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**The Bank Liquidity Channel of Financial  
(In)stability**

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# The Bank Liquidity Channel of Financial (In)stability\*

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## Abstract

We examine the system-wide effects of liquidity regulation on banks' balance sheets. In the general equilibrium model, banks have to hold liquid assets, and choose among illiquid assets varying in the extent to which they are difficult to value before maturity, e.g., structured securities. By improving the liquidity of interbank markets, tighter liquidity requirements induce banks to invest in such complex assets. We evaluate the welfare properties of combining liquidity regulation with other financial-stability policies, and show that it can complement ex-ante policies, such as asset-specific taxes, whereas it can undermine the benefits of ex-post interventions, such as quantitative easing.

**Keywords:** liquidity regulation, securitization, interbank markets, financial stability, quantitative easing

**JEL classification codes:** E44, G01, G21, G28

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# 1 Introduction

Banks' liquidity management takes the center stage in policy debates on financial stability. Their systemic importance as suppliers of liquidity to both the real and the remaining financial sector (e.g., [Kashyap, Rajan and Stein, 2002](#); [Gatev and Strahan, 2006](#); [Acharya and Plantin, 2021](#)) gives rise to the need for regulation with the goal of mitigating liquidity risk. In addition, the liquidity composition of banks' balance sheets is a relevant determinant of monetary-policy transmission (see, among others [Kashyap and Stein, 2000](#)). The failure of some banks to preserve a level of liquidity that would allow them to shield their operations from disruptions due to bank funding shocks has prompted tighter liquidity regulation around the globe, in the form of the Liquidity Coverage Ratio (LCR). While some commentators believe this new set of liquidity regulations to have improved the resilience of banks during the recent COVID-19 crisis ([Federal Reserve, 2020](#)), tighter liquidity regulation has also been associated with reduced liquidity creation in non-crisis times ([Roberts, Sarkar and Shachar, 2018](#)).

A relevant consideration affecting this trade-off is the way banks invest in assets that have limited eligibility for satisfying liquidity requirements. As securitization is an important channel through which banks seek to enhance their liquidity while accommodating risk taking in other asset classes, banks' investment in complex assets, such as structured securities, matters not only for their own solvency but also for other banks' ability to transfer credit risk. To shed light on the relationship between liquid and complex assets on the balance sheets in the banking system, this paper develops a general equilibrium model, and considers the effects of tighter liquidity regulation on banks' investment in complex assets, their provision of liquidity in the interbank market, and the implications for allocative efficiency arising from the interaction of liquidity regulation and other policies aimed at fostering financial stability.

In the model, banks maintain a required fraction of liquid assets, similar to the implementation of the U.S. Liquidity Coverage Ratio of 2013, which requires a subset of bank holding companies (BHCs) to hold an amount of high quality liquid assets (HQLA) that is sufficient to withstand their projected total net cash outflows over a 30-day period of significant stress.<sup>1</sup> They invest the remainder of their portfolios in long-term risky assets that differ only in terms of their complexity. Complex assets represent investments that are hard to value before maturity, such as non-

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<sup>1</sup>For example, the most liquid assets that can be used to satisfy the LCR without any discount include excess reserves, Treasury securities, government agency debt and MBS (not including government-sponsored agency debt and MBS), and sovereign debt with zero risk weights.

agency securitized assets and structured financial products.<sup>2</sup> In contrast, simple assets, such as corporate bonds, are relatively easy to value and exhibit an earlier resolution of the uncertainty regarding their payoffs. Some fraction of depositors of each bank demand liquidity depending on their intrinsic needs as well as their confidence in their bank, which in turn can depend on the opacity of their bank's assets and the state of the economy. Banks with excess liquidity or shortfalls relative to this demand can then trade in an interbank market.

Motivated by the observed illiquidity of complex assets during the crisis (Gorton and Metrick, 2010), we show the existence of an equilibrium in which complexity has two important implications for bank performance and the pattern of interbank trading. First, it increases a bank's exposure to aggregate shocks, resulting in a procyclical quality of liquidity provision to depositors. In good times, which corresponds to states in which risky assets yield a high expected return, banks that invest in complex assets perform better on average because depositors, who cannot observe the quality of the complex assets for an individual bank but are confident in the expected return, maintain their investment until maturity. Banks that invest in simple assets perform worse on average because depositors run on the subset of banks whose assets are revealed to be of low quality. In bad times, or crises, banks that invest in complex assets perform worse on average because uncertainty about the quality of their assets induces depositors to run. Banks that invest in simple assets perform better on average because depositors maintain their investments in the subset of banks whose assets are revealed to be of high quality.<sup>3</sup>

Second, complex-asset holdings also increase a bank's capacity to respond to liquidity stress by selling its long-term assets on the interbank market. This is because the symmetric opacity associated with complex assets reduces asymmetric information and facilitates trade (Dang, Gorton and Holmström, 2015). However, if a bank invests in simple assets that turn out to be of low quality, then it cannot sell them to raise liquidity. In this manner, our model implicitly takes into account the possibility for banks to securitize their illiquid loans, thereby making them liquid (interbank loans), as their ability to do so is spurred by their investment in complex assets.

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<sup>2</sup>Gorton and Metrick (2012) note that in the case of collateralized debt obligations, it is difficult to predict the payoff associated with each tranche. Additionally, Brunnermeier (2009) argues that the illiquidity of structured products during the crisis was associated with a loss of confidence in the ability to value these assets and in the reliability of ratings. For example, on August 9, 2007, BNP Paribas suspended valuations of three of its investment funds due to an inability to value assets that were exposed to the U.S. securitization market, eventually leading to a bank run on Northern Rock.

<sup>3</sup>Note that complex assets are not assumed to have procyclical inherent risk compared to simple assets. Their relatively procyclical character is solely due to how their opacity interacts with depositor sentiment in good versus bad times.

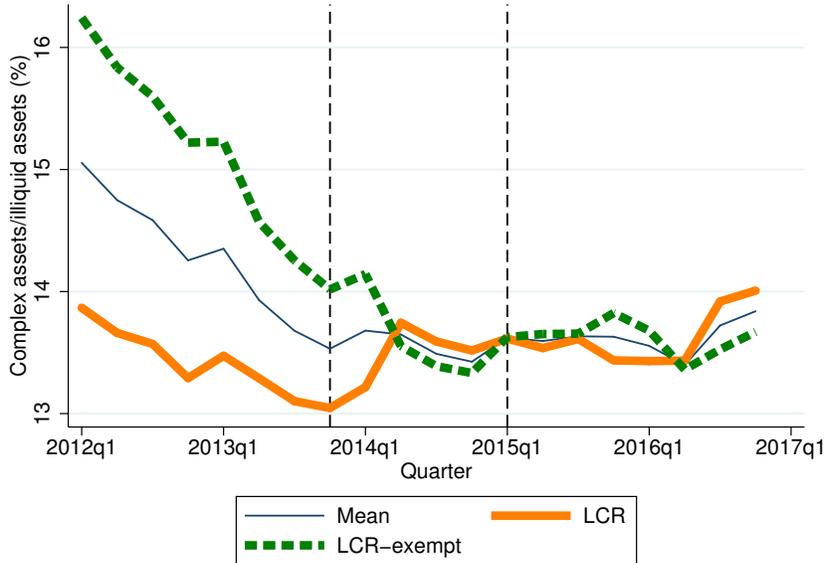


Figure 1: The effect of the Liquidity Coverage Ratio (LCR) on holdings of complex assets. This figure shows the mean ratio of complex assets to illiquid assets, separately for all bank holding companies that were subject to the LCR and those that were exempt from it. Illiquid assets are total assets minus liquid assets, where liquid assets consist of cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and government agency debt and MBS (not including government-sponsored agency debt and MBS). Complex assets consist of GSE MBS, non-agency MBS, asset-backed securities, and structured financial products. The first dashed line indicates the first quarter after the proposal of the LCR (2013Q4), and the second dashed line indicates the first quarter after the implementation of the LCR (2015Q1). Data source: FR Y-9C reports.

We use the model to analyze how liquidity regulation affects the degree to which banks invest in complex assets. We illustrate channels by which tighter liquidity regulation can either substitute for or complement investment in complex assets. On the one hand, requiring banks to hold greater liquidity buffers reduces the liquidity advantage of complexity in good times. On the other hand, it also increases the supply of liquidity in bad times, which leads to an increase in asset prices. Higher anticipated asset prices partially insure banks against runs associated with complex assets, which encourages greater ex-ante investment in the latter. When calibrating the model to the Great Financial Crisis, tighter liquidity regulation has a net positive effect on banks' investment in complex assets, which in turn dampens the effect of liquidity regulation in supporting asset prices during crises. This characterization of complex and liquid assets as complements matches our empirical evidence in Figure 1 that following the implementation of the LCR, affected

banks increase the portion of complex assets in their portfolio of illiquid assets, in comparison to banks that are exempt from the LCR.

To the extent that the availability and use of securitization, fitting our description of complex assets, has enabled lending to subprime borrowers, which is seen as a key precursor to the financial crisis (Mian and Sufi, 2009), our main result points to a potentially destabilizing effect on the financial system as an unintended consequence of liquidity regulation. Through the lens of our model, we then explore how liquidity regulation can be combined with other policies to counteract this effect and foster financial stability. Liquidity regulation can be used to complement ex-ante financial-stability policies, such as asset-specific taxes. In particular, the equilibrium degree of investment in complex assets is generically inefficient because the interbank lending market provides incomplete insurance, resulting in a distortionary pecuniary externality. Liquidity requirements determine how the equilibrium investment in complex assets compares to the level chosen by a constrained planner.<sup>4</sup> The constrained-efficient investment in complex assets can be induced via asset-specific taxes, but whether simple or complex assets should be taxed depends on the tightness of liquidity requirements.

As liquidity regulation affects banks' liquid-asset portfolio and their willingness to provide funds in the interbank market, the liquidity of which determines the pass-through of monetary-policy rates to interbank rates (see, e.g., Bianchi and Bigio, 2021), our model also links to monetary-policy transmission. Given central-bank purchases of illiquid assets in the course of quantitative easing (QE), we zoom in on the interaction between liquidity regulation and QE, which are concurrently implemented policies not only in the U.S. but also in the euro area. We show that tighter liquidity regulation can undermine the benefits of ex-post policies such as QE, i.e., asset purchases by the government in bad times. QE leads to higher asset prices to support solvent but illiquid banks, but it also involves a cost since the bond purchases must be financed with taxes. When undertaken as a surprise, QE always improves welfare. However, if QE is predictable, then banks respond by shifting their portfolios towards complex assets ex ante, which has an offsetting negative effect on the complex-asset price. Because of this attenuation, the gains from QE may wind up too small relative to its financing costs.

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<sup>4</sup>To be more precise, on the one hand, the planner may have a stronger incentive to invest in complex assets compared to the individual banks because it internalizes the full return of these assets in the bad state, whereas the individual banks that invest in complex assets receive only a fraction of this return based on the interbank market price. On the other hand, the planner may have a stronger incentive to invest in simple assets because it internalizes that this would effectively distribute more liquidity to the liquidity-shocked depositors of the distressed banks. If liquidity requirements are sufficiently tight, each safe bank has a large amount of excess liquidity that can be used to buy assets from the distressed banks, and the latter effect dominates.

**Relation to the literature.** In this paper, we set out to analyze how liquidity regulation affects banks' balance sheets, in particular the composition of illiquid assets that are not eligible to satisfy liquidity requirements imposed by rules such as the Liquidity Coverage Ratio. We further consider how this channel influences the effect of liquidity regulation on interbank debt markets, welfare, and the effectiveness of financial-stability policies in a general equilibrium model.

Allen and Gale (2017) provide a survey of the literature on liquidity regulation. They remark that there is little consensus regarding the specific nature of the market failures that it is intended to target. For example, liquidity regulations have been motivated on the basis of correcting for fire-sale externalities in short-term funding markets (Perotti and Suarez, 2011) as well as incomplete information of depositors about a bank's vulnerability to a run (Diamond and Kashyap, 2016). Dewatripont and Tirole (2018) analyze inconsistent shocks and interactions between liquidity regulation and solvency concerns. Lutz and Pichler (2021) study optimal liquidity regulation in an environment where banks face, unlike in our model, a liability choice and an asset choice, but only between liquid and illiquid investments, i.e., without any further differentiation among illiquid assets as in our model.

In terms of the empirical documentation of the effects of the LCR or very similar policies on banks' asset portfolio and interbank markets, Banerjee and Mio (2018) show that liquidity regulation in the UK led to higher investment in liquid assets and reduced reliance on short-term intra-financial loans and wholesale funding. Bonner and Eijffinger (2016) document that liquidity regulation in the Netherlands led to increased demand for long-term interbank loans. In the U.S., the LCR has been associated with reduced liquidity creation and fire-sale risk (Roberts, Sarkar and Shachar, 2018). Afonso et al. (2020) argue that liquidity regulations may have increased banks' desired level of reserves, potentially contributing to the high volatility in the U.S. repo market in September 2019. BIS (2017) argues that the LCR may lead to segmentation in repo markets by increasing the demand for trades that allow banks to maintain their regulatory ratios. In contrast to these existing findings, we present motivating evidence that the LCR has increased banks' ability to invest in complex, hard-to-value assets, which we rationalize in our model.

Our paper also contributes to a strand of the literature on policy interventions that are meant to support banks during crises, in particular quantitative easing. A natural connection to our model arises from the fact that, as pointed out by Chakraborty, Goldstein and MacKinlay (2020), quantitative easing interacts directly with banks' complex-asset holdings, e.g., structured securities, as the latter were targeted during two rounds of asset purchases in the U.S. Holmstrom and Tirole (1998) argue that government interventions to actively manage liquidity supply

can be welfare improving when liquidity shocks are correlated. However, [Farhi and Tirole \(2012\)](#) show that the anticipation of bailouts can induce banks to take excessive correlated risks.

