_{t+1}|s) > \overline{F}\,, \tag{3}$$

$$\beta\left(1-F(b_{t+1}|s)\right) = \left(1-F(b_{t+1}|s)\right)\cdot(1+L) - f(b_{t+1}|s)\cdot b_{t+1}\cdot(1+L) \qquad \text{if } F(b_{t+1}|s) \leq \overline{F}\,. \tag{4}$$

The eligibility requirement introduces a discontinuity into the first order condition. To make the implied discontinuity in the debt choice explicit, we refer to the debt levels satisfying (3) and (4) as non-eligible debt choice $b^n_{t+1}(s)$ and eligible debt choice $b^e_{t+1}(s)$, respectively. Non-eligible firms choose their bond issuance according to (3): the LHS of this expression reflects discounted expected repayment obligations from issuing another unit of bond, which has to equal the current revenue from issuing this bond net of debt dilution on the RHS. This case is consistent with the concept of *market discipline*, since the debt choice is determined solely by fundamentals. Collateral premia distort this trade-off by making debt issuance more attractive, since they increase the amount of funds raised per unit of bonds (first term on the RHS of (4)). At the same time, collateral premia increase the costs of debt dilution (second term on the RHS), which makes debt issuance less attractive.

Without further restrictions on the revenue distribution, the total effect of bond eligibility is ambiguous. However, guided by empirical evidence on increased debt issuance at the firm level in response to bond eligibility (Pelizzon et al., 2020) and consistent with the standard assumption in the literature (Bernanke et al., 1999), we assume that the distribution satisfies a monotonicity condition on the hazard rate $h(\mu_{t+1}|s) \equiv \frac{f(\mu_{t+1}|s)}{1-F(\mu_{t+1}|s)}$.

Proposition 1. If the revenue distribution satisfies $\frac{\partial(\mu_{t+1}h(\mu_{t+1}|s))}{\partial \mu_{t+1}} > 0$, the non-eligible debt choice is increasing in the productivity parameter $\frac{\partial b^n_{t+1}}{\partial s} > 0$, while the implied default risk decreases $\frac{\partial F(b^n_{t+1})}{\partial s} < 0$. Likewise, for eligible firms it holds that $\frac{\partial b^e_{t+1}}{\partial s} > 0$ and $\frac{\partial F(b^e_{t+1})}{\partial s} < 0$. Moreover, the optimal debt issuance of an eligible firm exceeds that of an otherwise identical non-eligible firm $b^e_{t+1}(s) > b^n_{t+1}(s)$.
Proof: See Appendix B.1.

Proposition 1 establishes two important results. First, more productive firms (higher s) issue more debt but their default risk falls compared to less productive firms. Because firms are risky, they increase debt issuance less than one-for-one with improving fundamentals. Second, collateral premia induce additional debt issuance of eligible firms. Those firms take advantage of their better bond valuation to front-load dividend payouts.

So far, we established differences between the non-eligible and eligible debt choice. Next, we focus on how firm choices are affected by eligibility requirements, since the eligibility status of firms is *endogenously* determined. It will be helpful to define the *eligible debt capacity* $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ as the highest debt choice for which the default probability of firm s does not exceed the threshold $\overline{F}$. Naturally, more productive firms have a higher eligible debt capacity. As shown in Proposition 2, the ex-ante heterogeneous revenue distribution determines how firms select themselves into eligible and non-eligible regions, taking the eligibility threshold as given.

Proposition 2. There are two cut-off values s_0, implicitly defined through $V^e(\widetilde{b}_{t+1}(s_0)|s_0) = V^n(b^n_{t+1}(s_0)|s_0)$, and s_2, defined through $b^e_{t+1}(s_2) = \widetilde{b}_{t+1}(s_2)$, such that

- firms with $s < s_0$ are *non-eligible* and choose $b^n_{t+1}(s)$ according to (3).
- firms with $s_0 < s < s_2$ are *constrained eligible* in the sense that they borrow up to their eligible debt capacity $\widetilde{b}_{t+1}(s)$.
- firms with $s > s_2$ are *unconstrained eligible* and choose $b^e_{t+1}(s)$ according to (4).

Proof: See Appendix B.2.

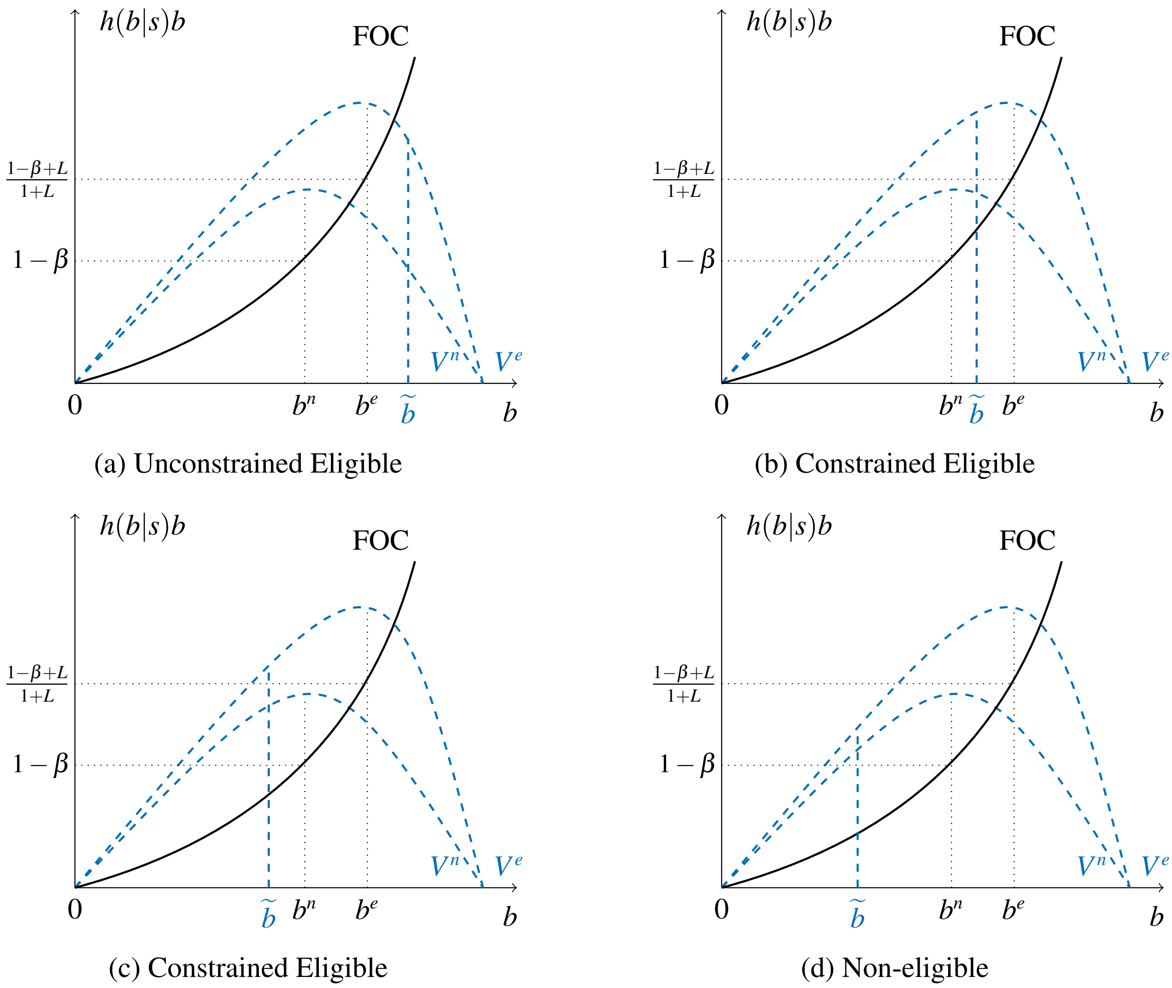
In Figure 1, we provide an illustration of Proposition 2. We plot the first order conditions in solid black lines, expressed in terms of the hazard rate $h(b|s)$. Objective functions for the case of non-eligibility and eligibility are denoted by V^n and V^e (blue dashed lines) and are obtained from evaluating (2) at the respective debt choices. There are four possible combinations of $b^n_{t+1}(s)$, $b^e_{t+1}(s)$, and $\widetilde{b}_{t+1}(s)$.

Figure 1a shows the case of a highly productive firm with a high draw of s so that $b^e_{t+1}(s) < \widetilde{b}_{t+1}(s)$. The eligible debt capacity of an *unconstrained eligible* firm is sufficiently high, such that it can satisfy (4). Figure 1b shows a firm with insufficient debt capacity to satisfy (4), whereas satisfying (3) would be possible, $b^n_{t+1}(s) < \widetilde{b}_{t+1}(s) < b^e_{t+1}(s)$. However, the value of the objective $V^e(\widetilde{b}_{t+1}(s)|s)$ exceeds the value $V^e(b^n_{t+1}(s)|s)$, because V^e is upward sloping for all $b < b^e_{t+1}(s)$. Thus, the firm chooses to be just eligible at debt level $\widetilde{b}_{t+1}(s)$. Such a firm is *constrained eligible*. Within the case of $\widetilde{b}_{t+1}(s) < b^n_{t+1}(s)$, there are two sub-cases: first, choosing $b^n_{t+1}(s)$ is feasible, but the firm can be better off by choosing $\widetilde{b}_{t+1}(s)$, since $V^n(b^n_{t+1}(s)|s) < V^e(\widetilde{b}_{t+1}(s)|s)$, as in Figure 1c. Such a firm chooses to be just eligible and is also classified as *constrained eligible*. Second, *non-eligible* firms with a sufficiently low s optimally choose $b^n_{t+1}(s)$, since the debt reduction required for eligibility is too large $V^e(\widetilde{b}_{t+1}(s)|s) < V^n(b^n_{t+1}(s)|s)$, as in Figure 1d.

2.3 Eligibility Requirements and Macroeconomic Aggregates

Having discussed how firm policies are characterized in the presence of collateral premia, we now turn to the effects of collateral easing. We consider an increase of the threshold default

Figure 1: Debt Choice Across Endogenous Firm Types



Notes: Endogenous partitioning of the firm space following Proposition 2. The blue lines denote the objective function (2) for each eligibility status. The black line represents the LHS of firms' first order conditions, rewritten as $h(b_{t+1}|s) \cdot b_{t+1}$, which maximizes V^e at b^e and V^n at b^n.

probability from a low value $\overline{F}^A$ to a higher value $\overline{F}^{BBB}$, akin to the ECB policy change in response to the 2008 financial crisis and also corresponding to our numerical experiments in Section 3. Formally, we characterize the change of aggregate collateral $\overline{B}$ in terms of the debt choice across the firm type space S. Let the cut-off values, which determine the partitioning of firms into constrained and unconstrained eligible, associated with $\overline{F}^A$ and $\overline{F}^{BBB}$ be denoted by (s_0^A, s_2^A) and (s_0^{BBB}, s_2^{BBB}), respectively. The threshold productivity levels partitioning firms into eligibility regions decrease in response to collateral easing, which we summarize in Lemma 1.

Lemma 1. Increasing the eligibility threshold from $\overline{F}^A$ to $\overline{F}^{BBB}$ decreases the threshold levels to $s_0^{BBB} < s_0^A$ and $s_2^{BBB} < s_2^A$.

Proof: See Appendix B.3.

We can use Lemma 1 to write the total effect of collateral easing on collateral supply as

$$\overline{B}^{BBB} - \overline{B}^{A} = (1+L)\Bigg(\int_{s_0^{BBB}}^{s_2^{BBB}} \Big(1 - F\big(\widetilde{b}_{t+1}^{BBB}(s)\big)\Big)\widetilde{b}_{t+1}^{BBB}(s)dG(s) + \int_{s_2^{BBB}}^{\infty} \Big(1 - F\big(b_{t+1}^{e}(s)\big)\Big)b_{t+1}^{e}(s)dG(s)$$
$$- \int_{s_0^{A}}^{s_2^{A}} \Big(1 - F\big(\widetilde{b}_{t+1}^{A}(s)\big)\Big)\widetilde{b}_{t+1}^{A}(s)dG(s) - \int_{s_2^{A}}^{\infty} \Big(1 - F\big(b_{t+1}^{e}(s)\big)\Big)b_{t+1}^{e}(s)dG(s)\Bigg). \quad (5)$$

Collateral supply, that is, the market value of eligible bonds, can be divided into two parts: the two integrals over $[s_0, s_2]$ contain all constrained eligible firms, respectively, while the integrals over $[s_2, \infty)$ summarize unconstrained eligible firms.

A central point of our framework is that such a policy has a mechanical effect by lowering the eligibility threshold and that it implies an endogenous firm response. To highlight this decomposition, we introduce a third threshold productivity s_1, where $s_0 < s_1 < s_2$. For the threshold firm s_1, the debt choice $b_{t+1}^{n}(s_1)$ equals its eligible debt capacity $\widetilde{b}_{t+1}(s_1)$. Mechanical effects are present if threshold levels satisfy $s_1^{BBB} < s_0^{A}$. This means that at least the firm exactly satisfying eligibility requirements after the policy change $F(b_{t+1}^{n}(s_1^{BBB})) = \overline{F}^{BBB}$ was not eligible before the policy change $\overline{F}^{A} < F(b_{t+1}^{n}(s_1^{BBB}))$. This firm was non-eligible under the tight policy, where it chooses $b_{t+1}^{n}(s)$, but becomes eligible without changing its debt issuance. To ease the exposition, we further restrict attention to 'small' changes to eligibility requirements and assume $s_0^{A} < s_2^{BBB}$. This implies that there is no firm directly switching from non-eligible to unconstrained eligible. We summarize the impact of collateral easing on collateral supply and default cost in Lemma 2.

Lemma 2. If $s_1^{BBB} < s_0^{A}$, the mechanical effect of collateral easing on collateral supply is positive and given by

$$\overline{B}^{BBB} - \overline{B}^{A}\Big|_{mech} = (1+L)\left(\int_{s_1^{BBB}}^{s_0^{A}} \Big(1 - F\big(b_{t+1}^{n}(s)\big)\Big)b_{t+1}^{n}(s)dG(s) \right). \quad (6)$$

If $s_0^A < s_2^{BBB}$, endogenous firm responses on collateral supply can be expressed as

$$\overline{B}^{BBB} - \overline{B}^{A}\Big|_{endo} = (1+L)\Big(\int_{s_0^{BBB}}^{s_1^{BBB}} \Big(1-F\big(\widetilde{b}_{t+1}^{BBB}(s)\big)\Big)\widetilde{b}_{t+1}^{BBB}(s)dG(s) \tag{7a}$$

$$+\int_{s_1^{BBB}}^{s_0^{A}} \Big(1-F\big(\widetilde{b}_{t+1}^{BBB}(s)\big)\Big)\widetilde{b}_{t+1}^{BBB}(s) - \Big(1-F\big(b_{t+1}^{n}(s)\big)\Big)b_{t+1}^{n}(s)dG(s) \tag{7b}$$

$$+\int_{s_0^{A}}^{s_2^{BBB}} \Big(1-F\big(\widetilde{b}_{t+1}^{BBB}(s)\big)\Big)\widetilde{b}_{t+1}^{BBB}(s) - \Big(1-F\big(\widetilde{b}_{t+1}^{A}(s)\big)\Big)\widetilde{b}_{t+1}^{A}(s)dG(s) \tag{7c}$$

$$+\int_{s_2^{BBB}}^{s_2^{A}} \Big(1-F\big(b_{t+1}^{e}(s)\big)\Big)b_{t+1}^{e}(s) - \Big(1-F\big(\widetilde{b}_{t+1}^{A}(s)\big)\Big)\widetilde{b}_{t+1}^{A}(s)dG(s)\Big)\,. \tag{7d}$$

Denoting the resource loss of firm s from defaulting by $M(b_{t+1}(s)|s) \equiv \int_{\underline{\mu}}^{b_{t+1}(s)} \mu_{t+1} dF(\mu_{t+1}|s)$, the change in aggregate default cost $\mathcal{M}_t$ can be expressed as follows

$$\mathcal{M}^{BBB} - \mathcal{M}^{A} = \int_{s_0^{BBB}}^{s_1^{BBB}} M\big(\widetilde{b}_{t+1}^{BBB}(s)\big) - M\big(b_{t+1}^{n}(s)|s\big)dG(s) \tag{8a}$$

$$+\int_{s_1^{BBB}}^{s_0^{A}} M\big(\widetilde{b}_{t+1}^{BBB}(s)\big) - M\big(b_{t+1}^{n}(s)\big)dG(s) \tag{8b}$$

$$+\int_{s_0^{A}}^{s_2^{BBB}} M\big(\widetilde{b}_{t+1}^{BBB}(s)\big) - M\big(\widetilde{b}_{t+1}^{A}(s)\big)dG(s) \tag{8c}$$

$$+\int_{s_2^{BBB}}^{s_2^{A}} M\big(b_{t+1}^{e}(s)\big) - M\big(\widetilde{b}_{t+1}^{A}(s)\big)dG(s)\,. \tag{8d}$$

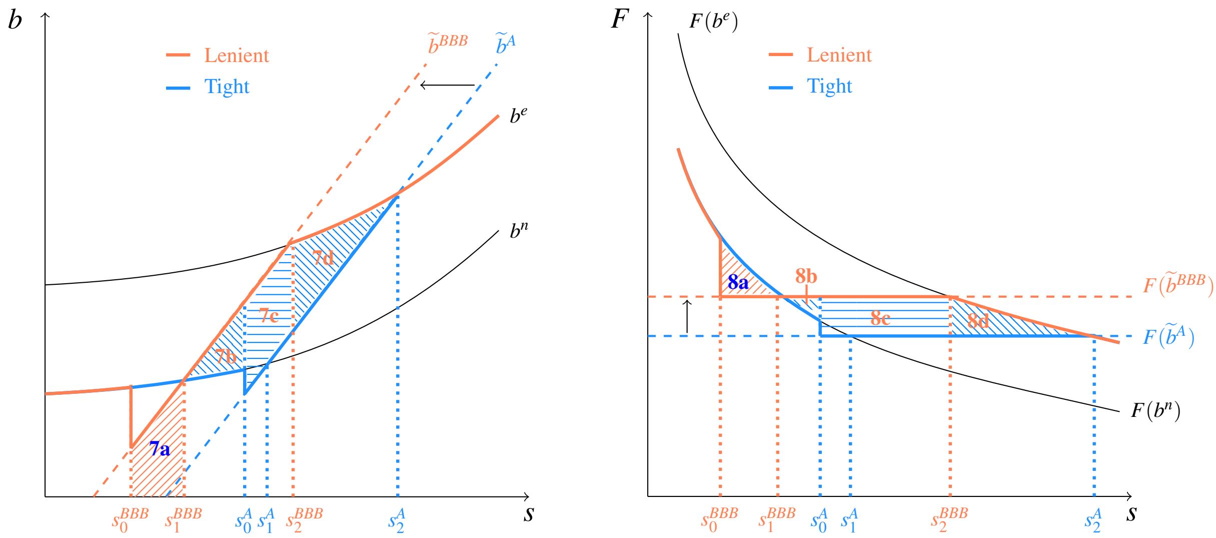
Proof: see Appendix B.4.

Equation (6) reflects the additionally eligible collateral under the assumption that firms do not change their debt choice. These firms were non-eligible under tight eligibility requirements and, therefore, issue bonds according to $b_{t+1}^{n}(s)$. The mechanical effect of collateral easing is positive.

The collateral supply effect of collateral easing associated to firm responses is given by (7). To aid intuition, Figure 2a shows the impact of collateral easing on firm debt issuance across firm types s. The lines b^n and b^e denote the debt choices that satisfy (3) and (4), respectively. As shown in Lemma 1 they increase in firm productivity. Dashed lines denote the eligible debt capacity under tight (blue) or lenient (orange) collateral policy. Under a lenient policy, the eligible debt capacity increases for every productivity level compared to a tight policy so that the line shifts to the left. Bold colored lines denote the actual firm debt choices.

For a given collateral policy, we can distinguish the three firm types from Proposition 2. Highly productive firms (above s_2) choose debt according to (4), i.e., they are unconstrained

Figure 2: Firm Responses After Collateral Easing



(a) Debt Choice Across Firm Types

(b) Default Risk Across Firm Types

Notes: We compare the change in debt issuance (left) and default risk (right) across firm types s after an easing from tight (blue) to lenient (orange) collateral requirements. Colored dashed lines represent the eligible debt capacities or the associated default risk under either policy. Black solid lines denote the debt choices (or the corresponding default risk) satisfying (3) and (4). Colored bold lines denote the firm debt choices and the related default risk. Colored dotted lines denote the threshold productivities as described in Proposition 2 and Lemma 1.

eligible. Firms of medium quality, between the jump at s_0 and the kink at s_2, choose their eligible debt capacity and are constrained eligible. Last, low productivity firms (below s_0) are non-eligible and choose debt b^n following (3). Risk-taking and disciplining effects for a given policy are related to the difference between the firm choice (bold lines) and b^n (solid black line). First, firms between s_0 and s_1 reduce debt issuance compared to non-eligibility, i.e., they *discipline* themselves to be eligible. Second, firms above s_1 issue more debt compared to b^n, which is a *risk-taking* effect.

Lowering the eligibility threshold, going from the blue to the orange lines, changes productivity cut-off values (see Lemma 2). The first integral (7a) relates to firms that reduce their debt issuance relative to the tight policy to benefit from being eligible, which is graphically represented by the red area in Figure 2a. It is associated with the disciplining effect across the firm distribution. All other parts of (7) relate to risk-taking effects (denoted as blue areas), i.e., firms that increase debt issuance compared to tight policy: the second integral (7b) corresponds to firms issuing debt at their eligible debt capacity, but above $b^n_{t+1}(s)$, which exceeds their borrowing under tight eligibility requirements. Likewise, the third integral (7c) captures firms that remain constrained eligible but with a higher eligible debt capacity $\widetilde{b}^{BBB} > \widetilde{b}^{A}$. Last, the fourth integral (7d) summarizes firms that switch from constrained to unconstrained eligible.

Disciplining and risk-taking have a positive collateral supply effect. Firms becoming newly eligible via the disciplining effect automatically increase collateral supply. For firms that take on more risk, this can be seen by noting that those firms will not issue debt beyond a point where debt dilution exceeds the funds raised by issuing an additional unit of debt.[10]

In contrast, effects on the aggregate cost from default (8) are ambiguous. We illustrate the change in firm default risk across types s in Figure 2b. Default risk associated to b^n and b^e (solid black lines) falls for more productive firms as seen from Lemma 2. The eligibility thresholds are given by the horizontal colored lines and default risk related to the firm debt choice is given by bold colored lines. As in Figure 2a, one can distinguish the three firm types of Proposition 2. The effect of collateral easing on aggregate default costs is closely related to the change in default risk across firm types: while disciplining effects in the first integral (8a) lead to a reduction in aggregate default cost (red area), the other three integrals (8b)-(8d) related to risk-taking effects (blue areas), imply an increase of aggregate default costs.

Our framework with short-term debt predicts that collateral easing has a positive mechanical impact on collateral supply, which is amplified by firm responses. At the same time, the effect of firm responses on aggregate default cost is ambiguous and depends on the relative strength of risk-taking and disciplining effects, making a heterogeneous firm model necessary to adequately study aggregate effects. To quantify the relevance of endogenous firm responses and determine their sign, we extend our framework with persistent revenue shocks and long-term debt in the next section. In this setting, there is a pronounced dampening effect on collateral supply, which has been described in several settings with long-term debt and default risk (see Gomes et al., 2016; Jungherr and Schott, 2022). This gives rise to a negative relationship between increasing collateral supply (which facilitates monetary policy implementation) and incentivizing risk-taking at the firm level (which increases resource losses of default). We discuss the implied central bank policy trade-off and a potential remedy in Section 4.

3 Application to the ECB Collateral Easing Policy

This section applies our framework to the ECB's collateral easing policy in response to the financial crisis of 2008. We extend the model by long-term bonds and persistent revenue shocks, solve it using global methods, and present its calibration to euro area data. We then use the

[10] Differentiating the market value of bonds outstanding for any eligible debt choice $\frac{\partial((1+L)(1-F(b^e_{t+1}))b^e_{t+1})}{\partial b^e_{t+1}} = (1+L)(1-h(b^e_{t+1})b^e_{t+1})(1-F(b^e_{t+1}))$. Using (4), this simplifies to $\beta(1-F(b^e_{t+1})) > 0$.

calibrated model to shed light on corporate bond spreads and study the aggregate impact of endogenous firm responses induced by collateral easing. The characterization of aggregate effects forms the basis for our analysis of collateral policy in Section 4.

3.1 Extended Model

Firm heterogeneity takes the form of persistent revenue shocks rather than permanent differences in the idiosyncratic firm revenue distribution. In addition, bonds are long-term and a firm defaults if it cannot repay the maturing share of outstanding bonds out of current revenues. As in the previous section, we maintain the assumption of no delays in restructuring so that the value of non-maturing bonds is not affected by a default event. This permits us to abstract from the firm credit status as a state variable.

Firms. There is a continuum of competitive firms, indexed by j. Firms receive random revenues $e^{\mu_t^j}$ following an log-AR(1) process

$$\mu_t^j = \rho \mu_{t-1}^j + \sqrt{\sigma}\varepsilon_t^j \quad \text{with } \varepsilon_t^j \sim N(0,1) \ .$$

The idiosyncratic revenue shock is independent across firms. We denote the (conditional) pdf of μ_{t+1}^j by $f(\mu_{t+1}^j|\mu_t^j)$ and the associated (conditional) cdf by $F(\mu_{t+1}^j|\mu_t^j)$. Firms issue long-term bonds b_{t+1}^j, which enables us to generate realistic debt ratios. Each period, a share π of outstanding bonds matures. The non-maturing share of bonds is valued like new issues at price q_t, according to the law of one price. Firms default on their current repayment obligation πb_t^j if they exceed current revenues $e^{\mu_t^j}$. We can write the default probability as

$$F(b_{t+1}^j|\mu_t^j) = \Phi\left(\frac{\log\left(\pi b_{t+1}^j\right) - \rho\mu_t^j}{\sigma}\right) \ . \tag{9}$$

Banks and Bond Pricing. Banks are modeled in a similar way as in Section 2. They are risk-neutral and discount the future at the constant rate r^{rf}. The per-unit price schedule for corporate bonds can be written

$$q(b_{t+1}^j,\mu_t^j) = \frac{1+\Psi(F(b_{t+1}^j|\mu_t^j))L}{1+r^{rf}}\left(\pi\left(1-F(b_{t+1}^j|\mu_t^j)\right)+(1-\pi)\mathbb{E}_t\left[q\left(\mathcal{B}(b_{t+1}^j,\mu_{t+1}^j),\mu_{t+1}^j\right)\right]\right) \ . \tag{10}$$

Note that the rollover value of bonds is obtained from evaluating the bond price schedule at next period's debt choice $\mathcal{B}(b_{t+1}^j,\mu_{t+1}^j)$, which we describe below. As in the baseline model from

Section 2, the total payoff contains a pecuniary part and a collateral premium L. The pecuniary part depends on default in $t+1$. If the firm repays, the maturing fraction π is redeemed and the remainder $1-\pi$ is rolled over at the next period's market price. In the case of default, banks lose the maturing fraction of the bond. Due to the assumption of immediate restructuring, the payoff still contains the rollover value of the non-maturing fraction.