Besides considering how liquidity regulation interacts with QE with and without commitment, we also consider ex-ante financial-stability policies such as asset-specific taxes. We show that they can be used to implement a constrained-efficient level of investment in complex assets. As such, our paper relates to [DiTella \(2019\)](#) and the characterization of optimal financial-regulation policy therein, showing that the socially optimal allocation can be implemented with a tax on asset holdings internalizing hidden-trade externalities.

More generally, our model is related to papers on financial crises, which the literature has argued to result from either weak fundamentals or panics ([Goldstein, 2012](#)). A self-fulfilling crisis can be caused by a panic among bank depositors, as in [Diamond and Dybvig \(1983\)](#), or among currency speculators, as in [Obstfeld \(1996\)](#). By contrast, fundamentals-based crises are analyzed by [Chari and Jagannathan \(1988\)](#), [Jacklin and Bhattacharya \(1988\)](#), [Allen and Gale \(1998\)](#), and [Baron, Verner and Xiong \(2021\)](#) for banks, and by [Krugman \(1979\)](#) for currency crises. Both of these views have also been considered in global coordination games by [Morris and Shin \(1998\)](#) and [Corsetti et al. \(2004\)](#) for currency attacks, and by [Morris and Shin \(2004\)](#) and [Corsetti, Guimarães and Roubini \(2006\)](#) for debt crises.

Specifically, the degree of “complexity” in bank portfolios in our setting is somewhat related to the model in [Dang, Gorton and Holmström \(2015\)](#), which focuses on optimal security design. They show that debt is welfare maximizing and information insensitive, and can give rise to crises. This is because when a bad systemic shock occurs, information-insensitive securities become more sensitive to information acquisition. In contrast, in our model there is no information acquisition. More than that, simple and complex securities are identical ex ante, and neither banks nor depositors obtain any information about complex assets prior to their maturity. Most importantly, rather than on security design, we focus on how liquidity regulation interacts with banks’ complex-asset holdings that are associated with greater informational uncertainty, with crucial repercussions for interbank markets, welfare, and monetary-policy transmission (quantitative easing in particular).

Our model’s implications regarding fire sales in the interbank market link to other theories of asset sell-offs during financial crises (e.g., [Shleifer and Vishny, 1992,9](#); [Kiyotaki and Moore, 1997](#)). Fire sales can be exacerbated by predatory trading ([Brunnermeier and Pedersen, 2005](#)). In addition, a run-up in either the repo or asset-backed commercial paper market can occur due to an increase in “money demand” ([Gorton and Metrick, 2012](#)) or global imbalances ([Caballero and Krishna-](#)

murthy, 2009). Adverse selection can also lead to fire sales in the interbank market. For example, under adverse selection in secondary debt markets (Gorton and Pennacchi, 1990), costly information acquisition (Ahnert and Kakhbod, 2018) and information production may be destabilizing (Dang, Gorton and Holmström, 2015; Gorton and Ordóñez, 2014). In contrast, and rather complementary, to these models, our fire-sale mechanism hinges on interactions between liquidity requirements and banks' choice to invest in complex or simple assets.

## 2 Model

This section introduces a model in which liquidity risk and liquidity regulation affect a bank's incentive to invest the portion of its portfolio that is ineligible for satisfying liquidity requirements in either complex or simple assets. We then characterize the equilibrium, and illustrate channels by which tighter liquidity requirements affect asset prices and investment in complex assets. Finally, we show that the equilibrium investment in complex assets can be either excessive or insufficient depending on the tightness of liquidity requirements.

### 2.1 Environment

**Overview.** There are three periods,  $t \in \{0, 1, 2\}$ . There is a mass one of limited-liability banks indexed by  $i \in [0, 1]$ . At date  $t = 0$ , each bank acquires funding from a mass one of depositors that each deposit one unit of capital. Liquidity regulations require banks to hold a fraction of their assets in liquid investments. Banks can invest their remaining assets in long-term, risky investments of varying complexity.

At date  $t = 1$ , the economic state  $\omega$ , which is publicly observed, is realized as either good,  $\omega = g$ , or bad,  $\omega = b$ . It is commonly known that the good state is realized with probability  $\eta$ . Subsequently, some depositors may withdraw early. Each bank may then have an excess or a shortfall of liquidity that it can trade against in an interbank market. At date  $t = 2$ , asset returns are realized, interbank trades are completed, and banks distribute any profits back to their depositors.

**Depositors.** There are two types of depositors. A *normal (late)* depositor has a constant marginal utility of 1 for all payoffs, regardless of when they are received. A *liquidity-shocked (early)* depositor experiences a liquidity shock at period 1, which is represented by having a marginal utility that is equal to  $\alpha > 1$  for the first  $\kappa > 1$  units of capital received in period 1, and that is equal to 1 for capital in excess of  $\kappa$  in period 1 or any payoffs received in period 2. A depositor's utility  $U(x)$  in period

1 from consuming  $x$  can thus be summarized as follows:

$$U(x) = x\mathbf{1}_{\text{shocked}}\left(\alpha\mathbf{1}_{\{x\leq\kappa\}} + \mathbf{1}_{\{x>\kappa\}}\right) + x\mathbf{1}_{\text{normal}}.$$

Each depositor's type is private information, but the fraction of liquidity-shocked depositors  $\phi \in (0, 1)$  is publicly known.

**Liquidity regulation.** In period 0, each bank must invest a fraction  $L$  of its assets in liquid investments that can be used to satisfy a regulatory liquidity requirement. In period 1, a bank can use its liquid assets to pay depositors who withdraw early.<sup>5</sup> We assume  $L > \kappa\phi$  to ensure that a bank has sufficient liquidity to meet the liquidity needs of the liquidity-shocked depositors. Liquid assets that are held until period 2 yield a return that is normalized to 1.

**A bank's portfolio choice.** The remaining fraction  $1 - L$  of a bank's portfolio can be invested in long-term, risky investments that mature in period 2. There are two types of investments that we denote by  $\theta$ . Complex assets, denoted by  $\theta = C$ , represent investments the quality of which is relatively difficult to evaluate before maturity, such as securitized assets and structured financial products. Simple assets, denoted by  $\theta = S$ , represent investments the quality of which can be evaluated relatively easily before maturity, such as corporate bonds. Specifically, the returns for simple assets become public knowledge in period 1, whereas the returns for complex assets are not known until they mature in period 2.

The two types of investments have identical return distributions that depend on the realization of the economic state  $\omega$ . Specifically, both yield a return of  $R > 0$  with probability  $\mu_\omega$  (depending on the economic state) and 0 otherwise, where  $\mu_g > \mu_b$ .

An important feature of our model is that any bank can choose at date  $t = 0$ , by deciding on its investment in either complex or simple assets, whether the return on its long-term, risky assets will become public knowledge in period 1 or not. Banks also have the option to invest their entire portfolio in liquid assets. A bank chooses its portfolio to maximize the expected utility of its depositors.<sup>6</sup> For simplicity of language, we refer to banks invested in simple, complex, or liquid investments as simple, complex, or liquid banks, respectively. A bank's portfolio choice is publicly observed.

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<sup>5</sup>This is consistent with the guidance for the implementation of the Liquidity Coverage Ratio articulated in [Basel Committee on Banking Supervision \(2013\)](#), which states that firms may temporarily break the requirement during periods of financial stress.

<sup>6</sup>Each bank can be understood as being mutually owned by its depositors, like in [Diamond and Dybvig \(1983\)](#).

**Debt contract.** In period 0, each bank promises to pay  $R_{d,i}$  to depositors that withdraw early in period 1, assuming it can meet the demand for liquidity. In period 2, the bank pays the remaining value of its assets to depositors that withdraw late. If the bank cannot meet the demand for liquidity in period 1, then it is said to experience a run. Specifically, the bank is liquidated in period 1, and each depositor receives a return in proportion to the bank's total value after liquidation.<sup>7</sup> Any remaining long-term assets that are not sold in the interbank market are liquidated and yield a return of zero. A bank chooses its early repayment to maximize the expected utility of its depositors.

**Interbank market.** In period 1, an interbank market allows banks with insufficient liquidity relative to the demand from early depositors to sell their long-term assets to banks with excess liquidity.

For convenience of notation, define a *normalized unit* of complex assets as the amount that yields an expected payoff of 1. In particular, a normalized unit of complex assets is equal to  $\frac{1}{\mu_\omega R}$  units of complex assets. Denote the state-dependent price for a normalized unit of complex assets by  $P_C(\omega)$ . Similarly, a normalized unit of simple assets with a high return is equal to  $\frac{1}{R}$  units of simple assets. Denote the price for a normalized unit of simple assets with a high return by  $P_S(\omega)$ . Note that simple assets with a low return cannot be sold since they are publicly observed to be worthless. Normalized units will be implicitly assumed for the rest of the paper.

The pattern of trade is as follows. If the mass of withdrawals in period 1 for bank  $i$  is equal to  $\alpha_i(\omega)$ , then the bank's net liquidity position in period 1 is given by  $y_i(\omega) = L - \alpha_i(\omega)R_{d,i}$ . If a bank has a liquidity shortfall, i.e.,  $y_i(\omega) < 0$ , then it would like to sell  $\frac{-y_i(\omega)}{P_\theta(\omega)}$  of its assets to generate enough liquidity to avoid a run. However, a bank can only sell up to  $1 - L$  units of long-term assets, which corresponds to  $\mu_\omega R(1 - L)$  normalized units of complex assets or  $R(1 - L)$  normalized units of simple assets. A bank's supply of assets on the interbank market can be summarized by

$$S_{B,i}(P_C(\omega)) = \left[ \frac{-y_i(\omega)}{P_C(\omega)} \wedge \left( \mathbf{1}_{\{\theta_i=C\}}\mu_\omega R + \mathbf{1}_{\{\theta_i=S \ \& \ R_i=R\}}R \right) (1 - L) \right]^+,$$

where  $A \wedge B$  denotes  $\min\{A, B\}$  and  $[A]^+$  denotes  $\max\{0, A\}$ .

If a bank has excess liquidity, i.e.,  $y_i(\omega) > 0$ , then its demand for long-term assets depends on how the return compares to the return of 1 on its liquid assets. Specifically, in the market for long-term assets of type  $\theta$ , the bank fully invests in  $\frac{y_i(\omega)}{P_\theta(\omega)}$  normalized units if  $P_\theta(\omega) < 1$ , it is indifferent if  $P_\theta(\omega) = 1$ , and it will hold on

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<sup>7</sup>It is mathematically equivalent to alternatively suppose that each depositor receives  $R_d$  with a uniform probability that depends on the bank's value after liquidation.

to its liquid assets if  $P_\theta(\omega) > 1$ . A bank's demand can thus be summarized by

$$D_{\theta,i}(P_\theta(\omega)) = \mathbf{1}_{\{P_\theta(\omega) < 1\}} \frac{y_i(\omega)}{P_\theta(\omega)} + \mathbf{1}_{\{P_\theta(\omega) = 1\}} [0, y_i(\omega)],$$

where  $[0, y_i(\omega)]$  indicates the respective range as the bank is indifferent between investing any amount up to  $y_i(\omega)$  if  $P_\theta(\omega) = 1$ .

The price is determined by the market-clearing condition:

$$\int D_{\theta,i}(P_\theta(\omega)) di = \int S_{\theta,i}(P_\theta(\omega)) di.$$

Note that the interbank market can also be interpreted as a repo market with a haircut of  $h_\theta(\omega) = 1 - P_\theta(\omega)$ , where in the repo-market interpretation  $P_\theta(\omega)$  represents the price of a bond backed by assets of type  $\theta$ . See Online Appendix A for details.

## 2.2 Equilibrium

We consider the following equilibrium.

**Proposition 1** (Equilibrium). *Assume the following parametric restrictions:*

$$\frac{\eta(\alpha\phi + 1 - \phi) + (1 - \eta)\alpha}{\eta\mu_g + (1 - \eta)\mu_b} < R \quad (1)$$

$$\frac{\eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) + (1 - \eta)\mu_b(1 - \phi - \alpha\phi(\kappa - 1))}{(1 - \eta)\mu_b(1 - \phi\kappa)} < R \quad (2)$$

$$R < \frac{L(1 - \phi)}{1 - L}(\alpha - 1) \quad (3)$$

$$R < \frac{L(1 - \phi)}{1 - L\phi}\alpha < \alpha \quad (4)$$

$$(L + \mu_b R(1 - L)) \left( 1 + \frac{(1 - \eta)(1 - \phi)}{\eta\phi} \right) < \kappa \quad (5)$$

$$\kappa < L + \mu_g R(1 - L). \quad (6)$$

Then there exists an equilibrium in which the following hold:

1. All banks invest in long-term assets and do not hold excess liquidity.
2. Banks pay depositors that withdraw early a return of  $R_d = \kappa$ .
3. Liquidity-shocked depositors always withdraw early, and normal depositors withdraw early if and only if
  - the bank is complex and the economic state is bad.

- *the bank is simple and its individual return is low.*

4. The price for simple assets is  $P_S^*(\omega) = 1$ , and the price for complex assets satisfies  $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < P_C^*(g) = 1$ .

The proofs of all propositions are relegated to the Appendix.

**Depositor choices.** The existence of an equilibrium in which depositors run under the described conditions follows from the stated assumptions. The assumptions in (3) and (4) ensure that the maximal return on long-term assets  $R$  is small enough relative to the liquidity shock  $\alpha$  that liquidity-shocked depositors always withdraw early. The assumptions in (5) and (6) ensure that the early payment  $R_d = \kappa$  is large enough that there exists an equilibrium in which normal depositors have an incentive to withdraw early under the described conditions, but small enough that there exists an equilibrium in which normal depositors withdraw late under the described conditions (see the proof of Lemma 2 in the Appendix for details).

Note that if all of a bank’s depositors withdraw early, then the bank experiences a run.<sup>8</sup> The feature that complex banks, i.e., banks invested in complex assets, experience a run when the economic state is bad is consistent with the observation that uncertainty regarding asset valuations was associated with illiquidity during the Great Financial Crisis (Gorton and Metrick, 2012).<sup>9</sup> The bank-run conditions for the two types of banks are summarized in Table 1 below.

Table 1: This table indicates when a run occurs for a complex bank (left panel) and a simple bank (right panel).

		(a) Complex		(b) Simple	
		Individual return		Individual return	
		High	Low	High	Low
State	Good				X
	Bad	X	X		X

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<sup>8</sup>For a complex bank, this follows from the assumption in (5). Specifically, (5) implies that the liquidity demand when all depositors withdraw early,  $\kappa$ , is greater than the sum of the bank’s liquid assets,  $L$ , and the funds that it can generate by selling complex assets in the bad state,  $P_C^*(b)\mu_b R(1-L)$ . Similarly, for a simple bank that draws a low return, the assumption  $L < 1 < \kappa$  implies that the liquidity demand when all depositors withdraw early,  $\kappa$ , is greater than sum of the bank’s liquid assets,  $L$ , and the funds that it can generate by selling assets, which in this case is zero.

<sup>9</sup>Note that in general, the equilibrium in which depositors run on complex banks in bad times is not unique. We choose to focus on that equilibrium to match the motivating evidence from Gorton and Metrick (2012). It can be interpreted as a “panic” among depositors in the bad state.

**Bank choices.** Banks optimally invest in one of the two types of long-term assets because they have a higher return compared to liquid assets, which is given by the assumption in (1).<sup>10</sup> Banks optimally pay a return of  $R_d = \kappa$  to depositors that withdraw early because the elevated marginal utility  $\alpha > 1$  of liquidity-shocked depositors creates an incentive to provide their full liquidity need  $\kappa$ .<sup>11</sup>

**Interbank market equilibrium.** The supply of simple assets is always equal to zero. This is because only simple banks with a low return experience a run, but they cannot sell their observably worthless assets. Therefore, the price is at the maximum level,  $P_S^*(g) = P_S^*(b) = 1$ .

For complex assets, the price depends on the economic state. In good times, the supply is equal to zero since complex banks do not experience a run. Therefore, the price is at the maximum level,  $P_C^*(g) = 1$ . In bad times, complex banks experience a run and need to raise funds by selling their assets. At the same time, simple banks with a positive individual return have excess liquidity. Thus, the equilibrium price  $P_C^*(b)$  may be less than 1.

More specifically, since banks are ex-ante identical and both types of risky assets are held in equilibrium, the equilibrium price is determined by the condition that banks are indifferent between investing in complex and simple assets.<sup>12</sup> Given the debt contract,  $R_d = \kappa$ , and bank-run conditions as described in Proposition 1, the expected utility from investing in complex assets as a function of  $P_C(b)$  can be written as

$$\begin{aligned} \mathbf{E}[U_C|P_C(b)] = & \eta \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} \quad + \underbrace{L - \kappa\phi + \mu_g R(1-L)}_{\text{return to normal dep.}} \right) \\ & + (1 - \eta) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \left( \underbrace{L + P_C(b) \mu_b R(1-L)}_{\text{liquidation value}} \right), \end{aligned} \quad (7)$$

<sup>10</sup>See the proof of Lemma 1 in the Appendix for details.

<sup>11</sup>The optimal debt contract is also supported by the assumption in (5), which ensures that the liquidity need  $\kappa$  is large enough that banks are willing to experience a run in bad times in order to meet the full need in good times. See the proof of Lemma 2 in the Appendix for details.

<sup>12</sup>Note that there does not exist an equilibrium in which all banks invest in just one of the two types of risky assets. See the proof of Lemma 3 in the Appendix for details.

and the expected utility from investing in simple assets can be written as

$$\begin{aligned}
\mathbf{E}[U_S|P_C(b)] = & \eta \left( \underbrace{\mu_g}_{\text{return to shocked dep.}} \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + R(1-L)}_{\text{return to normal dep.}} \right) \right. \\
& \left. + (1 - \mu_g) \left( \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} + \underbrace{L}_{\text{liquidation value}} \right) \right) \\
& \left. + (1 - \eta) \left( \underbrace{\mu_b}_{\text{return to shocked dep.}} \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{\frac{L - \kappa\phi}{P_C(b)}}_{\text{buy complex assets}} + R(1-L) \right) \right. \\
& \left. + (1 - \mu_b) \left( \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} + \underbrace{L}_{\text{liquidation value}} \right) \right), \tag{8}
\end{aligned}$$

where the blue terms correspond to cases where there is no bank run and the red terms correspond to cases where there is a bank run.

The relative benefit of investing in complex assets is then given by subtracting (8) from (7):

$$\begin{aligned}
\Delta(P_C(b)) & \equiv \mathbf{E}[U_C|P_C(b)] - \mathbf{E}[U_S|P_C(b)] \\
& = \eta\mu_g * 0 \\
& \quad + \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) \\
& \quad + (1 - \eta)\mu_b \left[ (\alpha\phi + 1 - \phi)(L + P_C(b)\mu_b R(1 - L)) - \left( \alpha\kappa\phi + \frac{L - \kappa\phi}{P_C(b)} + R(1 - L) \right) \right] \\
& \quad + (1 - \eta)(1 - \mu_b)(\alpha\phi + 1 - \phi)P_C(b)\mu_b R(1 - L). \tag{9}
\end{aligned}$$

The intuition is as follows. The first line of (9) reflects the fact that conditional on drawing a high return in the good state, complex and simple banks both achieve the same utility.

The second line reflects the fact that conditional on drawing a low return in the good state, complex banks achieve a higher utility because they can still service the full liquidity need of the liquidity-shocked depositors in period 1, whereas simple banks experience a run.

The third line reflects the fact that conditional on drawing a high return in the bad state, simple banks achieve a higher utility because they can service the full liquidity need of the liquidity-shocked depositors, earn a return on asset purchases

from the interbank market, and accrue the return on its long-term assets, whereas complex banks experience a run.

The fourth line reflects the fact that conditional on drawing a low return in the bad state, complex banks achieve a higher utility because they can sell assets to reduce the liquidity shortfall in a run.

Table 2 summarizes which asset has an advantage depending on the individual return and aggregate state.

Table 2: This table shows which type of asset (simple or complex) has an advantage depending on the individual return and aggregate state.

		Individual return	
		High	Low
State	Good	Neither	Complex
	Bad	Simple	Complex

The equilibrium complex-asset price is determined by equating the net advantage of simple banks in the bad state to the net advantage of complex banks in the good state. Lemma 3 in the Appendix shows that the price satisfies  $\frac{1}{R} < P_C^*(b) < 1$ . The relationship  $\frac{1}{R} < P_C^*(b)$  is supported by the assumption in (2). The feature that the asset price is lower in bad times compared to good times,  $P_C^*(b) < 1$ , is consistent with the drop in asset prices that was observed during the Great Financial Crisis (Gorton and Metrick, 2012), but also with the idea that complex banks made use of securitization to increase their liquidity during the run-up to the crisis.

**Total investment in complex assets.** Finally, the complex-asset price in bad times and the mass of investment in complex assets, which we dub the “volume of complex banks” and denote by  $V^*$ , are inversely related based on the market-clearing condition.

**Proposition 2** (Volume of complex banks). *The volume of complex banks is related to the complex-asset price in bad times as follows:*

$$\underbrace{V^*R(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{(1-V^*)\mu_b \frac{L-\kappa\phi}{P_C^*(b)}}_{\text{complex-asset demand}}. \quad (10)$$

### 2.3 The Effect of Tightening the Liquidity Requirements

This subsection illustrates channels by which tighter liquidity requirements affect the equilibrium complex-asset price and the degree of investment in complex assets.

First, note that the effect of tightening liquidity requirements on the equilibrium complex-asset price in bad times is inversely related to its effect on the incentive to invest in complex assets. In particular, if tightening liquidity requirements decreases the incentive to invest in complex assets, then the price must increase to restore the indifference between investing in complex and simple assets in equilibrium.

To elaborate, recall the relative advantage of complex assets  $\Delta$  as summarized by equation (9). Differentiating with respect to the liquidity requirement  $L$  at the equilibrium price  $P_C^*(b)$  obtains:

$$\frac{\partial \Delta}{\partial L} = -\eta(1 - \mu_g)\phi(\alpha - 1) + (1 - \eta)\mu_b(P_C^*(b)R - 1) \left[ -(\alpha\phi + 1 - \phi) + \frac{1}{P_C^*(b)} \right]. \quad (11)$$

The first term  $-\eta(1 - \mu_g)\phi(\alpha - 1) < 0$  reflects the fact that tightening liquidity requirements reduces complex banks' superior ability to provide liquidity to early depositors, by increasing the liquidity that simple banks with a low return can distribute back to investors when they experience a run.

The second term corresponding to the bad state has two subterms with opposite signs. Recall that  $P_C^*(b)R > 1$  (see Proposition 1). The first subterm,  $-(\alpha\phi + 1 - \phi)$ , is negative and reflects the fact that tightening liquidity requirements mitigates the advantage of complex banks relative to simple banks that draw a low return, which is their ability to mitigate runs by selling their long-term assets. Like in good times, tighter liquidity requirements lead to an increase in the liquidity that simple banks with a low return can distribute back to investors when they experience a run.

The second subterm,  $\frac{1}{P_C^*(b)}$ , is positive and reflects the fact that higher liquidity mitigates the disadvantage of complex banks relative to simple banks that draw a high return, which is their proneness to runs and subsequent inability to survive until period 2 to accrue the yield on their long-term assets. This is because tighter liquidity requirements lead to a reduction in the fraction of long-term assets that simple banks can invest in.

The net effect of tightening liquidity requirements on the complex-asset price is positive under a sufficient condition given by the following bound on the return probabilities.

**Proposition 3.** *If  $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \in \left[1, \frac{1-1/\kappa}{1-\phi}\right]$ , the equilibrium complex-asset price in bad times  $P_C^*(b)$  is increasing in the liquidity level  $L$ .*

The change in the complex-asset price is mediated by two mechanisms. First, liquidity requirements reduce the complex-asset supply for each individual complex bank while increasing the aggregate supply of liquidity, which directly increases the

complex-asset price. Second, depending on how this direct effect compares to the change in the price that is required to maintain indifference between investing in the two types of long-term assets, banks shift either towards or away from complex assets ex ante, which in general can lead to either a dampening or amplification of the price response.

## 2.4 Planner Solution

We next show that the equilibrium is generically inefficient, and that the pattern of inefficiency is monotonically related to the tightness of liquidity requirements.

Consider a regulator whose objective is to choose the volume of complex banks, denoted by  $V^W$ , to maximize the welfare in the economy, which is defined as the expected utility of depositors. The regulator is constrained to choices for which there is an equilibrium in which the privately optimal debt contract and bank-run conditions match the description in Proposition 1. The regulator also internalizes how the volume of complex banks affects the endogenous determination of the complex-asset price in interbank markets, which means that the complex-asset price in bad times  $P_C^W(b)$  is related to the volume of complex assets in a manner analogous to equation (10):

$$V^W R(1-L)\mu_b = \left(1 - V^W\right)\mu_b \frac{L - \kappa\phi}{P_C^W(b)}. \quad (12)$$

The welfare in the economy can then be written as

$$\mathcal{W}(V^W) = V^W \mathbf{E}\left[U_C | P_C(b) = P_C^W(b)\right] + (1 - V^W) \mathbf{E}\left[U_S | P_C(b) = P_C^W(b)\right]. \quad (13)$$

The equilibrium may exhibit excessive or insufficient investment in complex assets relative to the regulator's solution depending on the magnitude of liquidity requirements relative to a threshold level.

**Proposition 4** (Welfare-maximizing volume of complex banks). *Let  $\hat{L} = \kappa \left(1 - \frac{(1-\phi)(1-\eta)\mu_b}{(1-\mu_g)\eta}\right)$ .*

*When liquidity requirements are tight,  $L > \hat{L}$ , then there is excess investment in complex assets, i.e.,  $V^W < V^*$ . Moreover, the welfare-maximizing complex-asset price in bad times is equal to the maximum level of 1, i.e.,  $P_C^W(b) = 1 > P_C^*(b)$ . When liquidity requirements are loose,  $L < \hat{L}$ , then there is underinvestment in complex assets, i.e.,  $V^W > V^*$ . Moreover, the welfare-maximizing complex-asset price in bad times  $P_C^W(b)$  satisfies  $0 < P_C^W(b) < P_C^*(b)$ .*

The equilibrium is generically inefficient because the interbank lending market provides incomplete insurance. Banks do not take into account the impact of their port-

folio choice on the interbank complex-asset price, and how that affects the quality of insurance that can be achieved on the interbank market.<sup>13</sup>

The intuition for the pivotal role of the tightness of liquidity requirements is as follows. On the one hand, the planner may have a stronger incentive to invest in complex assets compared to the individual banks because it internalizes the full return of these assets in the bad state, whereas the individual banks that invest in complex assets receive only a fraction of this return based on the interbank market price. On the other hand, the planner may have a stronger incentive to invest in simple assets because it internalizes that this would effectively distribute more liquidity to the liquidity-shocked depositors of the distressed banks. If liquidity requirements are sufficiently tight, each safe bank has a large amount of excess liquidity that can be used to buy assets from the distressed banks, so the latter effect dominates. Otherwise, the former effect dominates.<sup>14</sup>

### 3 Calibration

As argued in Section 2.3, the relationship between tighter liquidity regulation and banks' investment in complex assets is ambiguous, and depends on which one of the following two effects dominates: a reduction in the comparative liquidity-provision advantage of complex banks or the greater insurance against runs associated with complex assets thanks to higher asset prices. To quantify the net effect of tighter liquidity regulation on banks' investment in complex assets, we calibrate our model to the Great Financial Crisis (GFC). At the calibrated parameters, the volume of complex assets and the interbank complex-asset price in bad times are increasing in the tightness of liquidity requirements, while welfare is decreasing.

We calibrate the eight parameters  $R$ ,  $L$ ,  $\kappa$ ,  $\eta$ ,  $\mu_g$ ,  $\mu_b$ ,  $\alpha$ ,  $\phi$  to satisfy the six

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<sup>13</sup>This is similar to the result in [Geanakoplos and Polemarchakis \(1985\)](#), which states that in the presence of incomplete markets, a competitive equilibrium is generically constrained inefficient.