Characterization of Debt Choices. Firms choose issue bonds b^j_{t+1} to maximize shareholder value, taken as given the bond price schedule (10). The maximization problem of firm j can be represented by the Bellman equation

$$W(b^j_t,\mu^j_t) = \max_{b^j_{t+1}} \quad V(b^j_{t+1},\mu^j_t) \quad \text{with} \tag{11}$$

$$V(b^j_{t+1},\mu^j_t) = \mathbb{1}\{e^{\mu^j_t} > \pi b^j_t\}\left(e^{\mu^j_t} - \pi b^j_t\right) + q(b^j_{t+1},\mu^j_t)\left(b^j_{t+1} - (1-\pi)b^j_t\right) + \beta \mathbb{E}_t\left[W(b^j_{t+1},\mu^j_{t+1})\right] . \tag{12}$$

Current dividends are given by revenues and debt service obligations $e^{\mu^j_t} - \pi b^j_t$, conditional on repayment, and net debt issuance $q(b^j_{t+1},\mu^j_t)\left(b^j_{t+1} - (1-\pi)b^j_t\right)$. Note that the debt choice b^j_{t+1} does not depend on a potential default in period t, which again follows from the assumption of immediate restructuring. A higher debt choice increases current dividends but reduces next period's dividends due to (i) higher default risk, (ii) elevated debt service conditional on no default, and (iii) increasing the rollover burden in the next period. Plugging in the bond pricing condition (10), the first order condition can be written as

$$\frac{\partial q(b^j_{t+1},\mu^j_t)}{\partial b^j_{t+1}}\left(b^j_{t+1} - (1-\pi)b^j_t\right) + q(b^j_{t+1},\mu^j_t) = \beta\left(\pi(1-F(b^j_{t+1}|\mu^j_t)) + (1-\pi)\mathbb{E}_t\left[q_{t+1}\right]\right) , \tag{13}$$

where the derivative of the bond price schedule (10) is given by

$$\frac{\partial q(b^j_{t+1},\mu^j_t)}{\partial b^j_{t+1}} = \begin{cases} -f(b^j_{t+1})\pi\frac{1}{1+r^{rf}} , & \text{if } F^j_{t+1} > \overline{F} , \\ -f(b^j_{t+1})\pi\frac{1+L}{1+r^{rf}} , & \text{if } F^j_{t+1} \leq \overline{F} . \end{cases} \tag{14}$$

Let the solution to (13) in the case without eligibility be denoted by $b^{j,n}_{t+1}$ and in the case of eligibility by $b^{j,e}_{t+1}$. The debt choice depends on the feasibility of $b^{j,e}_{t+1}$ and the value of the objective function (12) under both candidate debt choices. The eligible debt capacity in closed form is obtained from evaluating the default probability (9) at $\overline{F}$ and re-arranging to

$$\widetilde{b}^j_{t+1} = \frac{\exp\{\sigma\Phi^{-1}(\overline{F}) + \rho\mu^j_t\}}{\pi} , \tag{15}$$

which we can use to obtain the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$

$$\begin{aligned}\mathcal{B}(b_t^j, \mu_t^j) = \mathbb{1}\left\{V(b_{t+1}^{j,n}, \mu_t^j) \leq V(\min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\}, \mu_t^j)\right\} \cdot \min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\} \\ + \mathbb{1}\left\{V(b_{t+1}^{j,n}, \mu_t^j) > V(\min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\}, \mu_t^j)\right\} \cdot b_{t+1}^{j,n} \,.\end{aligned} \tag{16}$$

If $b_{t+1}^{j,e} > \widetilde{b}_{t+1}^j$, the eligible debt choice is not feasible. Therefore, the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$ depends on the value attained by exhausting the eligible debt capacity $V(\widetilde{b}_{t+1}^j, \mu_t^j)$ and the value of forgoing eligibility $V(b_{t+1}^{j,n}, \mu_t^j)$. Conversely, if $b_{t+1}^{j,e} < \widetilde{b}_{t+1}^j$, the firm can issue the optimal level of bonds without losing eligibility. Consistent with the one-period model in Section 2, the firm will issue $b_{t+1}^{j,e}$ in this case, since $b_{t+1}^{j,n} < b_{t+1}^{j,e}$ and $V(b_{t+1}^{j,n}, \mu_t^j) < V(b_{t+1}^{j,e}, \mu_t^j)$ by definition. Since there is no aggregate risk and banks' bond pricing condition is independent of the firm distribution, the debt choice of firms and the bond pricing condition of banks fully characterize the equilibrium of our model. The equilibrium bond price $\mathcal{Q}(b_t^j, \mu_t^j)$ obtains from evaluating the bond price schedule (10) at the debt choice (16)

$$\mathcal{Q}(b_t^j, \mu_t^j) = q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j) \,.$$

Recursive Competitive Equilibrium. A competitive equilibrium is given by the bond price schedule $q(b_{t+1}^j, \mu_t^j)$, the firm value function $W(b_t^j, \mu_t^j)$, and the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$ such that

- given the pricing schedules for bonds, the debt choice solves the firm problem (11).
- bonds are priced according to (10).
- the law of motion for the distribution of firms over bond holdings and firm-specific revenues follows

$$G_{t+1}(b_{t+1}, \mu_{t+1}) = \int\int \left[\mathbb{1}\{b_{t+1} = \mathcal{B}(b_t, \mu_t)\}\right] \times \mathbb{1}\{\mu_{t+1} = \rho\mu_t + \sigma\varepsilon_{t+1}\} \times G_t(b_t, \mu_t) f(\varepsilon_{t+1}) d\varepsilon_{t+1} db_{t+1} \,.$$

Numerical Solution Method. We solve the full model computationally using policy function iteration on a discrete grid for revenues and bond issuance. The algorithm contains four steps at each iteration: first, we compute both potentially optimal debt choices by solving (13), given the bond price schedule (10). Second, we compute the eligible debt capacity (15) and check whether the optimal debt choice under eligibility is feasible. If this is not the case, we replace it by the eligible debt capacity $\widetilde{b}$. We randomize over the value function under both candidate debt choices using Gumbel-distributed taste shocks as proposed by Gordon (2019) to compute the debt choice (16). Third, given these policies, we compute the distribution of firms over individual states. The fourth step consists of updating bond price schedules. For a detailed description of the algorithm and the parameters governing our numerical approximation, we refer to Appendix C.2.

3.2 Calibration

We calibrate the model to euro area data between 2004Q1, the earliest data with reliable corporate bond data, and 2008Q3, the last quarter before the ECB relaxed its collateral framework. One period corresponds to one quarter. Our calibration is divided into two parts: the first part contains parameters determining the pricing of bond payoffs and eligibility benefits by banks, while the second set of parameters is related to firm fundamentals and the payoff profile of corporate bonds. These two blocks are connected by the central bank eligibility requirement, which is the policy variable of interest. We consider two policies: the baseline calibration is associated with tight eligibility requirements (A-rating or higher) and collateral easing refers to a scenario with lenient eligibility requirements (BBB-rating or higher). These thresholds are based on the ECB policy before and after the financial crisis of 2008.

Eligibility Requirement. We begin with discussing the eligibility thresholds $\overline{F}^{A}$ and $\overline{F}^{BBB}$. The ECB's collateral framework is based on ratings by external credit assessment agencies that are difficult to model parsimoniously. Therefore, we adopt an indirect approach based on macroeconomic aggregates. Specifically, we obtain data from *IHS Markit* on the total fixed income securities universe in Europe and extract the subset for non-financial corporate bonds. Using data from September 2008, the last month prior to the relaxation of eligibility requirements, 50% of all corporate bonds in our sample carried a rating of A or higher and were formally eligible as collateral. To match this share of eligible bonds, we set the baseline eligibility threshold to $\overline{F}^{A} = 4.15\%$, expressed in annualized terms. Similarly, we choose the eligibility threshold for a BBB-rating as $\overline{F}^{BBB} = 18.59\%$ to match the share of bonds rated BBB or higher in the *IHS Markit* sample, which was 86% in September 2008. We interpret this observed 72% increase of collateral supply as measure of the *mechanical* effect, since it is based on data prior to the policy relaxation.

Collateral Premium. We proxy the time-invariant real risk-free interest rate by a short-term interbank rate from which we subtract the consumer price inflation rate. Specifically, we use the time-series average of the 3M-EURIBOR minus the euro area inflation and obtain $r^{rf} = 0.0035$.

The collateral premium L is based on the empirical findings from Pelizzon et al. (2020).[11] Their paper makes use of the ECB having discretion in including bonds that formally satisfy eligibility requirements in the list of eligible assets. This discretion generates a randomly selected control group of bonds that eventually become eligible but are not yet accepted. Depending on the econometric specification, they estimate a yield drop to surprise eligibility of 11-24bp. We pick the most conservative value of 11bp. Our structural model permits an explicit calculation of the yield effect of a surprise inclusion. We set $\Psi = 0$ when pricing the bond (holding firm behavior fixed) and compare this hypothetical price to the equilibrium bond price. The price of this hypothetical bond is given by

$$q^n(b_{t+1}^j, \mu_t^j) = \frac{1}{1+r^{rf}} \left(\left(1 - F(b_{t+1}^j | \mu_t^j)\right) \pi + (1-\pi)\mathbb{E}_t \left[q\left(b_{t+1}^j, \mu_{t+1}^j\right) \right] \right) , \tag{17}$$

and contains a collateral premium from $t+1$ onward via the bond continuation value. The yield-to-redemption $\tilde{r}^j$ is determined by the internal rate of return of a perpetuity with constant decay,

$$q_t^j = \sum_{\tau=t+1}^{\infty} \frac{\pi(1-\pi)^{\tau-1}}{(1+r_t^j)^\tau} = \frac{\pi}{\pi + r_t^j} .$$

It follows that $r_t^j = \pi / q_t^j - \pi$. The corporate bond spread is defined as $x_t^j \equiv r_t^j - r^{rf}$. Using an entirely analogous derivation, the yield on the hypothetical non-eligible bond is given by $r_t^{j,n}$ and the eligibility premium follows as $r_t^j - r_t^{j,n}$, which is always negative.

Firm Fundamentals. The second part of the calibration is related to firms, i.e., the parameters governing the idiosyncratic revenue process ρ and σ, the maturity parameter π characterizing the repayment profile of corporate bonds, and the discount factor of firms β affecting the relative impatience over investors. By setting β to a lower value than the time discount factor of banks $\frac{1}{1+r^{rf}}$, we ensure that even absent collateral premia, firms would have an incentive to issue bonds.

These parameters are chosen to match selected data moments characterizing the firm cross-section. We merge our corporate bond dataset from *IHS Markit* with company data available through *Compustat Global*. A description of the construction of our dataset is given in Appendix C. Specifically, we target the median debt/EBIT-ratio b_t^j / μ_t^j as a measure of firm indebtedness and the bond spread distribution, characterized by its quartiles. The time-series averages over the sample period 2004Q1-2008Q3 are $Q_{0.25}(x) = 24$bp, $Q_{0.50}(x) = 39$bp, and $Q_{0.75}(x) = 62$bp.

[11] Notably, L does not depend on aggregate collateral supply. We relax this assumption in Appendix D.4.

Table 1: Baseline Parameterization

Parameter	Value	Source
Bank discount rate r^{rf}	0.0035	Real risk-free rate
Borrower discount factor β	0.993	Calibrated
Maturity Parameter π	0.06	Calibrated
Collateral premium L	0.004	Calibrated
Revenue persistence ρ	0.93	Calibrated
Revenue shock std. dev. σ	0.027	Calibrated
(Annualized) A-eligibility threshold $\overline{F}^A$	4.15%	Calibrated
(Annualized) BBB-eligibility threshold $\overline{F}^{BBB}$	18.59%	Calibrated

Table 2: Targeted Moments

Moment	Data	Model
Collateral premium $\text{ave}(r - r^n)$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu\|\overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x\|\overline{F}^A)$	24	27
Bond spread $Q_{0.50}(x\|\overline{F}^A)$	39	52
Bond spread $Q_{0.75}(x\|\overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB)\|\overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB)\|\overline{F}^{BBB}$	86%	83%

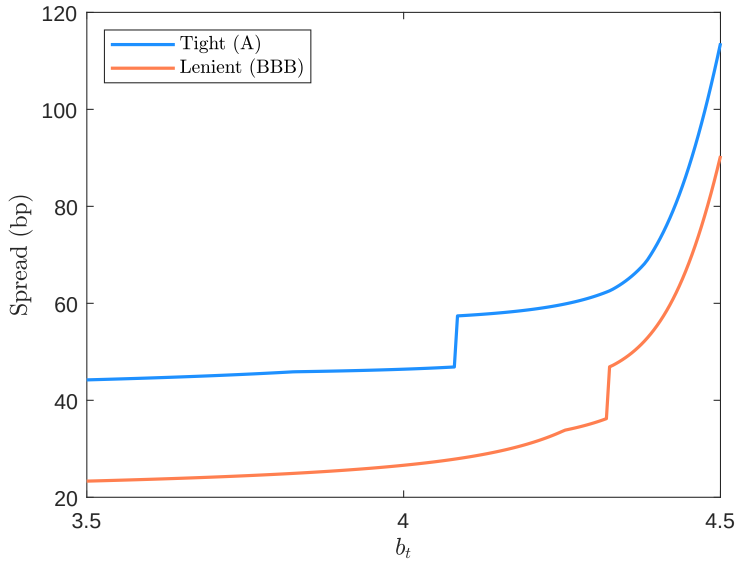
Notes: The collateral premium and spreads are annualized and expressed in basis points.