<sup>14</sup>An alternative explanation that is slightly more mathematical is the following. The regulator's incentive to increase the complex-asset price is increasing in the level of the equilibrium complex-asset price. This is because increasing the price improves the performance of complex banks, which sell assets, but decreases the performance of simple banks that draw a high return, which buy assets. The marginal benefit of increasing the price is constant since it enters linearly into the complex-bank return (see equation (7)), whereas the marginal cost is decreasing in the level of the price since it enters hyperbolically into the simple-bank return (see equation (8)). Hence, the marginal net benefit of increasing the price is increasing in the level of the price. Now, consider a case in which the equilibrium complex-asset price in bad times  $P_C^*(b)$  is increasing in the liquidity ratio  $L$  (see Proposition 3). If liquidity requirements exceed  $\hat{L}$ , then the equilibrium complex-asset price is high enough that the marginal net benefit of further increasing the complex-asset price is positive. In that case, the planner has an incentive to increase the complex-asset price relative to the equilibrium by reducing the volume of complex banks. An analogous argument holds if the liquidity level is lower than  $\hat{L}$ .

parametric restrictions in Proposition 1 and to match five empirical counterparts:

- The long-term return  $R$  is calibrated to match 1.067, which is approximately the mean of the 30-year fixed-rate mortgage rate in September 2008 (1.06)<sup>15</sup> and Moody’s medium-grade corporate bond yield in September 2008 (1.073).<sup>16</sup>
- The short-term interest rate  $R_D = \kappa$  is calibrated to match 1.018, which is the federal funds rate in September 2008.<sup>17</sup>
- The liquidity level  $L$  is calibrated to match 0.176, which is approximately the ratio of total liquid assets to total assets based on 2008Q3 FR Y-9C filings for bank holding companies. Liquid assets include cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and government agency debt and mortgage-backed securities (not including government-sponsored agency (GSE) debt and MBS).<sup>18</sup>
- The complex-asset price in bad times  $P_C^*(b)$ , which is also the ratio of the complex-asset price in bad times to the complex-asset price in good times, is calibrated to match 0.966, which corresponds to the ratio of the 3-month U.S. dollar LIBOR-OIS spread at its peak on October 10, 2008 ( $1/1.0364 \approx 0.965$ ) to its level just before the onset of the GFC in the summer of 2007 ( $1/1.0008 \approx 0.999$ ). The LIBOR-OIS spread corresponds to the premium for lending short-term funds in the interbank market relative to the expected policy rate (Gorton and Metrick, 2012).
- The fraction of complex assets  $V^*$  is calibrated to match 0.133, which is the ratio of complex assets to total illiquid assets based on 2008Q3 FR Y-9C filings. Illiquid assets are defined as assets minus liquid assets, as given above. Complex assets include GSE MBS, non-agency MBS, and asset-backed securities.<sup>19</sup>

Table 3 presents the calibrated parameters, and Table 4 compares the empirical and model-generated values for the observables.

In this calibration, the threshold level of liquidity is  $\hat{L} = 0.209$ , which is greater than  $L = 0.176$ . Therefore, Proposition 4 implies that, perhaps surprisingly, there

<sup>15</sup>Data: FRED series MORTGAGE30US.

<sup>16</sup>Data: FRED series BAA.

<sup>17</sup>Data: FRED series FEDFUNDS.

<sup>18</sup>This definition of liquid assets is an approximation for the set of (level 1) high quality liquid assets that can be used to satisfy the LCR without any discount, which includes excess reserves, Treasury securities, government agency debt and MBS (not including government-sponsored agency debt and MBS), and sovereign debt with zero risk-weights.

<sup>19</sup>Note that structured financial products are omitted for this exercise since banks were not required to report them at the time.

Table 3: Calibrated parameters.

Parameter	Value
High return ( $R$ )	1.067
Liquidity ratio ( $L$ )	0.176
Short-term return ( $\kappa$ )	1.018
Probability of good state ( $\eta$ )	0.999
Probability of high return in good state ( $\mu_g$ )	0.999
Probability of high return in bad state ( $\mu_b$ )	0.8
Fraction of liquidity-shocked depositors ( $\phi$ )	0.007
Marginal utility from liquidity shock ( $\alpha$ )	6.1

Table 4: Comparison of empirical and model-generated variables.

Variable	Empirical	Model
High return	1.067	1.067
Liquidity ratio	0.176	0.176
Short-term return	1.018	1.018
Price in bad times	0.966	0.964
Fraction of complex assets	0.133	0.166

was underinvestment in complex assets during this time. Following the reasoning in Section 2.4, this is because the gains for the buyers in the interbank market associated with increasing the volume of complex banks, and hence decreasing the complex-asset price during the crisis, was larger than the losses for the sellers.

Across rows and in this order, Figure 2 shows how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the volume of complex banks, and welfare vary with  $L$  in the equilibrium solution, the planner solution, and the difference between them (across columns).<sup>20</sup> Note that the figure only shows values of  $L$  that are greater than the calibrated value, since the calibrated  $L$  is at the boundary of the parameter space that is consistent with the restrictions stated in Proposition 1. The equilibrium price and the volume of complex banks are both increasing in  $L$  at these parameters, which is consistent with the motivating empirical observation in Figure 1.<sup>21</sup>

The last row of Figure 2 also indicates that the optimal level of liquidity requirements  $L$  that maximizes welfare  $\mathcal{W}$  is given by the left boundary in the figure.

<sup>20</sup>It also shows the haircut if the interbank market is interpreted as a repo market. See Online Appendix A for details about how the haircut is defined.

<sup>21</sup>In Online Appendix C.1, we present additional suggestive evidence that tighter liquidity requirements are associated with higher complex-asset prices in bad times.

This is true whether the fraction of complex assets is determined in equilibrium or chosen by the planner. This reflects in part the opportunity cost associated with holding low-yielding liquid assets in states where a bank does not experience a run, which is the most likely outcome given the high calibrated values for the probability of the good state  $\eta$  and the probability of receiving a high return in the good state  $\mu_g$ .

Figure 2: Variation in  $L$ . This figure shows how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in the equilibrium, the planner solution, and the difference between them.

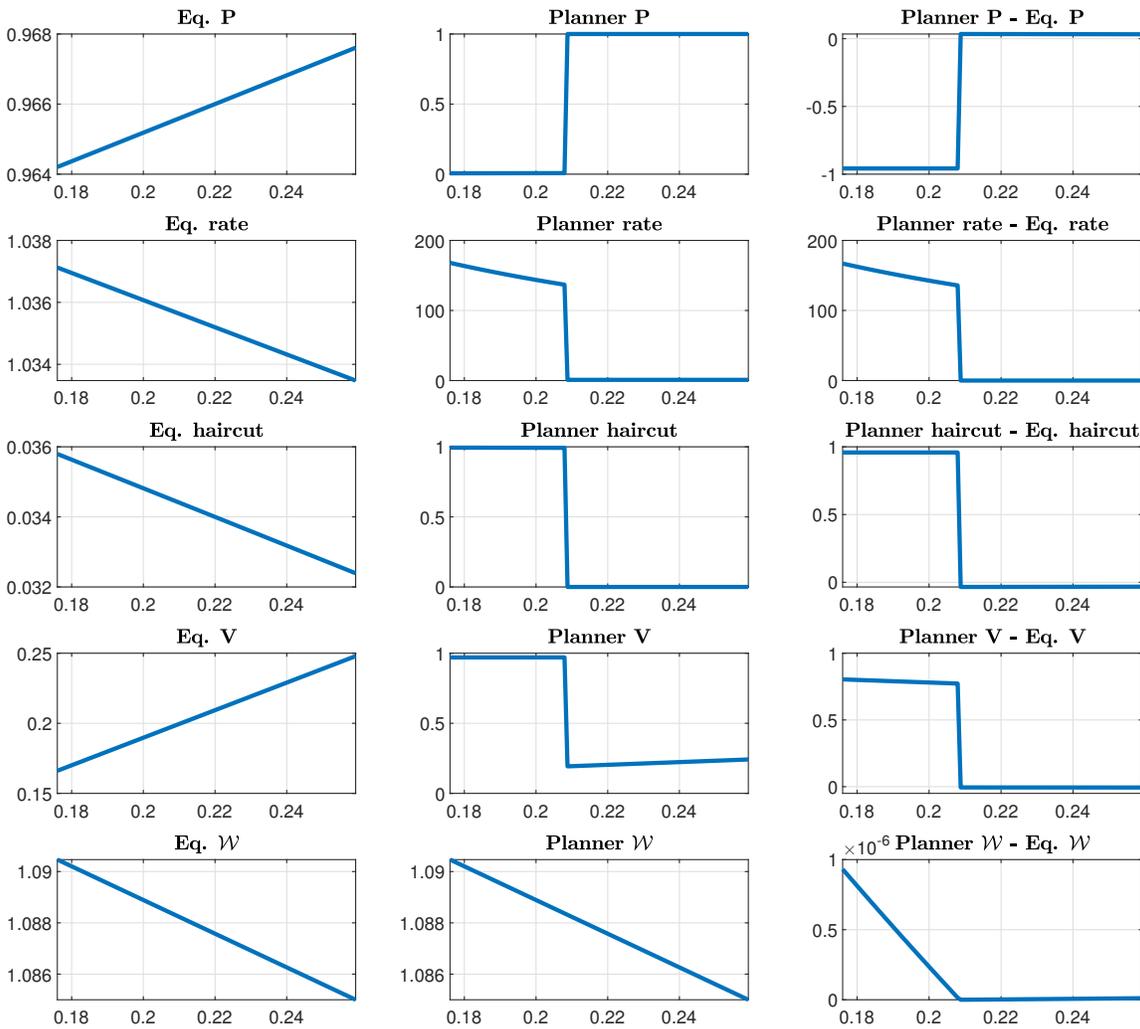
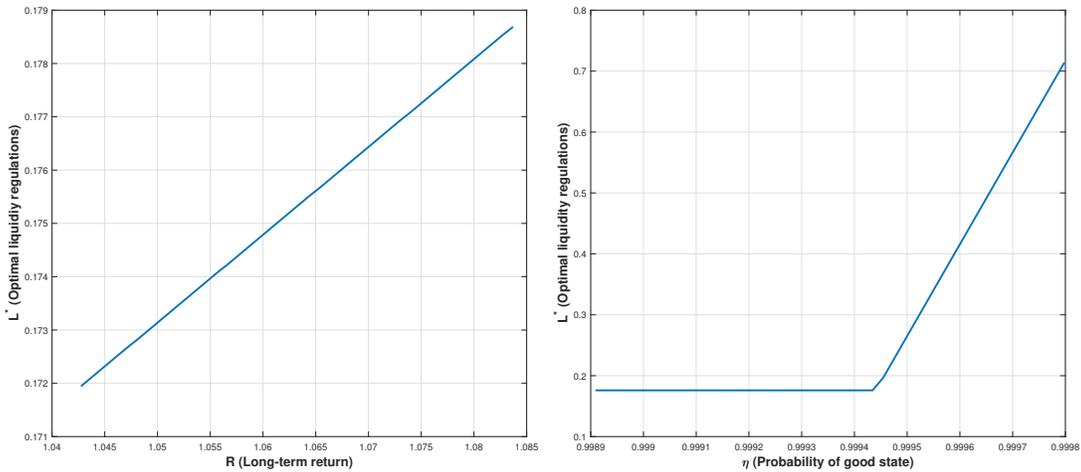


Figure 3 shows how the optimal liquidity level that maximizes welfare in the planner solution,  $L^*$ , varies with the long-term return  $R$  and the probability of the good state  $\eta$  locally around the calibrated values. The other parameters are fixed at the calibrated values. For all of the simulated parameter values, the optimal  $L^*$  is equal to the minimum value that is consistent with the parametric restrictions

described in Proposition 1. To understand how this boundary solution varies with the parameters, note that the restriction in (4) is binding for  $L^*$  at the calibrated parameters. This implies that  $L^*$  must increase with  $R$  to maintain this restriction. The intuition is that increasing  $R$  increases the incentive for liquidity-shocked depositors to withdraw late. This implies that  $L$  must simultaneously increase to reduce the payoff from withdrawing late in order to maintain an equilibrium in which depositors withdraw under the conditions described in Proposition 1.

Similarly,  $L^*$  is constant in  $\eta$  for values of  $\eta$  such that the binding restriction is (4). For  $\eta$  sufficiently high, however, (2) becomes the binding restriction, in which case  $L^*$  increases with  $\eta$  to maintain the restriction. The intuition is that increasing  $\eta$  increases the incentive to invest in complex banks due to their superior performance in the good state. This requires the equilibrium complex-asset price in bad times  $P_C^*(b)$  to decrease to maintain the indifference between investing in either type of long-term asset. If the equilibrium price is equal to the lower bound  $\frac{1}{R}$ , then  $L$  must alternatively increase to reduce the advantage of complex banks in good times, so that the equilibrium price satisfies the bound.

Figure 3: This panel shows the optimal liquidity level that maximizes welfare in the planner solution  $L^*$  as a function of the long-term return  $R$  (left) and the probability of the good state  $\eta$  (right).



Online Appendix B calibrates the model to the COVID-19 crisis. We compare the GFC and the COVID-19 crisis since they are two major crises that occurred before and after the introduction of the Liquidity Coverage Ratio. We find that there was overinvestment in complex assets during the COVID-19 crisis. However, the comparative statics with respect to  $L$  are qualitatively similar.

## 4 Financial-stability Implications of Different Policies

The previous analysis has uncovered the conditions under which tighter liquidity requirements give rise to greater investment in complex assets, with potential repercussions for financial stability. This section describes how liquidity regulation interacts with three alternative policies aimed at fostering financial stability: unconventional monetary policy in the form of quantitative easing, an ex-ante insurance system, and asset-specific taxes.

### 4.1 Quantitative Easing

By affecting interbank trading, our model naturally connects with the transmission of monetary policy through banks' funding costs, which are (at least partially) determined on the interbank market, and through the extent to which they are financially constrained, which is reflected by the liquidity composition of their asset side. Since the Great Financial Crisis, central banks around the world have responded by implementing unconventional monetary policies. In particular, quantitative easing (QE) refers to asset purchases by central banks. QE has been implemented by the Federal Reserve in the U.S. during both the Great Financial Crisis and the COVID-19 crisis to stabilize asset prices. This section analyzes how QE interacts with liquidity regulation, and describes conditions under which it may or may not improve welfare.

**QE general implementation.** To characterize the implementation of QE, we first enrich the model. We assume that at the beginning of period 1 each depositor randomly receives an income shock  $\hat{v} \in \{0, \nu\}$ , where it is commonly known that the probability of receiving  $\nu$  is equal to  $\delta$ . We assume  $\delta$  is sufficiently small, so that all of the previous results still hold. After potentially paying an income tax, the depositors deposit their income in banks.

The government only charges a tax if the aggregate state is bad. Specifically, the government requires all depositors with a positive income shock to pay a tax  $\tau$ , which creates a total tax revenue of  $\tau\delta$ . The government then uses the tax income to buy complex assets from the distressed banks. If the volume of complex banks at tax level  $\tau$  is equal to  $V(\tau)$ , then the resulting equilibrium complex-asset price  $P_C^\tau(b)$  satisfies

$$\underbrace{V(\tau)R(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{(1-V(\tau))\mu_b \frac{L-\kappa\phi}{P_C^\tau(b)}}_{\text{complex-asset demand from simple banks}} + \underbrace{\frac{\tau\delta}{P_C^\tau(b)}}_{\text{complex-asset purchase by gov.}} \quad (14)$$

Finally, in period 2 the government returns these assets to the late depositors as a lump sum. Given the debt contract and bank-run conditions in Proposition 1, the expected utility from investing in complex assets as a function of the complex-asset price in bad times  $P_C(b)$  can now be written as

$$\begin{aligned} \mathbf{E}[U_C|P_C(b)] = & \eta \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + \mu_g R(1-L)}_{\text{return to normal dep.}} + \underbrace{\delta v}_{\text{income}} \right) \\ & + (1-\eta) \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \left( \underbrace{L + P_C(b) \mu_b R(1-L) + \delta(v - \tau)}_{\text{liquidation value}} \right) + \underbrace{(1-\eta) \frac{\tau\delta}{P_C^\tau(b)}}_{\text{gov. payoff}}, \end{aligned} \quad (15)$$

and the expected utility from investing in simple assets can now be written as

$$\begin{aligned} \mathbf{E}[U_S|P_C(b)] = & \eta \left( \mu_g \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{L - \kappa\phi + R(1-L)}_{\text{return to normal dep.}} + \underbrace{\delta v}_{\text{income}} \right) \right. \\ & \left. + (1-\mu_g) \left( \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{(L + \delta v)}_{\text{liquidation value}} \right) \right) \\ & + (1-\eta) \left( \mu_b \left( \underbrace{\alpha\kappa\phi}_{\text{return to shocked dep.}} + \underbrace{\frac{L - \kappa\phi}{P_C(b)} + R(1-L) + \delta(v - \tau)}_{\text{return to normal dep.}} \right) \right. \\ & \left. + (1-\mu_b) \left( \underbrace{(\alpha\phi + 1 - \phi)}_{\text{proportional distribution}} \underbrace{(L + \delta(v - \tau))}_{\text{liquidation value}} \right) \right) + \underbrace{(1-\eta) \frac{\tau\delta}{P_C^\tau(b)}}_{\text{gov. payoff}}. \end{aligned} \quad (16)$$

We distinguish the welfare implications of QE based on whether it is undertaken with or without commitment.

**QE without commitment.** When implementing QE without commitment, or in a manner that comes as a surprise in period 1 after bank portfolios have already been determined, the government takes as given the volume of complex banks that would

occur if banks expected no tax,  $V(0)$ , and chooses the tax  $\tau$  to maximize welfare:

$$\mathcal{W}(\tau) = V(0)\mathbf{E}\left[U_C|P_C(b) = P_C^\tau(b)\right] + (1 - V(0))\mathbf{E}\left[U_S|P_C(b) = P_C^\tau(b)\right]. \quad (17)$$

Charging a higher tax rate allows the government to accrue more funds that it can use to buy complex assets, which in turn increases the complex-asset price.

**Proposition 5.** *If QE is undertaken without commitment and  $P_C^\tau(b) < 1$ , then the equilibrium complex-asset price is increasing in the tax  $\tau$ :  $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$ .*

Increasing the complex-asset price has the benefit of mitigating the severity of runs on complex banks during bad times, which always outweighs the cost of the tax when QE is undertaken without commitment.

**Proposition 6 (QE without commitment).** *If QE is implemented without commitment, then the optimal tax is positive and equal to the minimum of income  $\nu$  and the minimum tax necessary to increase the complex-asset price in bad times  $P_C^\tau(b)$  to 1.*

Figure 4 shows how features of the model vary with  $L$  in equilibrium, under optimal QE without commitment, and the difference between them. Note that  $\nu = 1$ ,  $\delta = 0.01$ , and the remaining parameters are the same as in the baseline calibration (see Table 3).

**QE with commitment.** When implementing QE with commitment, or in a manner that can be predicted when banks choose their portfolios in period 0, the government internalizes the fact that increasing the price of complex assets in bad times affects the volume of banks that invest in complex assets,  $V(\tau)$ . Welfare then becomes

$$\mathcal{W}(\tau) = V(\tau)\mathbf{E}\left[U_C|P_C(b) = P_C^\tau(b)\right] + (1 - V(\tau))\mathbf{E}\left[U_S|P_C(b) = P_C^\tau(b)\right]. \quad (18)$$

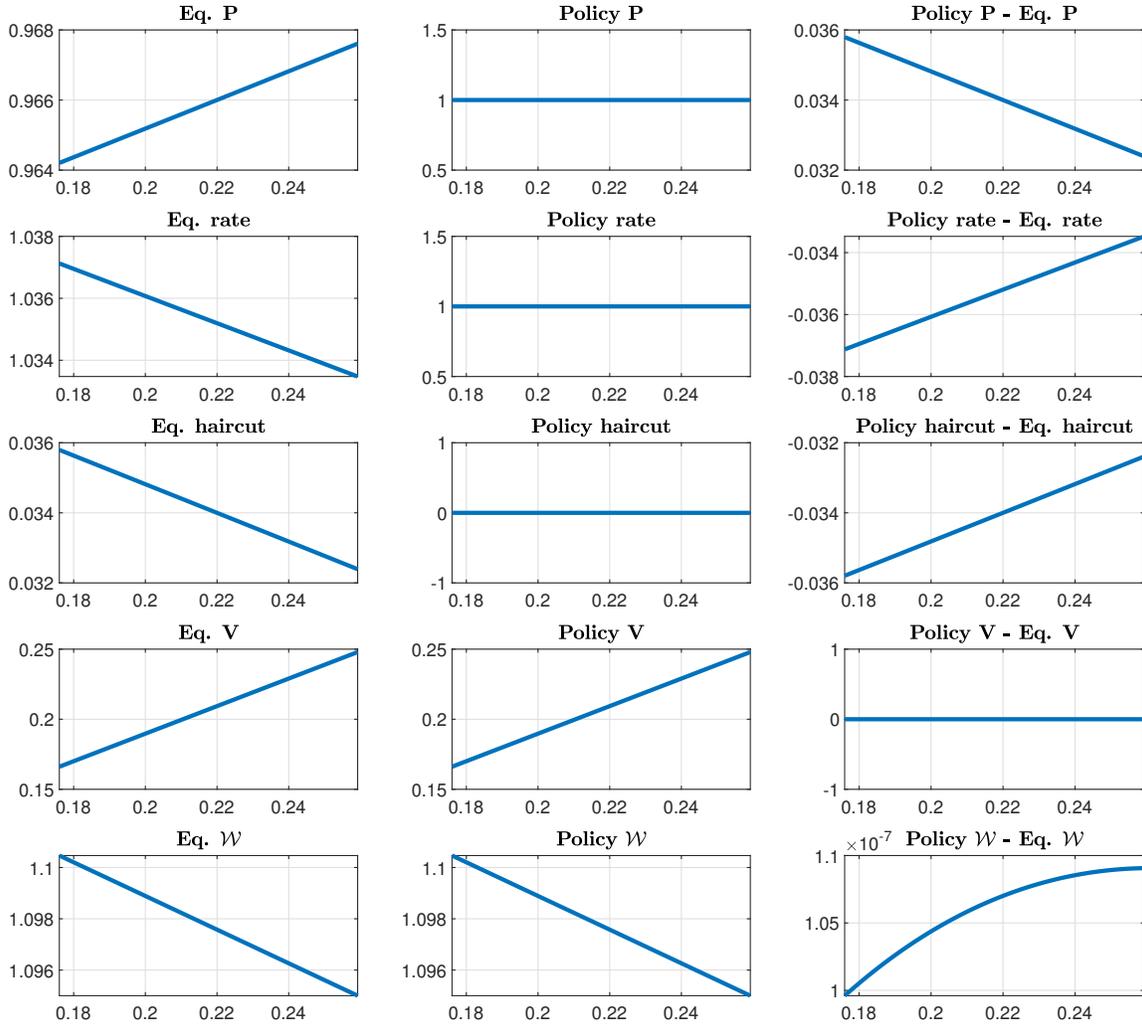
When undertaken with commitment, QE increases both the complex-asset price and the volume of complex banks.

**Proposition 7.** *If QE is undertaken with commitment and  $P_C^\tau(b) < 1$ , then*

- (a) *the equilibrium complex-asset price is increasing in the tax  $\tau$ :  $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$ , and*
- (b) *the equilibrium volume of complex banks is increasing in the tax  $\tau$ :  $\frac{\partial V(\tau)}{\partial \tau} > 0$ .*

Part (a) has a similar intuition as Proposition 5. Part (b) follows from the fact that increasing the complex-asset price partially insures against the runs experienced by complex banks in bad times. In contrast to the case without commitment, the

Figure 4: Variation in  $L$  under QE without commitment. This figure shows how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in equilibrium, under optimal QE without commitment, and the difference between them.



anticipation of QE strengthens the incentive for banks to invest in complex assets ex ante.

This, in turn, has an offsetting negative effect on the complex-asset price. Therefore, when QE is undertaken with commitment, the attenuated benefit of increasing the complex-asset price in bad times can be smaller than the cost associated with the tax.

**Proposition 8 (QE with commitment).** *Under QE with commitment, the optimal tax can in general be either positive or zero. If the liquidity level  $L$  is sufficiently high, then the optimal tax is zero.*

The intuition is that QE with commitment has a weaker effect on the complex-asset

price than QE without commitment since it encourages more banks to invest in complex assets. When liquidity requirements are tight, then this shift towards complex assets reduces welfare since there is overinvestment in complex assets in equilibrium (see Proposition 4).<sup>22</sup> Moreover, tightening liquidity requirements decreases the extent to which banks rely on interbank markets for managing their liquidity. For sufficiently tight liquidity requirements, the benefit of QE increasing the complex-asset price falls short of the financing costs.

Figure 5 shows how features of the model vary with  $L$  in equilibrium, under optimal QE with commitment, and the difference between them. Note that QE with commitment improves welfare for values of  $L$  near the calibrated value. Tighter liquidity requirements increase the effect of QE on asset prices but decrease the overall contribution to welfare, since banks are less reliant on asset prices as a means to respond to liquidity stress.<sup>23</sup>

## 4.2 Ex-ante Insurance

We next turn to an ex-ante insurance policy that always improves welfare. Consider the original environment as introduced in Section 2.2. If in period 1 the state is good, then in period 2 the government taxes high-return banks at the rate  $\tau$  and distributes the proceeds equally to low-return banks. The tax is predictable in period 0. The tax rate is  $\tau = 1 - \mu_g$ , which sets equal the after-tax long-term return in good times for all banks:

$$\underbrace{(1 - \tau)R(1 - L)}_{\text{return of a high return bank}} = \underbrace{\tau \frac{\mu_g}{1 - \mu_g} R(1 - L)}_{\text{return of a low return bank}} = \mu_g R(1 - L). \quad (19)$$

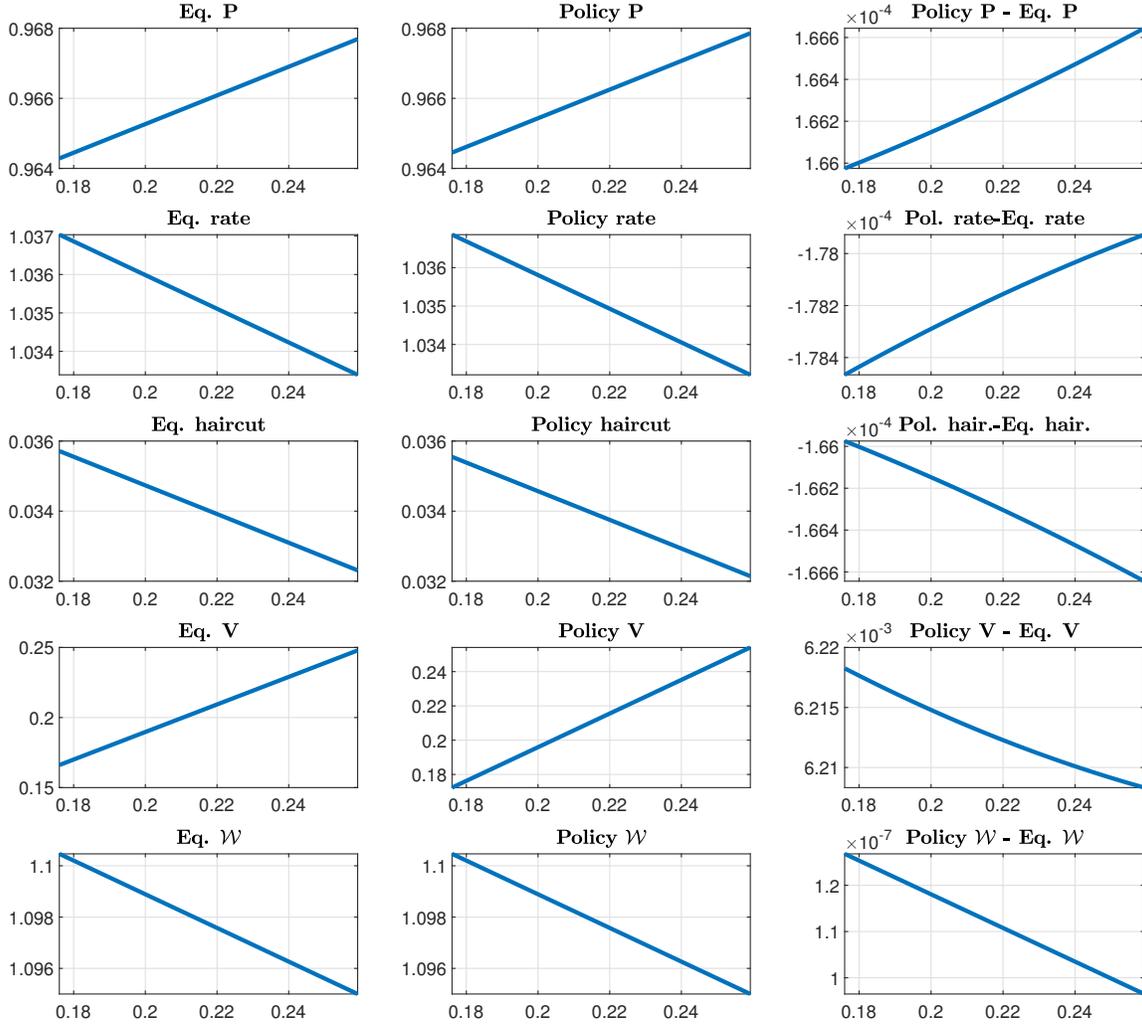
Note that the government must implement this arrangement since banks with a high realized return would have no incentive to honor a promise to pay the banks with a low realized return. This policy always improves welfare.