We conduct a sensitivity analysis with respect to a higher level of spreads computed over an extended sample period in Appendix D.3. Table 1 summarizes all parameters for our baseline calibration and Table 2 shows the targeted moments in our baseline calibration.

3.3 Corporate Bond Spreads

To illustrate how eligibility requirements affect the pricing of corporate bonds, we show the corporate bond spreads implied by banks' first order condition in Figure 3. Spreads are derived by evaluating the bond pricing condition at any candidate debt choice b^j_{t+1}, fixing revenues at their median. The baseline calibration (A-rating or higher) is marked in blue, while lenient eligibility requirements (BBB-rating or higher) are marked in orange. The discontinuity in each bond price schedule represents the location of the eligibility threshold. To the left of this point, bonds are currently eligible and investors are willing to pay collateral premia, which results

Figure 3: Corporate Bond Spreads



Notes: The blue (orange) line represents the corporate spread under a tight (lenient) eligibility requirement, conditional on firm legacy debt b_t^j, for a firm with median revenues.

in lower spreads. For a debt choice to the right of the discontinuity, bonds are not eligible and spreads jump upwards. The effect of relaxing eligibility requirements can be inferred from the location of the discontinuities. Intuitively, lenient eligibility requirements increase the eligible debt capacity, so that the discontinuity shifts to the right. Notably, since bonds are long-term, this also affects bond spreads away from the eligibility threshold: bonds are more likely to be eligible in future periods, which increases their price and lowers the spread already in the current period via the continuation value in (10). Hence, spreads under lenient eligibility requirements are uniformly lower.

3.4 Reconciling Cross-Sectional Evidence

Before discussing macroeconomic aggregates, we test the model's capability to replicate the (heterogeneous) impact of eligibility requirements across firms identified by several empirical papers. We run regressions on a simulated cross-section of firms, which is drawn from the equilibrium firm distribution under tight eligibility requirements. Regressions based on the firm distribution associated with lenient eligibility requirements yield similar results. We run

Table 3: Cross-Sectional Regression Results

	Empirical Literature			Model		
Coefficient	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$
Eligibility	-	+	+	-	+	+
Indebtedness $\times$ Eligibility	+	-	-	+	-	-

Notes: Signs in the left panel are taken from the empirical literature. Signs in the right panel are obtained from running (18) on the simulated firm cross-section. Model-implied coefficient signs are independent of the tightness of eligibility requirements.

the following (cross-sectional) regression

$$y_t^j = \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \cdot \frac{b_t^j}{\mu_t^j} + \varepsilon_t^j \ , \tag{18}$$

for three different specifications, that differ in the outcome variable. First, we examine the bond yield reaction to surprise eligibility $r_t^j - r_t^{j,n}$, following Pelizzon et al. (2020). Since we can control for firm indebtedness as a measure of default risk, this approach is similar to Grosse-Rueschkamp et al. (2019), Mota (2021), and Todorov (2020). Second, we evaluate the effects of a surprise inclusion on debt issuance $\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$ and, third, on dividends $\mathcal{D}_t^j - d_t^{j,n}$.[12]

The sign of the model-implied regression coefficients in all three specifications are collected in the right panel of Table 3. Since we sample from a parsimonious structural model, all coefficients are highly significant. We benchmark the model-implied coefficients against findings from the literature, reported in the left panel. The eligibility premium $r_t^j - r_t^{j,n}$ is a calibration target. Therefore, the coefficient on eligibility is negative by construction. The positive impact of eligibility on debt issuance is consistent with findings by Pelizzon et al. (2020), while the positive effect of eligibility on dividends has been described in Todorov (2020).

The coefficient signs of the interaction terms are informative about heterogeneous firm responses. Consistent with our theory, debt issuance and dividend payouts respond more strongly for less risky firms in the model, as the negative coefficients on the interaction term of eligibility and beginning-of-period indebtedness demonstrate. This negative relationship is consistent

[12] The equilibrium dividend in period t is given by

$$\mathcal{D}(b_t^j, \mu_t^j) = e^{\mu_t^j} - \pi b_t^j + q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j)\left(\mathcal{B}(b_t^j, \mu_t^j) - (1-\pi) b_t^j\right) \ ,$$

while the dividend of an non-eligible, but otherwise identical, firm can be written as

$$d^n(b_t^j, \mu_t^j) = e^{\mu_t^j} - \pi b_t^j + q(b^n(b_t^j, \mu_t^j), \mu_t^j)\left(b^n(b_t^j, \mu_t^j) - (1-\pi) b_t^j\right) \ .$$

with the findings of Mota (2021), who documents a positive relationship of eligibility premia, debt issuance, and dividend payouts with firm safety as measured by ratings. The negative coefficient on the interaction term eligibility $\times$ indebtedness is also in line with the results of Grosse-Rueschkamp et al. (2019). They report that firms rated A or higher increased their leverage ratio by 1.8 percentage points in response to CSPP-eligibility, as opposed to eligible BBB-rated firms, which only weakly increase leverage by 0.8 percentage points. Taken together, our model can capture the impact of eligibility requirements on multiple firm outcome variables documented in the data.

3.5 Aggregate Effects

We now turn to the impact of collateral easing on macroeconomic aggregates. As demonstrated Appendix D.1, the heterogeneous risk-taking and disciplining effects from Section 2 carry over to the case of long-term debt and persistent revenue shocks and we will organize our discussion around these two effects as well. The changes to the cross-sectional firm distribution induced by collateral easing are relegated to Appendix D.2.

Similar to the one-period bond model in Section 2, our discussion is based on a decomposition of collateral supply into a *mechanical effect* and endogenous *firm responses*. Formally, this decomposition obtains from expanding the total effect as follows:

$$\begin{aligned}
\overline{B}^{BBB} - \overline{B}^{A} &\equiv \int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{A}\} q^{A} b^{A} dG^{A}(\mu, b) \\
&= \underbrace{\int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{BBB}\} q^{A} b^{A} dG^{A}(\mu, b)}_{\text{Firm Response}} \\
&\quad + \underbrace{\int \mathbb{1}\{F^{A} < \overline{F}^{BBB}\} q^{A} b^{A} dG^{A}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{A}\} q^{A} b^{A} dG^{A}(\mu, b)}_{\text{Mechanical Effect}} .
\end{aligned}$$

The total effect on collateral supply is given by the difference between the market value of bonds issued by all eligible firms under either policy. The mechanical effect in the third line keeps firm behavior and the cross-sectional distribution at the baseline calibration, varying only the eligibility requirement. Firm responses are given residually and encompass changes in the market value of bonds and default risk but evaluate the eligibility status at the same minimum rating requirement $\overline{F}^{BBB}$.

In the first panel of Table 4, we apply this decomposition to the collateral easing experiment. It stands out that the percentage change of collateral supply from the (targeted) mechanical effect (+71%) exceeds the total effect (+62%). In contrast to Section 2, firm responses *dampen* the impact of eligibility requirements on collateral supply. This result is associated with the shares of firms being subject to risk-taking and disciplining effects. In particular, the share of firms disciplining themselves to be eligible falls from 19% under tight to 3% under lenient collateral policy. At the same time, the share of firms engaging in risk-taking rises from 51% to 79%. Consequently, there are also adverse effects on the corporate bond market as measured by a 8% increase of default costs.[13]

Intuitively, under lenient policy corporate bonds are eligible for worse fundamentals, which reduces disciplining incentives at the expense of risk-taking. The dampening effect of risk-taking on collateral supply is directly related to the persistence of revenue shocks and the stickiness of indebtedness: high-revenue firms find it optimal to increase their debt issuance and increase current dividends. If revenues are sufficiently persistent and firm managers sufficiently impatient, this only leads to a modest increase in default risk in the current period. Ultimately, however, firms will receive adverse revenue shocks and, due to the inherent stickiness of indebtedness, find themselves with a large amount of debt outstanding, which makes default more likely (see Jungherr and Schott, 2022, or Gomes et al., 2016). Default risk not only leads to a drop in the market value of eligible bonds but may also imply that those firms default. Both effects lower collateral supply. This feature is not present in our setting of Section 2 with i.i.d. shocks and one-period bonds. In such a setting, it is never optimal for firms to increase debt issuance beyond a point where it decreases the market value of bonds outstanding. The overall dampening requires the central bank to relax eligibility requirements more aggressively to achieve a specific increase in collateral supply. In addition, collateral easing induces adverse side effects on the corporate bond market in our model. In practice, higher prevalence of default risk can directly increase restructuring costs or inefficient liquidation of firms and indirectly make the financial system fragile, e.g., due to counterparty default risk.

4 Eligibility Covenants and the Central Bank Policy Frontier

In the previous section, we discussed the macroeconomic effects of changing the eligibility requirement. Firm responses increase resource losses from default and dampen the positive reaction on collateral supply. In this section, we extend the central bank toolkit by an eligibility covenant, which targets the large risk-taking effects associated with long-term debt and persistent revenues.

We embed our previous results in a discussion of optimal central bank collateral policy and

[13]There are no mechanical effects on aggregate default costs by construction.

Table 4: Macroeconomic Effects of Collateral Easing

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+62%	+71%
Default Costs $\mathcal{M}$	+8%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	19%	51%
Lenient (BBB)	3%	79%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

assume that the central bank aims to minimize *violations of market discipline*, i.e., incentivizing firm risk-taking, while ensuring sufficient collateral to *facilitate monetary policy implementation*. Even though our model is too simplistic to quantitatively assess optimal policy, it is still useful to outline the key policy trade-off arising from our analysis and to discuss its potential implications for optimal collateral policy. In assuming a trade-off between violating market discipline and increasing collateral supply, we follow central banks' stated objectives (see Bindseil et al., 2017) and the literature on risky assets in the central bank collateral portfolio. In Koulischer and Struyven (2014), lenient central bank collateral policy increases credit supply and output in the private sector but implies central bank losses, because central banks are second-best user of collateral in case of a counterparty default. Similarly, Choi et al. (2021) offer a macroprudential approach to collateral policy. In their model, accepting low-quality collateral has a positive effect on bank lending, because banks can use high-quality collateral on the interbank market instead. At the same time, this exposes the central bank to potential losses.

While we do not specifically model the sources of collateral demand, we are consistent with these papers in so far that larger collateral supply is desirable but comes at a cost if this implies accepting risky collateral. Our analysis offers a complementary view, since the risk-taking decision is made at the firm level in our model. Therefore, we propose eligibility covenants to mitigate adverse collateral supply effects from firm risk-taking, while at the same time maintaining a positive quantity effect. We stress the microprudential nature of this instrument due to the absence of aggregate shocks to collateral supply and demand in our analysis.