**Proposition 9** (Ex-ante insurance). *Implementing the ex-ante insurance policy (i) increases the equilibrium complex-asset price in bad times, (ii) decreases the volume of*

<sup>22</sup>Note that when liquidity requirements are loose, this shift towards complex assets improves welfare since there is underinvestment in complex assets in equilibrium.

<sup>23</sup>Tightening liquidity requirements amplifies the effect of QE on asset prices when QE is implemented with commitment since it attenuates the complementarity between QE and investment in complex assets. In particular, it reduces the volume of complex assets an individual bank can hold, which reduces the benefit of the anticipated price support associated with QE. The weaker degree of substitution with complex assets therefore leads to a weaker reduction in the price relative to the direct effect of QE. Online Appendix C.2 presents suggestive evidence that asset prices were more responsive to QE announcements after the implementation of the LCR compared to before, consistent with this mechanism.

Figure 5: Variation in  $L$  under QE with commitment. This figure shows how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut (or “hair.”), the volume of complex banks, and welfare vary with  $L$  in equilibrium, under optimal QE with commitment (“Policy” or “Pol.”), and the difference between them.



complex banks, and (iii) increases overall welfare.

The intuition is as follows. The ex-ante insurance policy increases the period-2 income of simple banks with a low return such that they no longer experience a run. This directly increases the expected utility from investing in simple assets since by avoiding runs in the good state, it shifts a greater share of the expected return of a simple bank to liquidity-shocked depositors with a higher marginal utility.

Since the policy is predictable, it additionally motivates banks to switch to simple assets ex ante, which allows a greater fraction of banks to benefit from the redistribution in the good state.<sup>24</sup> This shift away from complex assets, in turn,

<sup>24</sup>Note that as more banks switch to simple assets, the price for complex assets in bad times in-

leads to a reduction in liquidity demand during bad times and, thus, an increase in the equilibrium complex-asset price. Note that there is no incentive to implement an analogous redistribution in the bad state because the average return is less than the promised repayment to the early depositors due to the parametric assumption in (5). In particular, committing to redistribute in the bad state would trigger a run on all banks.

Figure 6 shows how features of the model vary with  $L$  in equilibrium, under the ex-ante insurance policy, and the difference between them. Note that this exercise is conducted using the calibrated parameters (see Table 3).

### 4.3 Implementation through Asset-specific Taxes

Finally, we consider asset-specific taxes as a means of implementing the constrained-efficient volume of complex banks (similar to Dávila and Korinek, 2018). For this purpose, recall that the equilibrium is generically inefficient and that the degree of investment in complex assets can be greater or less than in the planner solution depending on whether liquidity requirements are tighter or looser than a threshold level  $\hat{L}$ , respectively (Proposition 4). This subsection first shows that QE and ex-ante insurance cannot always be used to implement the constrained-efficient volume of complex banks. It then provides conditions under which the constrained-efficient volume of complex banks can be implemented with a tax on either complex or simple assets.

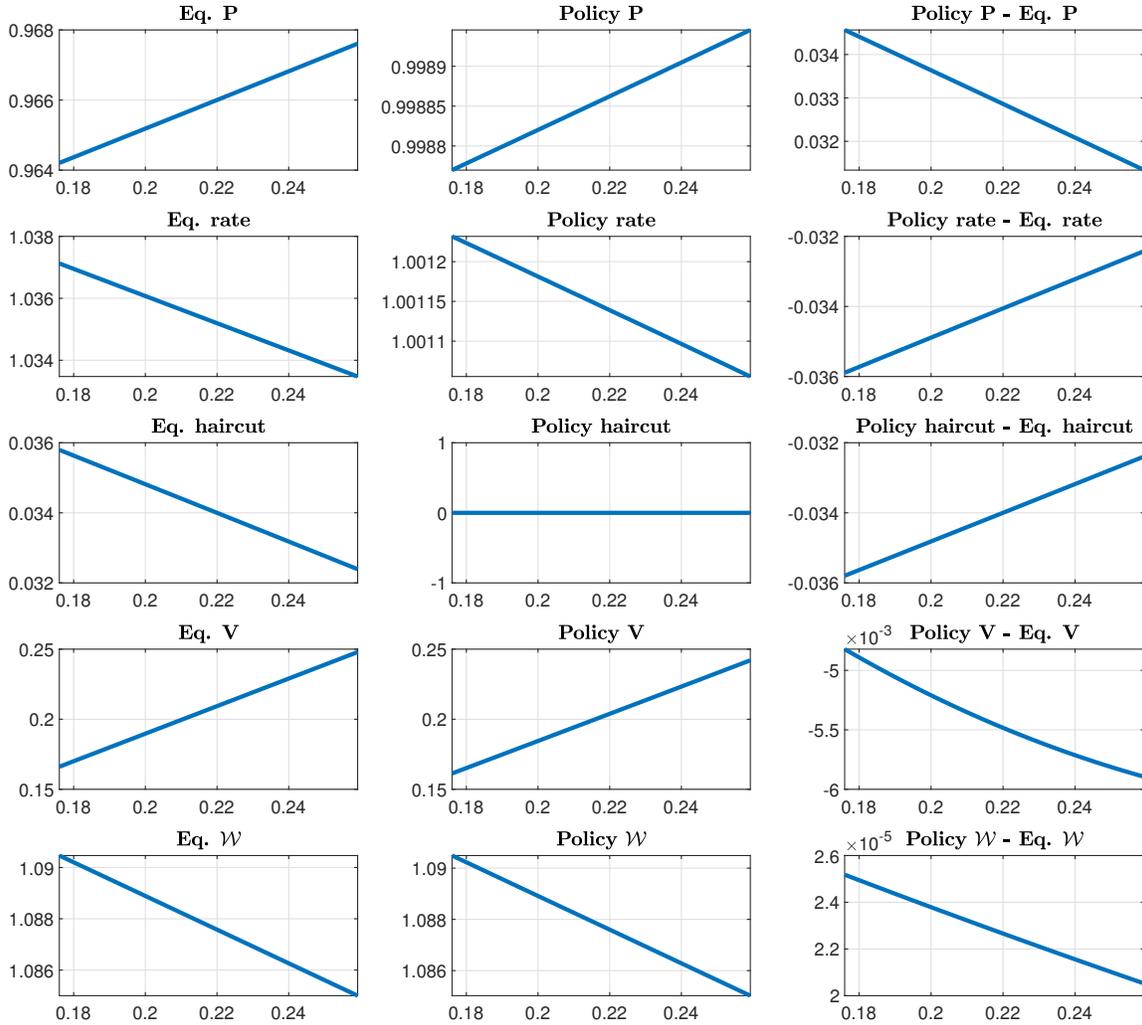
**Proposition 10.** *If  $L < \hat{L}$  and  $v$  is sufficiently large, then the constrained-efficient volume of complex banks can be implemented via QE with commitment. However, the tax that implements the constrained-efficient volume of complex banks may not be welfare-optimizing. If  $L > \hat{L}$ , then neither QE nor the ex-ante insurance policy can implement the constrained-efficient volume of complex banks.*

The intuition is as follows. If  $L < \hat{L}$ , then the constrained-efficient volume of complex banks is greater than under the equilibrium solution (Proposition 4). Recall that QE with commitment increases the incentive to invest in complex assets since it supports the complex-asset price in bad times (Proposition 7). In particular, there is a tax that implements the constrained-efficient volume of complex banks. However, this tax level does not necessarily maximize welfare because QE also affects welfare through channels other than the volume of complex banks. To illustrate

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creases, which decreases the expected utility of simple banks relative to the case where there is no adjustment of bank portfolios. However, the net effect of the adjustment on the expected utility across all banks is clearly positive because banks only switch to simple assets when the expected utility is greater compared to sticking with complex assets.

Figure 6: Variation in  $L$  under ex-ante insurance policy. This figure shows how the complex-asset price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in equilibrium, under the ex-ante insurance policy, and the difference between them.

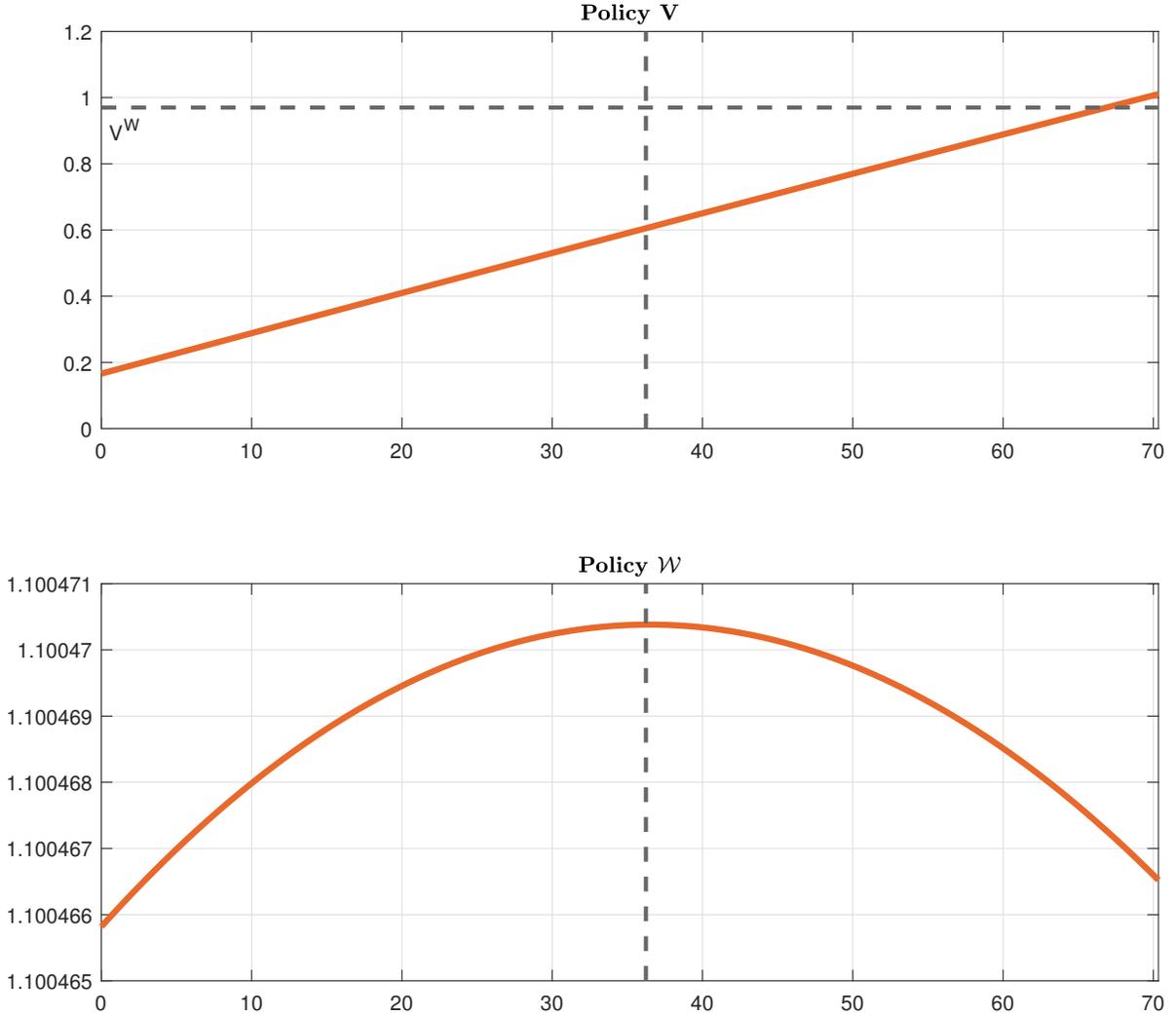


this, Figure 7 shows how the volume of complex banks and total welfare vary with the tax  $\tau$  used to implement QE with commitment at the calibrated liquidity level  $L$ .

If  $L > \hat{L}$ , then the constrained-efficient volume of complex banks is less than that in the equilibrium solution (Proposition 4). The constrained-efficient volume of complex banks cannot be implemented with QE, since QE without commitment has no effect on the volume of complex banks and QE with commitment can only increase the volume of complex banks. The ex-ante insurance policy decreases the volume of complex banks (Proposition 9), but not enough to implement the constrained-efficient level.