Leverage-Based Covenants. Covenants restrict the eligible debt capacity of firms in addition to the default risk threshold $\overline{F}$ that applies uniformly to all firms. We condition covenants on firm-specific states and focus on debt-based covenants in the following. Since debt outstanding is common knowledge for firms that are sufficiently large to issue marketable debt securities, such a policy is in principle implementable. However, it still leaves us with all functions mapping from the debt state space into the binary eligibility indicator $\Psi \in \{0,1\}$. In the following, we focus on the exponential class, parameterized by $\gamma > 0$, such that the eligible debt capacity is decreasing in debt outstanding

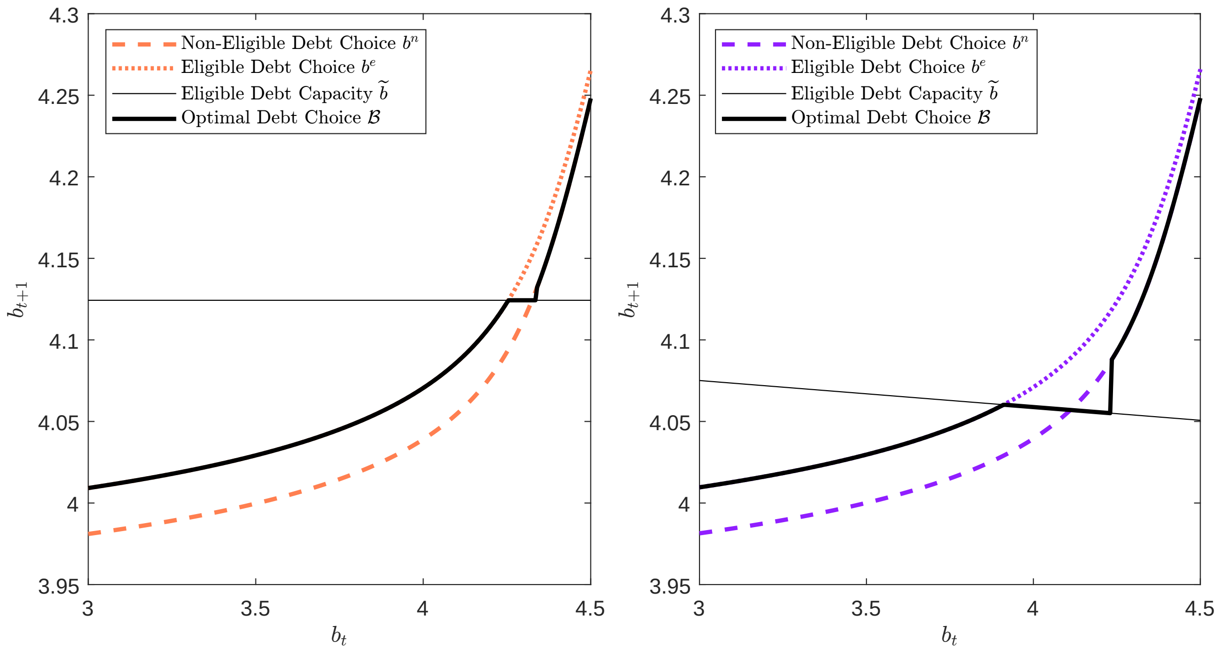
$$\widetilde{b}_{t+1}^{j} = \exp\{-\gamma b_t^j\} \cdot \frac{\exp\{\sigma \Phi^{-1}(\overline{F}) + \rho \mu_t^j\}}{\pi} \, . \tag{19}$$

The eligibility covenant $\exp\{-\gamma b_t^j\}$ effectively lowers the eligible debt capacity of firms with high debt outstanding and, thereby, provides deleveraging incentives. We fix revenues at the median and eligibility requirements at the BBB-level to visualize the impact of an eligibility covenant on the firm debt choice in Figure 4. The left panel shows the case *without* an eligibility covenant. The bold black line denotes the debt choice $\mathcal{B}_{t+1}$ for a firm with median revenues under lenient eligibility requirements. This function maps bonds outstanding b_t into (gross) bond issuance b_{t+1} and exhibits a kink and a jump. These points are associated with the debt levels where firms switch from non-eligible to constrained eligible and, then, to unconstrained eligible (see Proposition 2). The orange dashed (dotted) line represents the debt choice if the firm is non-eligible (eligible) and the horizontal black lines denotes the firm's eligible debt capacity.

Firm risk-taking and disciplining effects are related to the difference between the orange dashed line b_{t+1}^n and the equilibrium debt choice $\mathcal{B}_{t+1}$ (bold black line). The disciplining effect is represented by firms reducing their debt issuance below b_{t+1}^n, which applies to firms located near the jump of the policy function. The risk-taking effect is reflected by firms issuing debt according to $\mathcal{B}_{t+1} > b_{t+1}^n$, that is, wherever the bold black line is above the dashed orange one. Compared to the mass of firms being disciplined by collateral eligibility, the risk-taking effect is sizable.

The right panel represents the case *with* an eligibility covenant. Intuitively, introducing an eligibility covenant reduces the eligible debt capacity (light black line) if firms enter the period with large legacy debt. Compared to the debt choice without covenants, the downward sloping shape of the eligible debt capacity induces a larger disciplining effect, the optimal debt choice $\mathcal{B}_{t+1}^{j,n}$ is located below $b_{t+1}^{j,n}$ for a broader range of legacy debt stocks, and, conversely, reduces the risk-taking effect. As a result, the dampening effect of firm responses on collateral supply will be limited in the presence of an eligibility covenant.

Figure 4: Debt Choice with Eligibility Covenant

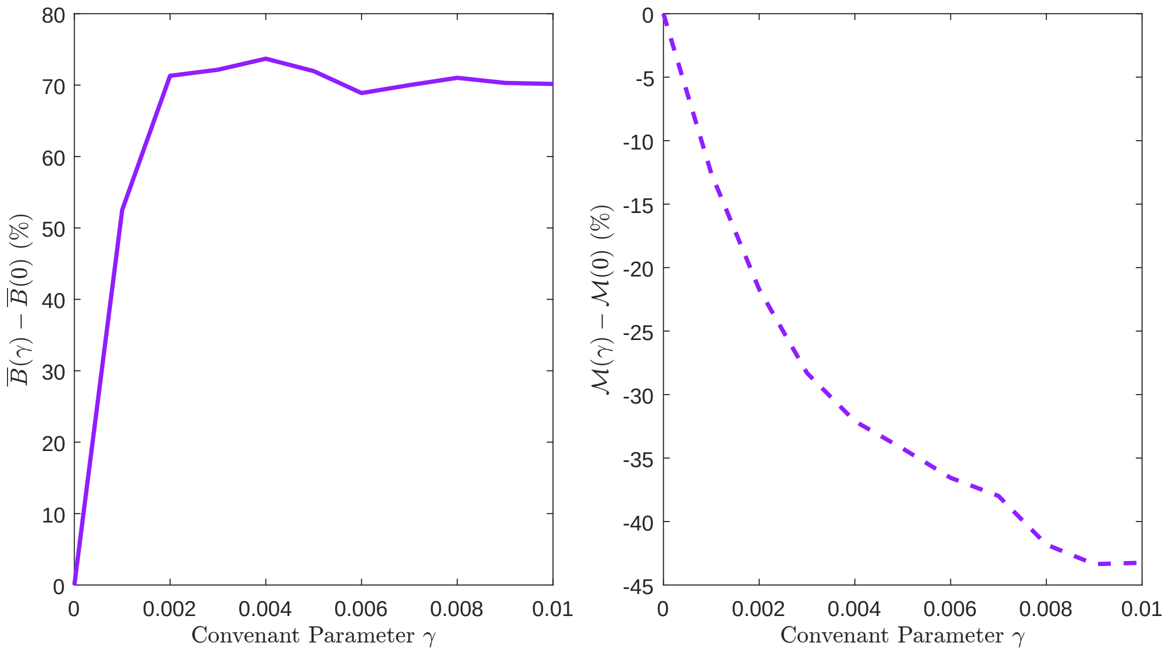


Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (16)). The purple and orange lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case without (with) covenant.

Optimal Covenants Given a Minimum Rating. We now turn to how the eligibility covenant influences collateral supply and aggregate default cost for a given eligibility requirement. The covenant has an ambiguous collateral supply impact. On the one hand, setting an overly harsh covenant (a large γ) reduces collateral supply, since it dis-incentives firms from issuing bonds. On the other hand, an overly lenient covenant (a small γ) fails to limit the risk-taking by eligible firms. We make the dependency of collateral supply $\overline{B}(\overline{F},\gamma)$ and aggregate default cost $\mathcal{M}(\overline{F},\gamma)$ on both policy parameters explicit in the following, compute them for different covenant parameters and show the results in Figure 5. The covenant gives rise to a collateral Laffer curve, that reflects this trade-off. We observe that the covenant increases collateral supply by *up to* 74% (left panel), which is very similar to the 72% increase induced by collateral easing from $\overline{F}^A$ to $\overline{F}^{BBB}$. At the same time, we observe a potential reduction in aggregate default cost of *up to* 42% (right panel). Thus, already for a fixed collateral eligibility policy, the covenant has powerful impacts.

Eligibility Covenants and the Collateral Policy Frontier. Next, we investigate how adding covenants to the central bank toolkit, in addition to the eligibility threshold, affects the collateral policy trade-off between high collateral supply and maintaining a high level of market disci-

Figure 5: Aggregate Effects of Eligibility Covenant



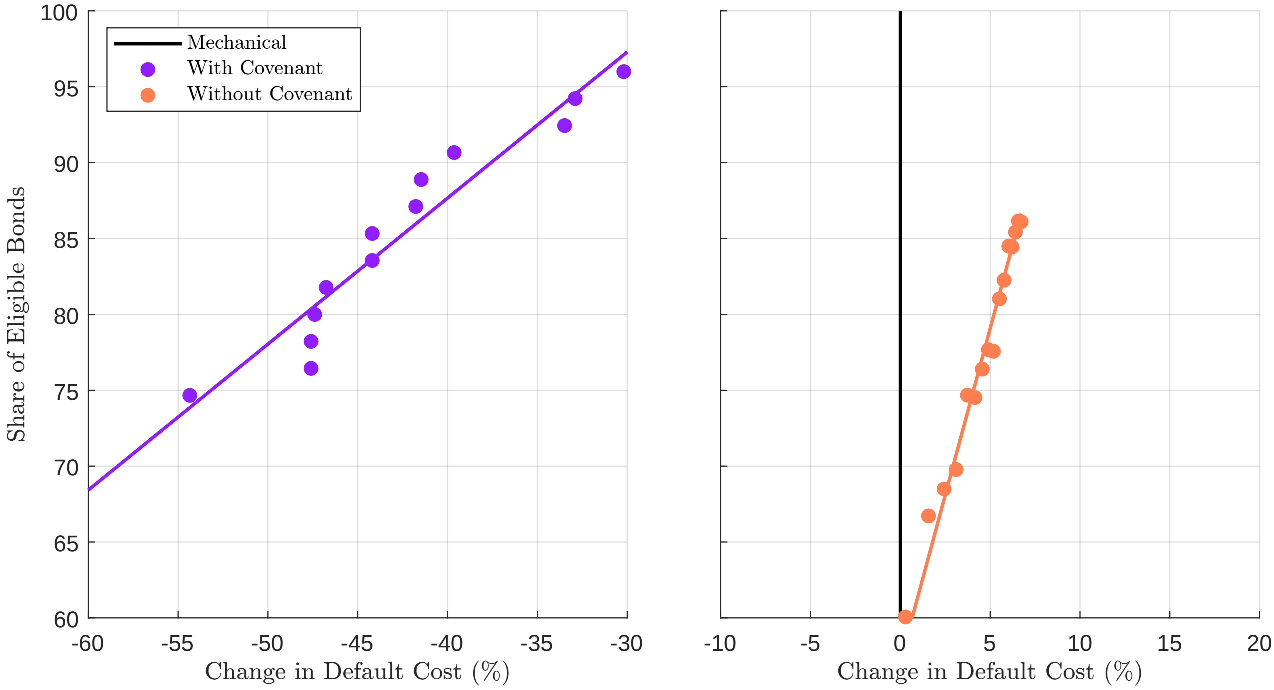
Notes: We show the effect of varying the covenant parameter γ for lenient eligibility requirements $\overline{F}^{BBB}$. The y-axis in the left (right) panel shows the percentage-increase in collateral supply (aggregate default cost) relative to the no-covenant case $\gamma = 0$.

pline. We define the cost of violating market discipline as additional default cost that derive from making corporate bonds eligible, i.e., default cost are expressed relative to an economy without bond eligibility, where all firms would issue debt according to $b_{t+1}^{j,n}$.

Figure 6 shows the results in terms of the collateral policy frontier. Each dot is associated with a fixed collateral supply target on the x-axis. For a given supply target, we then choose the policy parameters to minimize the *additional default cost* relative to the case of market discipline, which the central bank has to allow to satisfy its supply target, as shown on the y-axis. We distinguish between the baseline case with eligibility thresholds only (orange) and the extended central bank toolkit that also comprises covenants (purple). The vertical black line indicates the benchmark with no firm responses, where default cost are the same as in the case of market discipline. In this setting, the central bank could simply pick a collateral supply target and set $\overline{F}$ accordingly without adverse effects on the corporate bond market. Endogenizing firm responses gives rise to a trade-off between collateral supply and default cost, which is reflected by the positive slope of the policy frontier.

In the case without covenant (right panel), increasing the collateral supply necessitates increasing $\overline{F}$, which leads to additional default cost compared to the market discipline case. With covenant (left panel), the positive relation between collateral supply and default risk persists,

Figure 6: Collateral Policy Frontier



Notes: Both panels display the additional default cost (relative to market discipline) that are necessary to satisfy a given collateral supply target. The collateral supply target is expressed as the share of eligible bonds $\overline{B}$ relative to the market value of all bonds under market discipline. In the left panel, we vary the eligibility threshold $\overline{F}$ and the covenant parameter γ. In the right panel, the covenant parameter is fixed at $\gamma = 0$.

but the overall level of default cost is significantly lower and even falls compared to the market discipline case. At the same time, the associated collateral supply levels are substantially higher, since the covenant incentivizes firms to issue more eligible *and* default less frequently. Consequently, the collateral policy frontier is shifted outward, making this instrument a potentially powerful extension of collateral frameworks.

Implementation. In our model, the eligibility covenant is expressed in terms of firm-specific eligible debt capacities, which are negatively dependent on debt outstanding $\partial \widetilde{b}^j_{t+1} / \partial b^j_t < 0$. In practice, implementing such covenants would require information on the indebtedness of firms, i.e., about current revenues and its dynamics, as well as the maturity structure of outstanding liabilities. However, revenue dynamics and debt repayment schedules of large firms are often difficult to determine, particularly if firms have multiple subsidiaries. Therefore, several collateral frameworks (the ECB's among them) are based on credit assessments by external rating agencies.