The constrained-efficient volume of complex banks for any level of  $L$  can be

Figure 7: Variation in  $\tau$  under QE with commitment. This figure shows how the volume of complex banks and welfare vary with  $\tau$  under QE with commitment. The vertical dashed line corresponds to the welfare-optimizing tax. The horizontal dashed line corresponds to the constrained-efficient volume of complex banks.



implemented with a tax on either complex or simple assets. The tax can be described as follows: if in period 1 the state is good, then in period 2 the government taxes high-return complex (or simple) banks at a rate of  $\tau$ , and distributes the proceeds equally to the high-return simple (or complex) banks.

**Proposition 11.** Denote by  $\Delta(P_C(b)) = \mathbf{E}[U_C|P_C(b)] - \mathbf{E}[U_S|P_C(b)]$  the relative benefit of investing in complex assets without the tax as expressed in equation (9), by  $V^W$  the constrained-efficient volume of complex banks, and by  $P_C^W(b)$  the complex-asset price in bad times for the constrained-efficient allocation. Then the following hold:

- If  $L < \hat{L}$  and  $\frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)} < \frac{R(1-L) - (\kappa-L)}{R(1-L)}$ , then the constrained-efficient volume of

complex banks can be implemented by transferring from simple to complex banks via a tax at the rate  $\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}$ .

- If  $L > \hat{L}$  and  $\frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)} < \frac{\mu_g R(1-L)-(L-\kappa)}{\mu_g R(1-L)}$ , then the constrained-efficient volume of complex banks can be implemented by transferring from complex to simple banks via a tax at the rate  $\tau^* = \frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)}$ .

Additionally, the tax level that implements the constrained-efficient volume of complex banks also maximizes welfare.

Note that the parametric assumptions in this proposition ensure that the tax is consistent with the incentive-compatibility conditions supporting an equilibrium of the form as described in Proposition 1.

## 4.4 Comparison of Policies

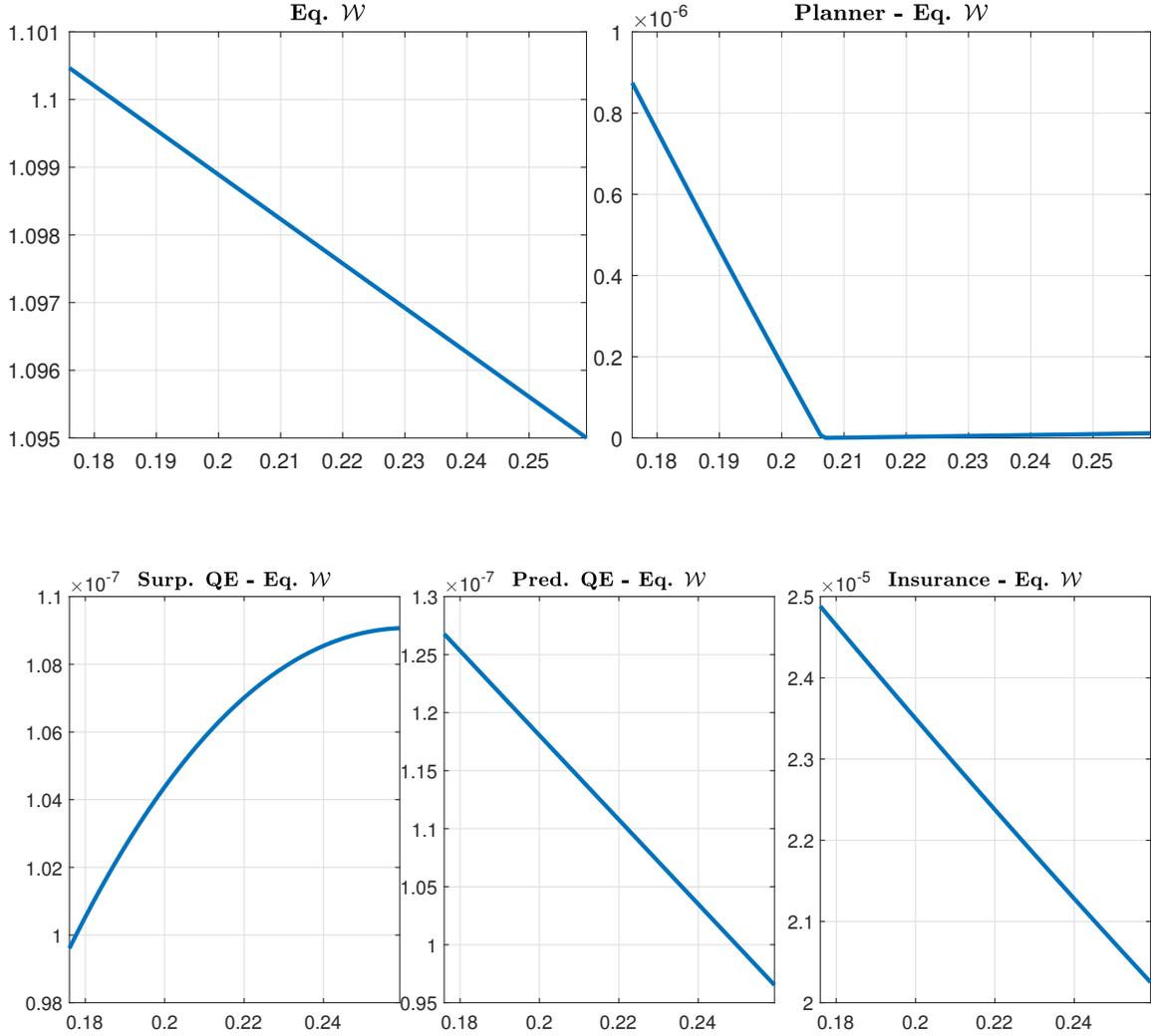
Figure 8 compares welfare as a function of the liquidity level  $L$  for the various policy scenarios. The first row shows welfare in the baseline equilibrium in the version of the model with income shocks, as introduced at the beginning of Section 4.1. Note that  $\nu = 1$ ,  $\delta = 0.01$ , and the remaining parameters are the same as in the baseline calibration (see Table 3). It also shows the improvement in utility associated with the constrained-efficient volume of complex banks, which can be implemented using asset-specific taxes (see Proposition 11).

The second row shows the welfare gains associated with QE without commitment, QE with commitment, and ex-ante insurance. At the calibrated liquidity level  $L = 0.176$ , the ex-ante insurance policy achieves the greatest welfare gain, followed by the planner solution, QE with commitment, and QE without commitment. For tighter liquidity requirements, the ex-ante insurance policy continues to achieve the greatest welfare gain, but QE without commitment may become more effective than the planner solution and QE with commitment.

## 5 Conclusion

The Liquidity Coverage Ratio, alongside the Net Stable Funding Ratio, has been put in place to foster financial stability by forcing large banks to maintain sufficient liquidity on their balance sheets. This paper shows under what conditions tighter liquidity requirements substitute for or complement banks' investment in complex assets, such as structured securities, that may contribute to destabilizing trends in the economy.

Figure 8: Comparison of welfare gains under different policies. This figure shows welfare as a function of the liquidity level  $L$  in the baseline equilibrium with income shocks as well as the improvement in utility associated with QE without commitment (“Surp. QE”), QE with commitment (“Pred. QE”), and the ex-ante insurance policy.



In our model, the symmetric opacity associated with complex assets supports bank liquidity in good times, but it has a mixed effect on liquidity during crises. On the one hand, it causes panic-stricken depositors to run on banks that may turn out to be solvent. On the other hand, it also allows banks to draw liquidity from interbank lending markets. The model shows that tighter liquidity requirements can support asset prices during crises by increasing the supply of liquidity on interbank markets, but by doing so, it can also encourage greater investment in complex assets beforehand.

We provide a rich assessment of the welfare properties of the interaction of liquidity regulation and other policies aimed at fostering financial stability. First,

the degree of investment in complex assets can be inefficiently high or low depending on liquidity requirements. Therefore, the tightness of liquidity requirements determines the asset-specific taxes that can be used to implement the constrained-efficient investment in complex assets. Second, liquidity regulation can undermine the benefit of ex-post interventions such as unconventional monetary policy, in particular quantitative easing (QE). This is more likely to be true if QE is implemented in a predictable manner, in which case the benefit of QE in supporting asset prices is offset by higher ex-ante investment in complex assets.

We rely on parameters calibrated to the Great Financial Crisis. Doing so, we find that there was potentially underinvestment in complex assets, which suggests that the dry-up of the market for mortgage-backed securities following the crisis may have been excessive, or that information frictions outside of our model are at play (Daley and Green, 2016). Furthermore, at the calibrated liquidity level, the constrained-efficient level of total investment in complex assets under an ex-ante insurance policy achieves the greatest welfare gain compared to quantitative easing. These considerations may give rise to a more general model of how the regulation of different portions of banks' balance sheets affects the extent of pecuniary externalities that, in turn, determine the efficacy of different financial-market interventions, which we leave for future research.

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# Proofs

## Proof of Proposition 1

**Proposition 1** (Equilibrium). *Assume the following parametric restrictions:*

$$\frac{\eta(\alpha\phi + 1 - \phi) + (1 - \eta)\alpha}{\eta\mu_g + (1 - \eta)\mu_b} < R \quad (1)$$

$$\frac{\eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) + (1 - \eta)\mu_b(1 - \phi - \alpha\phi(\kappa - 1))}{(1 - \eta)\mu_b(1 - \phi\kappa)} < R \quad (2)$$

$$R < \frac{L(1 - \phi)}{1 - L}(\alpha - 1) \quad (3)$$

$$R < \frac{L(1 - \phi)}{1 - L\phi}\alpha < \alpha \quad (4)$$

$$(L + \mu_b R(1 - L)) \left( 1 + \frac{(1 - \eta)(1 - \phi)}{\eta\phi} \right) < \kappa \quad (5)$$

$$\kappa < L + \mu_g R(1 - L). \quad (6)$$

Then there exists an equilibrium in which the following hold:

1. All banks invest in long-term assets and do not hold excess liquidity.
2. Banks pay depositors that withdraw early a return of  $R_d = \kappa$ .
3. Liquidity-shocked depositors always withdraw early, and normal depositors withdraw early if and only if
  - the bank is complex and the economic state is bad.
  - the bank is simple and its individual return is low.
4. The price for simple assets is  $P_S^*(\omega) = 1$ , and the price for complex assets satisfies  $\frac{1}{a} < \frac{1}{R} < P_C^*(b) < P_C^*(g) = 1$ .

We prove this via a lemma for each part taking the other parts as given.

**Lemma 1** (Preference for risky assets). *All banks invest in long-term assets and do not hold excess liquidity.*

*Proof.* We will show that holding only liquid assets is a dominated portfolio. To determine the optimal early repayment for a bank fully invested in liquid assets, we consider three cases depending on the magnitude of the early repayment  $R_d$ .

**Case 1:** If  $R_d > 1$ , we consider an equilibrium in which all depositors withdraw early. If all depositors withdraw from the bank in period 1, then the bank defaults since the maximum liquidity it can supply, 1, is less than the demand  $R_d$ . Therefore the best response for an individual depositor of either type is to withdraw early since the payment from withdrawing early, which is the total liquidation value of 1 since the bank experiences a run, is greater than the payment from withdrawing late,

which is zero. Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\mathbf{E}[U_I] = \alpha\phi + 1 - \phi. \quad (20)$$

**Case 2:** If  $\frac{1}{\alpha P_C^*(b)(1-\phi)+\phi} \leq R_d \leq 1$ , we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.<sup>25</sup> Note that this case is well-defined since  $\alpha P_C^*(b) > 1$  implies  $\frac{1}{\alpha P_C^*(b)(1-\phi)+\phi} < 1$ .

If the economic state in period 1 is  $\omega$ , a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1 and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the expected payment from withdrawing late is  $\frac{(1-R_d\phi)/P_C^*(\omega)}{1-\phi} > 1 \geq R_d$ , which implies that the best response for an individual normal depositor is to withdraw late. Moreover,  $\frac{(1-R_d\phi)/P_C^*(\omega)}{1-\phi} \leq \alpha R_d$ , which implies that the best response for a liquidity-shocked depositor is to withdraw early. Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\mathbf{E}[U_I] = \eta \left( \alpha R_d \phi + 1 - R_d \phi \right) + (1 - \eta) \left( \alpha R_d \phi + \frac{1 - R_d \phi}{P_C^*(b)} \right).$$

The locally optimal  $R_d$  is the upper bound of 1 since  $\alpha > 1$  and  $\alpha P_C^*(b) > 1$ . The maximum expected utility is then

$$\mathbf{E}[U_I | R_d = 1] = \eta \left( \alpha \phi + 1 - \phi \right) + (1 - \eta) \left( \alpha \phi + \frac{1 - \phi}{P_C^*(b)} \right). \quad (21)$$

Since  $P_C^*(b) < 1$ , it is clear to see that the expected utility from Case 2 (equation (21)) is greater than the expected utility from Case 1 (equation (20)).

**Case 3:** If  $R_d \leq \frac{1}{\alpha P_C^*(b)(1-\phi)+\phi}$ , then there is no equilibrium in which liquidity-shocked depositors withdraw early in the bad state.<sup>26</sup> If liquidity-shocked depositors withdraw late in the bad state, then the utility of the bank in the bad state,  $\frac{1}{P_C^*(b)}$ , is less than the utility in the bad state from Case 2 from equation (21) since  $\alpha P_C^*(b) > 1$ . Similarly, if liquidity-shocked depositors withdraw late in the good state then the utility of the bank in the good state, 1, is less than the utility in the good

<sup>25</sup>Note that under this condition there is no equilibrium in which normal depositors withdraw early. In particular, the bank cannot default in period 1 even if all depositors withdraw early. As a result, if all other depositors withdraw early, then the best response for an individual depositor is to withdraw late because the individual payoff is infinite.