While we abstract from modeling credit ratings and we assume that revenues and debt outstanding at the firm level is common knowledge, it is of practical importance that eligibility covenants in our model can be expressed in terms of firm-specific eligibility thresholds, which negatively dependent on debt outstanding $\partial \overline{F}_t^j / \partial b_t^j < 0$. T For sufficiently large firms, the eligibility status could be made dependent on a measure of debt outstanding *and* CDS-spreads. This would allow for a more granular classification of firms into different eligibility categories based on spreads and debt outstanding.[14] An alternative way to implement covenants is to condition eligibility on rating notches combined with debt outstanding or on the *rating outlook*, if these take into account the sustainability of firm debt in a satisfactory way. Firms rated A but with negative outlook can for example be interpreted as being on a financially unsustainable path and could, therefore, be subjected to a tighter eligibility requirement than a firm rated BB+ but with a positive outlook. This would be especially useful for firms on which no CDS are actively traded.

Last, note that collateral frameworks not only comprise eligibility thresholds but also haircuts on eligible assets. From the firm's point of view, a higher haircut reduces the collateral value of its bonds and could in principle be made dependent on debt outstanding, which would also reduce risk-taking incentives. However, covenants provide much more salient deleveraging incentives, if a firm would observe its eligible debt capacity through the investment bank handling the underwriting process of new bond issues. In contrast, a haircut would still leave new bond issues eligible. From the central bank's point of view, haircuts are often set to account for losses in asset liquidation in the event of counterparty default. Haircuts typically address a different form of risk so that the covenant remains a potentially useful extension of collateral frameworks.

5 Conclusion

This paper evaluates the effects of central bank eligibility requirements on the debt and default decision of firms, i.e., the collateral supply side. Adding collateral premia and eligibility requirements to a heterogeneous firm model with default risk reveals that firms can be affected in different ways: low-risk firms increase their debt issuance and risk-taking, whereas medium-risk firms are disciplined by the prospect of benefiting from collateral premia. Both effects increase aggregate collateral supply, while they have opposing effects on cost from corporate default. Which of these two effects is the dominating force is, therefore, a numerical question. Consistent with empirical evidence at the firm level, our numerical findings suggest that risk-taking is the dominating force in the aggregate. Endogenous firm responses are quantitatively

[14] A discussion regarding the usage of market-based credit risk assessments, such as CDS-spreads, is given by Nyborg (2017).

relevant and substantially dampen the impact of collateral easing on collateral supply. Eligibility covenants are suitable instruments to alleviate adverse risk-taking effects on collateral supply and aggregate default cost.

Our work can be extended along multiple dimensions. Interacting endogenous collateral supply with frictions on the collateral demand side, such as aggregate liquidity risk, can potentially generate interesting interactions with implications for the conduct of collateral policy. It should also be stressed that we take investment opportunities as exogenous. A model with endogenous investment allows to study real effects of eligibility requirements using a richer trade-off between distributing cashflows as dividends and investment. We also do not account for bank loans as alternative source of financing, which is also a margin affected by eligibility requirements. All extensions add additional layers of complexity to our present framework and we leave them to future research.

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A Corporate Bond Eligibility in Collateral Frameworks

This section reviews the eligibility of corporate bonds in central bank collateral frameworks. As we show in Table A.1, eligibility of corporate bonds as collateral in central bank operations varies across countries and over time. The Eurosystem stands out due to its acceptance of corporate collateral before the financial crisis.

Table A.1: Non-Financial Corporate Bonds in Various Collateral Frameworks

Country	Pre 2008 (Min. Rating)	Post 2008 (Min. Rating)	Post Covid-19 (Min. Rating)
Australia	No	Yes (AAA)	Yes (BBB)
Eurosystem	Yes (A)	Yes (BBB)	Yes (BB)*
Japan	Yes (A)	Yes (BBB)†	Yes (BBB)
Switzerland	Yes (AA)	Yes (AA)	Yes (AA)
United Kingdom	No	Yes (A)	Yes (A)
United States††	Yes (AAA)	Yes (AAA)	Yes (BBB)

Notes: †: Multiple changes after Financial Crisis; *: For the duration of PEPP; ††: Only allowed in the discount window. Source: Bank for International Settlements (2013) & national CBs.

Table A.2 gives an overview of changes in the ECB collateral framework since 2007. Corporate bonds were eligible prior to the 2008 crises at a minimum rating requirement of A. In response to the financial crises, the minimum requirements were reduced from A to BBB, which substantially extended the amount of eligible assets and, thereby, broadened financial intermediaries' access to central bank liquidity. The smaller changes in 2011 and 2013 suggest that some fine-tuning was necessary after the initial relaxation. Nevertheless, the reduction of the minimum rating requirement was by far the largest adjustment, which motivates our choice of modeling collateral policy as a step function.

Table A.2: Corporate Bonds in the ECB Collateral Framework

Timespan	Regime	Haircut: A- or higher	Haircut: BBB- to BBB+
01 Jan 2007 - 24 Oct 2008	Fitch, S&P, and Moody's are accepted ECAI, minimum requirement A-.	4.5 %	100 %
25 Oct 2008 - 31 Dec 2010	DBRS accepted as ECAI, minimum requirement BBB-.	4.5 %	9.5 %
01 Jan 2011 - 30 Sep 2013	Tightening of haircuts.	5 %	25.5 %
01 Oct 2013 - 01 Dec 2019	Relaxation of haircuts.	3 %	22.5 %

Notes: Haircuts on a corporate bond with fixed coupon and maturity of 3 to 5 years; DBRS: Dominion Bond Rating Service, ECAI: external credit assessment institutions.

B Proofs

This section contains the proofs of Propositions 1 and 2 and Lemma 1 and 2.

B.1 Proof of Proposition 1

To see why the most productive firms have the lowest default risk, we differentiate default risk with respect to s and obtain $\frac{\partial F(b^n_{t+1}|s)}{\partial s} = \frac{\partial F(b^n_{t+1}-s)}{\partial s} = f(b^n_{t+1}|s)[-1+\frac{\partial b^n_{t+1}}{\partial s}]$. We can use (B.6), which we derive below, to show that this term is unambiguously negative. Using analogous steps and (B.7), we show the same for $F(b^e_{t+1}|s)$.

We rewrite the first order conditions (3) and (4) in terms of the hazard rate as

$$h(b_{t+1}|s)\cdot b_{t+1} = 1-\beta \qquad \text{if} \quad F(b_{t+1}|s) > \overline{F}\,, \tag{B.1}$$

$$h(b_{t+1}|s)\cdot b_{t+1} = \frac{1-\beta+L}{1+L} \qquad \text{if} \quad F(b_{t+1}|s) \leq \overline{F}\,. \tag{B.2}$$

The first order condition (B.1) implies that the (debt-weighted) marginal default risk $h(b^n_{t+1}|s)\cdot b^n_{t+1}$ is identical for all s. The productivity parameter shifts the revenue distribution to the right: holding debt issuance constant, we have $\frac{\partial F(b^n_{t+1}|s)}{\partial s} < 0$ and also $\frac{\partial f(b^n_{t+1}|s)}{\partial s} < 0$ by the monotone hazard rate property so that $h(b^n_{t+1}|s)$ falls. Since the RHS of (3) is constant, increasing the productivity parameter implies that the debt choice has to increase $\frac{\partial b^n_{t+1}}{\partial s} > 0$. We perform analogous steps to show the same for b^e_{t+1}.

Finally, since $\frac{1-\beta+L}{1+L} > 1-\beta$ and using the monotonicity assumption on $h(\mu_{t+1})\mu_{t+1}$, we have that an eligible firm issues more debt than an otherwise non-eligible firm. $\square$

B.2 Proof of Proposition 2

The partitioning of firms into different groups (unconstrained eligible, constrained eligible, and non-eligible) uses the fact that there are three *potentially* optimal debt choices for every s. The first possibility is to issue bonds $\widetilde{b}_{t+1}(s)$ to be exactly at the eligibility threshold. By the strict monotonicity of $F(b_{t+1}|s)$ in b_{t+1}, there is a unique $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ for which the corporate bond is just eligible. Second, there is a debt level b^n_{t+1} satisfying the first order condition (B.1) for the case of non-eligibility. Third, the level b^e_{t+1} solves (B.2), the first order condition in the eligibility case. Under the monotonicity assumption on $h(b_{t+1}) \cdot b_{t+1}$, both conditions are satisfied by a unique b^n_{t+1} and b^e_{t+1}, respectively. The remainder of the proof characterizes which of these three debt levels is optimal, given the type parameter s.

Existence of Type Space Partitions. There is a positive mass of unproductive firms, such that $\widetilde{b}_{t+1}(s) = 0 < b^n_{t+1}(s) < b^e_{t+1}(s)$, which holds at least for $s = s^-$ by assumption. These firms are not able to issue any bonds without exceeding the minimum quality requirement $\overline{F}$, i.e., their eligible debt capacity is zero. On the other hand, there are firms with positive eligible debt capacity. This can be shown by finding values s_1 and s_2 such that $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ and $b^e_{t+1}(s_2) = \widetilde{b}_{t+1}(s_2)$, i.e., firms are able to issue debt according to (B.1) and (B.2) without losing eligibility. We then show that the cut-off values satisfy $s^- < s_1 < s_2 < \infty$.

From the mass-shifting property of s, we can express the eligible debt capacity as

$$\widetilde{b}_{t+1}(s) = F^{-1}(\overline{F}) + s\,. \tag{B.3}$$

Define the hypothetical value functions for a never eligible firm $V^n(b_{t+1}|s)$ and an always eligible firm as $V^e(b_{t+1}|s)$. Plugging (B.3) into the first order conditions (3) and (4), we get

$$\left.\frac{\partial V^n(s)}{\partial b}\right|_{\widetilde{b}_{t+1}(s)} = (1-\beta)(1-\overline{F}) - \left(F^{-1}(\overline{F})+s\right) f(F^{-1}(\overline{F}))\,, \tag{B.4}$$

$$\left.\frac{\partial V^e(s)}{\partial b}\right|_{\widetilde{b}_{t+1}(s)} = \frac{1-\beta+L}{1+L}(1-\overline{F}) - \left(F^{-1}(\overline{F})+s\right) f(F^{-1}(\overline{F}))\,. \tag{B.5}$$

For a sufficiently productive firm with a large s, the eligible debt capacity $\widetilde{b}_{t+1}(s)$ lies on the downward sloping part of the objective function. Since the objective is concave by the monotone hazard rate assumption, $\widetilde{b}_{t+1}(s)$ is not optimal and such a firm voluntarily issues less debt than it could without losing eligibility. From $1-\beta < \frac{1-\beta+L}{1+L}$ and noting that for s_1 and s_2 (B.4) and (B.5) evaluate to zero, it follows that $s_1 < s_2$.

We can exploit the monotonicity of the first order conditions in s and monotonicity of the eligible debt capacity $\frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} = 1$. Implicitly differentiating (B.1) and (B.2) with respect to s, we have

$$\frac{\partial b^n_{t+1}(s)}{\partial s} = \frac{(1-F(b^n_{t+1}|s))\, f'(b^n_{t+1}|s) b^n_{t+1} + f(b^n_{t+1}|s)^2 b^n_{t+1}}{(1-F(b^n_{t+1}|s))\, [f'(b^n_{t+1}|s) b^n_{t+1} + f(b^n_{t+1}|s)] + f(b^n_{t+1}|s)^2 b^n_{t+1}} < 1\,, \tag{B.6}$$

$$\frac{\partial b^e_{t+1}(s)}{\partial s} = \frac{(1-F(b^e_{t+1}|s))\, f'(b^e_{t+1}|s) b^e_{t+1} + f(b^e_{t+1}|s)^2 b^e_{t+1}}{(1-F(b^e_{t+1}|s))\, [f'(b^e_{t+1}|s) b^e_{t+1} + f(b^e_{t+1}|s)] + f(b^e_{t+1}|s)^2 b^e_{t+1}} < 1\,. \tag{B.7}$$

Since firms are risky by the first order conditions (3) and (4), we have $f(b^n_{t+1}|s) > 0$ and $f(b^e_{t+1}|s) > 0$ and the denominator is larger than the numerator, respectively. Therefore, the partial derivatives $\frac{\partial b^n_{t+1}(s)}{\partial s}$ and $\frac{\partial b^e_{t+1}(s)}{\partial s}$ are strictly smaller than one. Since by assumption $\widetilde{b}_{t+1}(s^-) = 0$ and $b^n_{t+1}(s^-) > 0$, $s_1 > s^-$ follows from the implicit definition $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$. Furthermore, we can conclude that the cut-off values s_1 and s_2 are unique.

Characterizing Debt Choices. For every $s > s_2$, firms issue less debt than they could issue without losing eligibility. All firms with $s > s_2$ choose debt issuance according to their first order condition and are called *unconstrained eligible*.

Consider next firms which cannot choose their optimal borrowing without losing eligibility, i.e., firms with $s < s_2$. All firms between s_1 and s_2 choose to be just eligible and lever up until $\tilde{b}_{t+1}(s)$, since for them $V^e(b^e_{t+1}(s)|s)$ is not feasible and $V^n(b^n_{t+1}(s)) < V^e(b^n_{t+1}(s)) < V^e(\widetilde{b}_{t+1}(s))$. The first inequality follows from $V^e(b_{t+1}|s) > V^n(b_{t+1}|s)$ for all b_{t+1}, holding s constant. The second inequality follows from the fact that V^e is increasing between $b^n_{t+1}(s)$ and $\widetilde{b}_{t+1}(s)$.

Finally, there is a threshold $s_0 < s_1$, below which firms choose $b^n_{t+1}(s)$ and are not eligible. All firms between s_0 and s_1 also choose $\widetilde{b}_{t+1}(s)$. The value s_0 is implicitly defined through the indifference condition $V^e(\widetilde{b}_{t+1}|s_0) = V^n(b^n_{t+1}|s_0)$. The assumptions on the revenue distribution will imply the existence of exactly one s_0 by the intermediate value theorem. To see this, consider their difference

$$\Delta(s) \equiv V^e(\widetilde{b}_{t+1}(s)|s) - V^n(b^n_{t+1}(s)|s)\,. \tag{B.8}$$

Obviously $\Delta(s_1) > 0$, because $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ and $V^e(\widetilde{b}_{t+1}(s_1)|s_1) > V^n(\widetilde{b}_{t+1}(s_1)|s_1)$. In addition, there exists a level s^- where $F(0|s^-) > \overline{F}$ by assumption. At this level $V^e(\widetilde{b}_{t+1}(s^-)|s^-) - V^n(b^n_{t+1}(s^-)|s^-) < 0$. Note that $\widetilde{b}_{t+1}(s^-) = 0$ and that $V^e(\widetilde{b}_{t+1}(s^-)|s^-)$ is the value of the unlevered firm. Choosing $b_{t+1} = 0$ would violate (B.1) and therefore $V^n(b^n_{t+1}(s)|s)$ exceeds the value of an unlevered firm for every s. Together with continuity of s, this already implies existence of at least one s_0 by the intermediate value theorem. To establish uniqueness, we differentiate $\Delta(s)$ with respect to s. The first part of $\Delta(s)$ can be written as

$$V^e(\widetilde{b}_{t+1}(s)|s) =(1-\overline{F})(1+L)\widetilde{b}_{t+1} + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - \widetilde{b}_{t+1}(s) dF(\mu_{t+1}|s)\,,$$

and its total derivative is given by

$$\begin{aligned}
\frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} &= \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial \widetilde{b}_{t+1}} \frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} + \left.\frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s}\right|_{\widetilde{b}_{t+1}} \\
&= \left((1-\overline{F})(1+L) + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} (-1) dF(\mu_{t+1}|s)\right) \frac{\partial \widetilde{b}_{t+1}}{\partial s} + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1} - \widetilde{b}_{t+1}(s)) df(\mu_{t+1}|s) \\
&= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) - \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} df(\mu_{t+1}|s) + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \widetilde{b}_{t+1}(s) df(\mu_{t+1}|s) \\
&= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) - \beta \Big(f(\overline{\mu})\overline{\mu} - f(\widetilde{b}_{t+1}(s)|s)\widetilde{b}_{t+1}(s) \\
&\quad - (1-F(\widetilde{b}_{t+1}(s)|s))\Big) + \beta \widetilde{b}_{t+1}(s) \left(f(\overline{\mu}) - f(\widetilde{b}_{t+1}(s)|s)\right) \\
&= (1-\overline{F})(1+L)\,.
\end{aligned} \tag{B.9}$$

We used again that $\frac{\partial \widetilde{b}_{t+1}}{\partial s} = 1$ and $f(\overline{\mu}) = 0$.

The second part of $\Delta(s)$ is given by

$$V^n(b^n_{t+1}(s)|s) =(1-F(b^n_{t+1}(s)|s)b^n_{t+1}(s) + \beta \int_{b^n_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - b^n_{t+1}(s) dF(\mu_{t+1}|s)\,.$$

The derivative of the second part of (B.8) is given by $\left.\frac{\partial V^n(b_{t+1}(s),s)}{\partial s}\right|_{b^n_{t+1}}$, since $\frac{\partial V^n(b_{t+1}(s),s)}{\partial b_{t+1}} = 0$ by the principle of optimality, when totally differentiating $V^n(b_{t+1}(s)|s)$ with respect to s. Specifically,

$$\begin{aligned}
\frac{\partial V^n(b^n_{t+1}(s)|s)}{\partial s} &= f(b^n_{t+1}(s)|s) \cdot b^n_{t+1}(s) + \beta \int_{b^n_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1} - b^n_{t+1}(s)) df(\mu_{t+1}|s) \\
&= (1-\beta)\,(1-F(b^n_{t+1}(s)|s)) + \beta\,(1-F(b^n_{t+1}(s)|s)) \\
&= 1 - F(b^n_{t+1}(s)|s)\,.
\end{aligned} \tag{B.10}$$

In the second line, we directly used the first order condition (B.1). Putting both parts together

$$\frac{\partial \Delta(s)}{\partial s} = \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} - \frac{\partial V^n(b^n_{t+1}(s)|s)}{\partial s} = (1-\overline{F})(1+L) - (1-F(b^n_{t+1}(s)|s)) > 0\,.$$

The sign follows from the fact that $\widetilde{b}_{t+1}(s) < b^n_{t+1}(s)$ holds in the region of interest. This implies that the default probability at $b^n_{t+1}(s)$ exceeds the eligibility threshold, i.e., $F(b^n_{t+1}(s)|s) > \overline{F}$. The inequality follows from $1-F(b^n_{t+1}(s)|s) < 1-\overline{F}$ and $L > 0$. Since $\Delta(s)$ is continuous and monotonically increasing, there exists an unique s_0 where the firm is indifferent between constrained eligibility and non-eligibility by the intermediate value theorem. All firms between s_0 and s_2 are called *constrained eligible*, firms below s_0 are *non-eligible*. □

B.3 Proof of Lemma 1

Lemma 1 can be shown by noting that collateral easing increases the eligible debt capacity across firms and that $b^n_{t+1}(s)$ and $b^e_{t+1}(s)$ are independent of the eligibility thresholds. To see that $\frac{\partial s_0}{\partial \overline{F}} < 0$, consider the indifference condition (B.8). The value of being constrained eligible $V^e(\widetilde{b}_{t+1}(s)|s)$ increases in $\overline{F}$. Differentiating the eligible debt capacity $\widetilde{b}_{t+1}(s)$ with respect to the eligibility threshold yields

$$\frac{\partial \widetilde{b}_{t+1}(s)}{\partial \overline{F}} = \frac{\partial F^{-1}\left(\overline{F}|s\right)}{\partial \overline{F}} = \frac{1}{f\left(F^{-1}(\overline{F}|s)\right)} > 0\,, \tag{B.11}$$

where the last step follows from the inverse function theorem. A constrained eligible firm will be better off after a relaxation of eligibility requirements $V^e(\widetilde{b}^A_{t+1}(s^A_0)|s^A_0) < V^e(\widetilde{b}^{BBB}_{t+1}(s^{BBB}_0)|s^A_0)$. Note also that the value of being non-eligible $V^n(b^n_{t+1}(s)|s)$ does not depend on the eligibility threshold. Taken together, we have

$$V^e(\widetilde{b}^{BBB}_{t+1}(s^{BBB}_0)|s^A_0) > V^n(b^n_{t+1}(s^A_0)|s^A_0) = V^n(b^n_{t+1}(s^{BBB}_0)|s^A_0)\,.$$

Furthermore, for a given policy s_0 has to satisfy $V^e(\widetilde{b}_{t+1}(s)|s) = V^n(b^n_{t+1}(s)|s)$. We showed in (B.9) that the value of a constrained eligible firm is increasing in the shifting parameter. Thus, the indifference point s^{BBB}_0 shifts to the left: $s^{BBB}_0 < s^A_0$. To see the effect of eligibility thresholds on s_2, it suffices to note that $V^e(b^e_{t+1}(s)|s)$ is independent of $\overline{F}$ and restrict attention to the condition pinning down the eligible debt capacity $F(\widetilde{b}_{t+1} - s) = \overline{F}$. Rearranging for s and differentiating w.r.t. $\overline{F}$ yields $\frac{\partial s_2}{\partial \overline{F}} = -\frac{1}{f\left(F^{-1}(\overline{F})\right)} < 0$. □

B.4 Proof of Lemma 2

Endogenous firm responses are residually given by subtracting the mechanical effect (6) from the total effect (5)

$$
\begin{aligned}
\overline{B}^{BBB} - \overline{B}^{A}\Big|_{endo} = (1+L)\Bigg(& \underbrace{\int_{s_0^{BBB}}^{s_2^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right)\right)\widetilde{b}_{t+1}^{BBB}(s)dG(s)}_{B.12.1} + \underbrace{\int_{s_2^{BBB}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right)\right)b_{t+1}^{e}(s)dG(s)}_{B.12.2} \\
& - \underbrace{\int_{s_0^{A}}^{s_2^{A}} \left(1 - F\left(\widetilde{b}_{t+1}^{A}(s)\right)\right)\widetilde{b}_{t+1}^{A}(s)dG(s)}_{B.12.3} - \underbrace{\int_{s_2^{A}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right)\right)b_{t+1}^{e}(s)dG(s)}_{B.12.4} \\
& - \underbrace{\int_{s_1^{BBB}}^{s_0^{A}} \left(1 - F\left(b_{t+1}^{n}(s)\right)\right)b_{t+1}^{n}(s)dG(s)}_{B.12.5}\Bigg) . \qquad \text{(B.12)}
\end{aligned}
$$

Since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms B.12.2 and B.12.4 reduce to

$$
\int_{s_2^{BBB}}^{s_2^{A}} \left(1 - F\left(b_{t+1}^{e}(s)\right)\right)b_{t+1}^{e}(s)dG(s) . \qquad \text{(B.13)}
$$

Due to the assumption $s_0^A < s_2^{BBB}$ we can split B.12.3 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, respectively. The second sub-integral can be combined with (B.13) and yields the last line of (7). The first sub-integral ranging from $[s_0^A, s_2^{BBB}]$ is used in the next step.

Note that the ordering of threshold productivity values arising from our assumptions and Lemma 1 is $s_0^{BBB} < s_1^{BBB} < s_0^A < s_2^{BBB}$. As a result, we can split B.12.1 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. The third of these sub-integrals and the remaining sub-integral from B.12.3 are combined to line three in (7). Moreover, we combine the second sub-integral of B.12.1 with B.12.5 to obtain the second line in (7). Finally, the first sub-integral of B.12.1 corresponds to the first line in (7).

The aggregate default cost can be decomposed in a similar way. Notably, it contains *all* bonds and not only eligible ones:

$$
\begin{aligned}
\mathcal{M}^{BBB} - \mathcal{M}^{A} = & \underbrace{\int_{s^-}^{s_0^{BBB}} M\left(b_{t+1}^{n}(s)\right)dG(s)}_{B.14.1} + \underbrace{\int_{s_0^{BBB}}^{s_2^{BBB}} M\left(\widetilde{b}_{t+1}^{BBB}(s)\right)dG(s)}_{B.14.2} + \underbrace{\int_{s_2^{BBB}}^{\infty} M\left(b_{t+1}^{e}(s)\right)dG(s)}_{B.14.3} \\
& - \underbrace{\int_{s^-}^{s_0^{A}} M\left(b_{t+1}^{n}(s)\right)dG(s)}_{B.14.4} - \underbrace{\int_{s_0^{A}}^{s_2^{A}} M\left(\widetilde{b}_{t+1}^{A}(s)\right)dG(s)}_{B.14.5} - \underbrace{\int_{s_2^{A}}^{\infty} M\left(b_{t+1}^{e}(s)\right)dG(s)}_{B.14.6} . \qquad \text{(B.14)}
\end{aligned}
$$

Again, since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms B.14.3 and B.14.6 reduce to

$$\int_{s_2^{BBB}}^{s_2^A} M\big(b_{t+1}^e(s)\big)dG(s)\ . \tag{B.15}$$

Splitting B.14.5 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, we can combine the second of these with (B.15) to obtain the last line of (8).

Given the ordering of threshold productivity values arising from our assumptions and Lemma 1, we can split B.14.2 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. Combining the last of these with the remaining sub-integral of B.14.5 yields the third line of (8).

Since $s_0^{BBB} < s_0^A$, we can summarize B.14.1 and B.14.4 to

$$-\int_{s_0^{BBB}}^{s_0^A} M\big(b_{t+1}^n(s)\big)dG(s) = -\int_{s_0^{BBB}}^{s_1^{BBB}} M\big(b_{t+1}^n(s)\big)dG(s) - \int_{s_1^{BBB}}^{s_0^A} M\big(b_{t+1}^n(s)\big)dG(s)\ .$$

Combining these two integrals with the remaining two sub-integrals of B.14.2 yields the first and second lines of (8). □

C Data and Computation

C.1 Corporate Bond Data

We merge monthly data on the corporate bond universe in Europe from the iBoxx High Yield and Investment Grade Index families, provided by *IHS Markit*. We apply the following inclusion criteria:

1. Bond issuers are head-quartered in euro area member countries.
2. Issuers are non-financial firms.
3. The bond is denominated in euro, senior, not callable, uncollateralized, and fixed coupon.
4. The issuer is part of the constituent list for at least 48 months.

Bond issuers are provided by *Markit* and we consider only the parent company level, since it can be reasonably assumed that dedicated financial management subsidiaries are identical from an economic perspective to the respective parent company.

Company Data. We match company names to their unique *Compustat* identifier (`gvkey`) and drop all companies which are not represented in the *Compustat Global* database. For the remaining firms we query *Compustat* for long-term liabilities (`dltt`) in the firmq database and EBIT (`ebit`) in the firma database.

C.2 Computational Algorithm

We solve the individual firm problem using policy function iteration over a discrete set of collocation points using piecewise linear interpolation. The revenue shock is discretized using the method of Tauchen on an equi-spaced grid with $n_\mu = 25$ points over the interval $[-3\hat{\sigma}, +3\hat{\sigma}]$ with $\hat{\sigma} = \frac{\sigma}{1-\rho^2}$ denoting the unconditional variance of the revenue process. We denote the corresponding transition matrix Π_μ. Debt is discretized on an equispaced grid with $n_b = 21$ points over the interval $[5.5, 15.5]$.

To overcome the typical convergence issues in models with long-term debt and default, we use taste shocks when computing the debt choice (16), as proposed by Gordon (2019). The mass shifter for endogenous states follows immediately from the debt choice and is denoted Π_b. This matrix maps the current idiosyncratic state (μ_t^j, b_t^j), into next period's endogenous state b_{t+1}^j, i.e., has dimension $n_\mu \cdot n_b \times n_\mu \cdot n_b$.

Together with the transition matrix of idiosyncratic revenues, the combined mass shifter is given by $\Pi_g = \Pi_b \otimes \Pi_\mu$. The mass shifter implicitly defines the firm distribution G via $G^T = G^T \Pi_g$, where G denotes the firm distribution. Extracting the distribution, thus, boils down to computing the right eigenvector to Π_g.

Starting with a guess for firm policies and bond prices, each iteration ι consists of four different steps:

(i) Solve the firm problem taken as given the bond price schedule and value function from the previous iteration.

(ii) Compute the eligible debt capacity (15), the associated values of the objective function, and determine the debt choice according to (16).

(iii) Obtain the ensuing mass shifter Π_g from the policy functions and the transition matrix for revenue shock Π_μ and update the distribution G by iterating on $G^T = G^T \Pi_g$.

(iv) Update bond price schedules and value functions.

We then iterate on the policy functions until convergence, i.e., $||\mathcal{B}^\iota(b_t^j, \mu_t^j) - \mathcal{B}^{\iota-1}(b_t^j, \mu_t^j)||_\infty < 10^{-5}$. The standard deviation of the taste shock is set to 0.01 to ensure convergence. This is typically achieved within 200 iterations.

D Additional Numerical Results

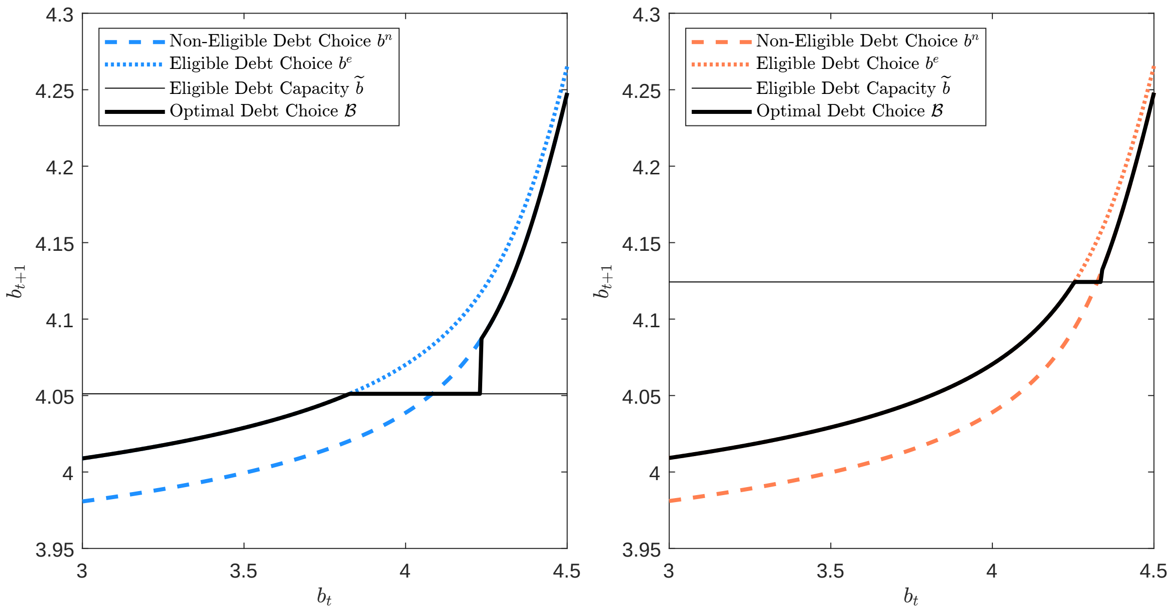
This section contains supplementary numerical results to our quantitative policy analysis. In Appendix D.1 we compare optimal debt choices under tight and lenient collateral policy. Appendix D.2 provides details on the distribution of bond spreads and default risk across firms. Appendix D.3 presents a robustness check that also includes data from the financial crisis of 2008 and consequently has a higher level of default risk. Appendix D.4 endogenizes the size of collateral premia.

D.1 Firm Debt Choices

We now illustrate how the characterization of firm debt choices carries over to the case of long-term debt. The black solid line in each panel of Figure D.1 denotes the debt choice given current debt b_t for a firm with median revenues under tight (left) or lenient (right) eligibility requirements. The colored dashed and dotted lines in either panel denote the debt choice if a firm is non-eligible or eligible, respectively. The firms' eligible debt capacity (15), which is independent of legacy debt b_t, is given by the horizontal black line. The debt choice exhibits a kink and a jump that represent the debt levels where firms change type (from non-eligible to constrained eligible and, eventually, to unconstrained eligible). The optimal debt choice (bold black line) is equal to b^e_{t+1} until it reaches its eligible debt capacity (first kink). For legacy debt levels between the kink and the jump, the firm exhausts its eligible debt capacity and is constrained eligible. Last, for debt outstanding above those at the jump, firms choose b^n_{t+1}. Similar to the one-period bond model, the effects of bond eligibility correspond to the difference between the non-eligible debt choice b^n_{t+1} and the equilibrium debt choice $\mathcal{B}_{t+1}$ (bold black line). Firms subject to risk-taking choose debt above b^n_{t+1}. Disciplined firms choose debt lower than b^n_{t+1} instead.

Comparing the left to the right panel, we observe that under lenient eligibility requirements, where the eligible debt capacity shifts upwards, the risk-taking effect becomes more prominent, while the relative size of the disciplining effect falls.

Figure D.1: Debt Choice with Collateral Easing



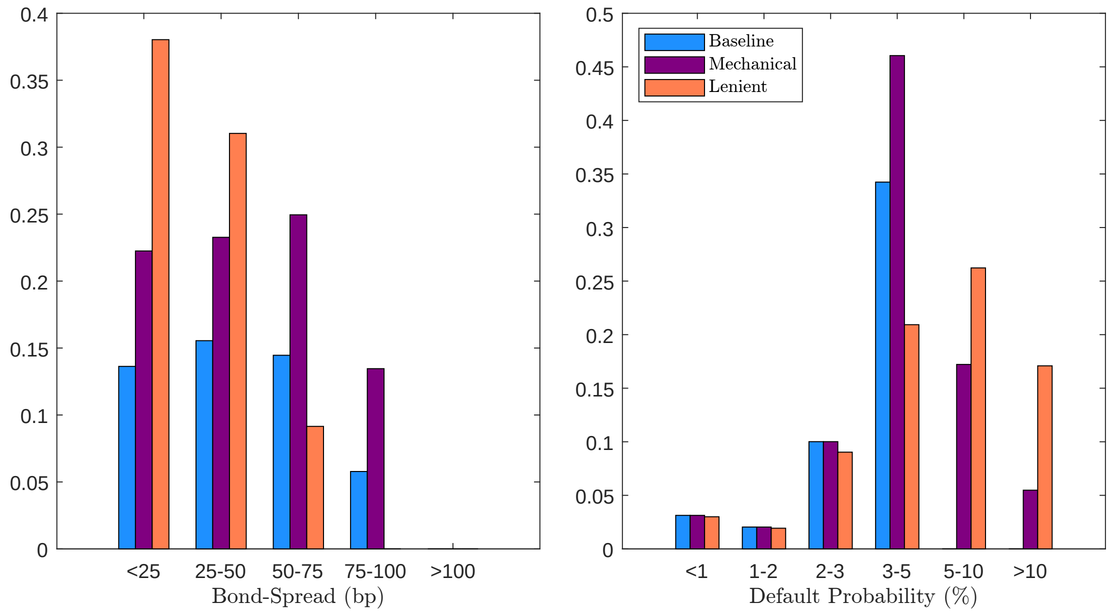
Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (16)). The colored lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case of tight (lenient) collateral policy.

D.2 Firm Distribution

While Section 3.5 condenses firm responses into the shares of risk-taking and disciplined firms, this section provides supplementary information on the firm distribution. Specifically, we compare the bond spread and default risk distributions of eligible firms for the baseline calibration (blue) to those under lenient eligibility requirements (orange), and to those under lenient eligibility requirements with constant firm behavior (purple). Differentiating between the full equilibrium response and the share of eligible bonds with constant firm behavior allows us to decompose the total collateral supply response into *mechanical* and firm effects.

The left panel of Figure D.2 divides eligible firms into different spread buckets. For the mechanical effect, we observe a rightward shift of bond spreads compared to the baseline of tight eligibility requirements, corresponding to newly eligible risky firms. Accordingly, in the right panel we observe a similar rightward shift of the distribution of eligible firms' default probabilities. Taking firms responses into account markedly increases the share of firms in the left tail of the spread distribution. This follows from the high likelihood of satisfying the minimum rating requirement in future periods, which is associated with low bond spreads. However, the default probability distribution in the right panel reveals that firm responses raise

Figure D.2: Eligible Firm Distribution over Bond Spreads



Notes: We show bond spread (left) and default risk (right) distributions across firms. Blue bars denote the economy with tight collateral policy, purple bars an economy with lenient collateral policy but fixed firm responses, and the orange bars an economy with lenient collateral policy.

the mass of eligible firms in the higher risk buckets, reflecting risk-taking effects.

D.3 Extended Sample Period

As a robustness check, we recalibrate the model and target the higher spread level over a sample encompassing the financial crisis of 2008. To match the elevated level of spreads, we set $\pi = 0.058$ and $\rho = 0.94$ to match the higher debt/EBIT-ratio as well as the increased level and cross-sectional dispersion of spreads. We calibrate $\overline{F}^A = 1.7\%$ and $\overline{F}^{BBB} = 18.5\%$ to recover the share of eligible bonds before and after relaxing eligibility requirements.

In Table D.2, we observe that firm responses dampen the impact of eligibility requirements to a similar extent as in the baseline calibration, but the mechanical and total effect are of smaller magnitude: since the firm distribution over default risk exhibits a larger dispersion, collateral easing increases $\overline{B}$ in a less effective way. However, the shares of risk-taking and disciplining firms under either policy are similar to the baseline calibration, suggesting that our characterization of endogenous firm responses does not crucially depend on the aggregate level of default risk.

Table D.1: Targeted Moments, Extended Sample

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu\|\overline{F}^A)$	4.1	3.06
Bond spread $Q_{0.25}(x\|\overline{F}^A)$	45	58
Bond spread $Q_{0.50}(x\|\overline{F}^A)$	72	92
Bond spread $Q_{0.75}(x\|\overline{F}^A)$	115	118
Eligible bond share $\overline{B}/(QB)\|\overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB)\|\overline{F}^{BBB}$	86%	77%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

Table D.2: Macroeconomic Effects of Collateral Easing, Extended Sample

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+58%	+67%
Default Costs $\mathcal{M}$	+7%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	16%	52%
Lenient (BBB)	0%	77%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

D.4 Endogenous Size of Collateral Premia

This section presents a robustness check of our results by endogenizing the *size* of collateral premia. While these have been fixed to a constant L in the baseline, we make them dependent on aggregate collateral supply. In this case, collateral premia decline after collateral easing, which reduces both risk-taking incentives for eligible firms and disciplining effects for firms slightly below the eligibility requirement. Whether and how this affects the macroeconomic effects of collateral easing can, therefore, only be assessed numerically. Assume that banks directly draw utility from holding collateral. For numerical and analytical tractability, we impose a CARA-functional form

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1 \overline{B}\right\} . \tag{D.1}$$

Table D.3: Targeted Moments, Endogenous L

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu \vert \overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x \vert \overline{F}^A)$	24	25
Bond spread $Q_{0.50}(x \vert \overline{F}^A)$	39	49
Bond spread $Q_{0.75}(x \vert \overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB) \vert \overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB) \vert \overline{F}^{BBB}$	86%	85%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

Table D.4: Macroeconomic Effects of Collateral Easing, Endogenous L

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+53%	+66%
Default Costs $\mathcal{M}$	-2%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	17%	51%
Lenient (BBB)	0%	82%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

The collateral premium in this case is given by $L = l_0 \exp\{-l_1 \overline{B}\}$. While we calibrate l_0 to match the eligibility premium of -11bp, the CARA-parameter l_1 governs the curvature of (D.1) and will be normalized to $l_1 = 1$. In Table D.3 we show the model fit corresponding to a parameter choice of $\beta = 0.994$, $\rho = 0.93$, $\sigma = 0.03$, and and $l_0 = 8.25$, while the (annualized) threshold default risk levels are given by $\overline{F}^A = 1.4\%$ and $\overline{F}^{BBB} = 18.5\%$.

Different to the baseline model with constant collateral premia, the large increase in collateral supply induces a drastic decline of the collateral premium to $L \approx 1$ bp in response to collateral easing, which decreases the *extent* of risk-taking in our model. Even though risk-taking effects still have a dampening effect on collateral supply, this is smaller than in the baseline calibration (see Table 4). Furthermore, default costs experience a slight *decline*: firms take on more risk and are less likely to be eligible, but they default less often.