<sup>26</sup>There is no equilibrium in which liquidity-shocked depositors withdraw early in the bad state since the utility from withdrawing early  $\alpha R_d$  is less than the payment from withdrawing late (conditional on the other liquidity-shocked depositors withdrawing early),  $\frac{(1-R_d\phi)/P_C^*(b)}{1-\phi}$ . Moreover, to have an equilibrium in which liquidity-shocked depositors withdraw late, the payment from withdrawing late (conditional on the other liquidity-shocked depositors withdrawing late),  $1/P_C^*(b)$ , must be larger than the payment from withdrawing early,  $\alpha R_d$ , which requires  $R_d \leq \frac{1}{\alpha P_C^*(b)}$ . Note that  $\frac{1}{\alpha P_C^*(b)}$  is less than the bound  $\frac{1}{\alpha(1-\phi)P_C^*(b)+\phi}$  since  $\alpha P_C^*(b) > 1$ . Hence, there is no equilibrium for  $R_d \in \left[ \frac{1}{\alpha P_C^*(b)}, \frac{1}{\alpha(1-\phi)P_C^*(b)+\phi} \right]$ .

state from Case 2 from equation (21) since  $\alpha > 1$ . Therefore Case 3 is dominated by Case 2.

Since Case 2 dominates Case 1 and Case 3, the globally optimal early repayment  $R_d$  is the local optimum from Case 2,  $R_d = 1$ , and the maximum utility is given by equation (21).

Now, the portfolio from Case 2 is dominated by investing a fraction  $1 - L$  of the bank's assets in complex assets and setting  $R_d = L$ . To see this, consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.<sup>27</sup> If the economic state in period 1 is  $\omega$ , a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1, and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is  $\frac{(L-\phi L)/P_C^*(\omega)+\mu_\omega R(1-L)}{1-\phi} > L = R_d$ . Moreover,  $\frac{(L-\phi L)/P_C^*(\omega)+\mu_\omega R(1-L)}{1-\phi} < \alpha L = \alpha R_d$  by  $\frac{1}{P_C^*(b)} < R$  and the assumption in (4), which implies that the best response for a liquidity-shocked depositor is to withdraw early. Therefore, there is an equilibrium as described. In this equilibrium the utility of the bank is

$$\begin{aligned} \mathbf{E}[U_C | R_d = L] &= \eta \left( \alpha L \phi + L - \phi L + \mu_g R (1 - L) \right) \\ &\quad + (1 - \eta) \left( \alpha L \phi + \frac{L - \phi L}{P_C^*(b)} + \mu_b R (1 - L) \right). \end{aligned} \quad (22)$$

Then, subtracting (21) from (22) obtains

$$\begin{aligned} &(1 - L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1 - L)(1 - \eta) \left( \mu_b R - \left\{ \alpha\phi + \frac{1 - \phi}{P_C^*(b)} \right\} \right) \\ &> (1 - L)\eta(\mu_g R - \{\alpha\phi + 1 - \phi\}) + (1 - L)(1 - \eta)(\mu_b R - \alpha) \\ &> 0. \end{aligned}$$

The penultimate line follows from  $\alpha P_C^*(b) > 1$  and the last line follows from the assumption in (1). This shows that holding only liquid assets is a dominated portfolio.  $\square$

**Lemma 2** (Debt contract). *Banks pay depositors that withdraw early a return of  $R_d = \kappa$ .*

*Proof.* Since liquidity-shocked depositors only have elevated marginal utility for the first  $\kappa$  of payments, it is clear that a bank has no incentive to pay more than  $\kappa$ .<sup>28</sup> Next, we show that it is also not optimal for banks to offer a rate lower than  $\kappa$ . For the rest of the proof, assume  $R_d \leq \kappa$ . Note that the assumption  $L > \kappa\phi$  implies that, if  $R_d \leq \kappa$  and only the mass  $\phi$  of liquidity-shocked depositors withdraws early, then a

<sup>27</sup>Note that there does not exist an equilibrium in which normal depositors withdraw early by similar reasoning as in Case 2 above.

<sup>28</sup>Note that an individual bank is indifferent to paying slightly higher amounts than  $\kappa$  as long as normal depositors do not have an incentive to withdraw early. However, we rule out these equilibria because they are socially less efficient due to their effects on the equilibrium bond price.

bank has excess liquidity  $L - R_d\phi > 0$ , which can be used to buy up to  $(L - R_d\phi)/P_C^*(\omega)$  bonds.

**Complex banks.** Consider a complex bank. We consider three cases depending on the magnitude of the early repayment  $R_d$ .

**Case 1:** If  $L + P_C^*(b)\mu_b R(1 - L) < R_d \leq \kappa$ , we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors withdraw early only in the bad state. Note that this case is well-defined since  $L + P_C^*(b)\mu_b R(1 - L) < \kappa$  follows from the assumption in (5).

If the economic state in period 1 is good, a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1, and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is  $\frac{L - R_d\phi + \mu_g R(1 - L)}{1 - \phi}$ . Therefore the best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} \frac{L - R_d\phi + \mu_g R(1 - L)}{1 - \phi} &> R_d \\ \iff L + \mu_g R(1 - L) &> R_d. \end{aligned} \quad (23)$$

This is satisfied since  $R_d \leq \kappa < L + \mu_g R(1 - L)$  by the assumption in (6). The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} \frac{L - R_d\phi + \mu_g R(1 - L)}{1 - \phi} &< \alpha R_d \\ \iff \frac{L + \mu_g R(1 - L)}{\alpha(1 - \phi) + \phi} &< R_d. \end{aligned} \quad (24)$$

This is satisfied since  $R_d > L + P_C^*(b)\mu_b R(1 - L) > \frac{L + \mu_g R(1 - L)}{\alpha(1 - \phi) + \phi}$  by the assumption in (3).

If the economic state in period 1 is bad and all depositors withdraw early, then the bank defaults since the maximum liquidity it can supply by paying out of its liquid assets,  $L$ , and by selling bonds,  $\mu_b P_C^*(b)R(1 - L)$ , is less than the demand,  $R_d$ . Therefore the best response for an individual normal depositor is to withdraw early since the payment from withdrawing early, which is the total liquidation value of the bank  $L + P_C^*(b)\mu_b R(1 - L)$  since the bank experiences a run, is greater than the payment from withdrawing late, which is zero.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\begin{aligned} \mathbf{E}[U_C] &= \eta \left( \alpha R_d \phi + L - R_d \phi + \mu_g R(1 - L) \right) \\ &\quad + (1 - \eta) \left( \alpha \phi + 1 - \phi \right) \left( L + P_C^*(b)\mu_b R(1 - L) \right). \end{aligned}$$

The locally optimal  $R_d$  is the upper bound  $\kappa$  since  $\alpha > 1$ . The maximum expected

utility is then

$$\begin{aligned} \mathbf{E}[U_C | R_d = \kappa] &= \eta(\alpha\kappa\phi + L - \kappa\phi + \mu_g R(1-L)) \\ &\quad + (1-\eta)\left(\alpha\phi + 1 - \phi\right)\left(L + P_C^*(b)\mu_b R(1-L)\right). \end{aligned} \quad (25)$$

**Case 2:** If  $\max\left\{\frac{L/P_C^*(\omega) + \mu_\omega R(1-L)}{\alpha(1-\phi) + \phi/P_C^*(\omega)}\right\}_{\omega=b,g} \leq R_d \leq L + P_C^*(b)\mu_b(1-L)R$ , we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.<sup>29</sup> Note that this case is well-defined since  $\frac{L + \mu_g R(1-L)}{\alpha(1-\phi) + \phi} \leq L + P_C^*(b)\mu_b(1-L)R$  follows from the assumption in (3), and  $\frac{L/P_C^*(b) + \mu_b R(1-L)}{\alpha(1-\phi) + \phi/P_C^*(b)} < L + P_C^*(b)\mu_b(1-L)R$  follows from  $\alpha P_C^*(b) > 1$ .

If the economic state in period 1 is good and the equilibrium is as described, then the best response for an individual normal depositor is to withdraw late since  $R_d \leq \kappa$  implies that the condition in equation (23) is satisfied. The best response for an individual liquidity-shocked depositor is to withdraw early since  $R_d > \frac{L + \mu_g R(1-L)}{\alpha(1-\phi) + \phi}$  implies that the condition in equation (27) is satisfied.

If the economic state in period 1 is bad, a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1, and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is  $\frac{(L - R_d\phi)/P_C^*(b) + \mu_b R(1-L)}{1-\phi}$ . The best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} \frac{(L - R_d\phi)/P_C^*(b) + \mu_b R(1-L)}{1-\phi} &> R_d \\ \iff \frac{L/P_C^*(b) + \mu_b R(1-L)}{1-\phi + \phi/P_C^*(b)} &> R_d. \end{aligned} \quad (26)$$

This is satisfied since  $R_d \leq L + \mu_b P_C^*(b)(1-L)R < \frac{L/P_C^*(b) + \mu_b R(1-L)}{1-\phi + \phi/P_C^*(b)}$ , which follows from  $P_C^*(b) < 1$ . The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} \frac{(L - R_d\phi)/P_C^*(b) + \mu_b R(1-L)}{1-\phi} &\leq \alpha R_d \\ \iff \frac{L/P_C^*(b) + \mu_b R(1-L)}{\alpha(1-\phi) + \phi/P_C^*(b)} &\leq R_d. \end{aligned} \quad (27)$$

This is satisfied since  $R_d \geq \frac{L/P_C^*(b) + \mu_b R(1-L)}{\alpha(1-\phi) + \phi/P_C^*(b)}$ .

Therefore, there is an equilibrium as described. In this equilibrium, the utility

<sup>29</sup>Note that there is no equilibrium in which normal depositors withdraw early. To see this, note that the bank cannot default in period 1 even if all depositors withdraw early. If all other depositors withdraw early, then the best response for an individual normal depositor is to withdraw late because the individual payoff is infinite.

of the bank is

$$\begin{aligned} \mathbb{E}[U_C] &= \eta(\alpha R_d \phi + L - R_d \phi + \mu_g R(1 - L)) \\ &\quad + (1 - \eta) \left( \alpha R_d \phi + \frac{L - R_d \phi}{P_C^*(b)} + \mu_b R(1 - L) \right). \end{aligned} \quad (28)$$

The locally optimal  $R_d$  is the upper bound  $L + \mu_b P_C^*(b)(1 - L)R$  since  $\alpha > 1$  and  $\alpha P_C^*(b) > 1$ . The maximum expected utility can then be written as

$$\begin{aligned} \mathbb{E}[U_C | R_d = L + \mu_b P_C^*(b)(1 - L)R] &= \eta(\phi(\alpha - 1)[L + P_C^*(b)\mu_b(1 - L)R] + L + \mu_g R(1 - L)) \\ &\quad + (1 - \eta)[L + P_C^*(b)\mu_b(1 - L)R] \left( \alpha \phi + \frac{1 - \phi}{P_C^*(b)} \right). \end{aligned} \quad (29)$$

**Case 3:** If  $R_d < \max\left\{\frac{L/P_C^*(\omega) + \mu_\omega R(1 - L)}{\alpha(1 - \phi) + \phi/P_C^*(\omega)}\right\}_{\omega=b,g}$ , then there is no equilibrium in which liquidity-shocked depositors withdraw early in both states since at least one of (24) or (27) is violated. If liquidity-shocked depositors withdraw early in the bad state, then the utility of the bank in the bad state,  $\frac{L}{P_C^*(b)} + \mu_b R(1 - L)$ , is less than the utility in the bad state from Case 2 in equation (28) since  $\alpha P_C^*(b) > 1$ . Similarly, if liquidity-shocked depositors withdraw early in the good state then the utility of the bank in the good state,  $L + \mu_g R(1 - L)$ , is less than the utility in the good state from Case 2 in equation (28) since  $\alpha > 1$ . Therefore Case 3 is dominated by Case 2.

To determine the globally optimal payment  $R_d$ , we compare the maximum utility between Case 1 and Case 2. The expected utility from Case 1 (equation (25)) minus the expected utility from Case 2 (equation (29)) is

$$\eta\phi(\alpha - 1)\left(\kappa - [L + P_C^*(b)\mu_b R(1 - L)]\right) - (1 - \eta)(1 - \phi)[L + P_C^*(b)\mu_b R(1 - L)]\left(\frac{1}{P_C^*(b)} - 1\right) > 0, \quad (30)$$

where the inequality follows by  $1 > P_C^*(b) > \frac{1}{\alpha}$  and the assumption in (5).<sup>30</sup> This implies the globally optimal payment is  $R_d = \kappa$ .

**Simple banks.** Consider a simple bank. We consider three cases depending on the magnitude of the early repayment  $R_d$ .

**Case 1:** If  $L < R_d \leq \kappa$ , we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors withdraw early only when the bank's return is low. Note that this case is well-defined since  $L < \kappa$ .

If return is high, the economic state is  $\omega$ , a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1, and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the payment in period 2 for an individual normal depositor is

<sup>30</sup>To see this, denote the left hand side of (5) by  $F$ . Note that  $L + P_C^*(b)\mu_b R(1 - L) < L + \mu_b R(1 - L)$  since  $P_C^*(b) < 1$ . Let  $x \equiv L + \mu_b R(1 - L)$ . Therefore  $F \geq \eta\phi(\alpha - 1)(\kappa - x) - (1 - \eta)(1 - \phi)x\left(\frac{1}{P_C^*(b)} - 1\right)$ . Then note that  $\frac{1}{P_C^*(b)} < \alpha$ , which implies  $F \geq \eta\phi(\alpha - 1)(\kappa - x) - (1 - \eta)(1 - \phi)x(\alpha - 1)$ . Then the result follows from rearranging and applying (5).

$\frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi}$ . Therefore the best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} & \frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi} > R_d \\ \iff & \frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)} > R_d. \end{aligned} \quad (31)$$

This is satisfied since  $R_d \leq \kappa < \frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)}$ .<sup>31</sup> The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} & \frac{(L-R_d\phi)/P_C^*(\omega)+R(1-L)}{1-\phi} < \alpha R_d \\ \iff & \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} < R_d. \end{aligned} \quad (32)$$

This is satisfied since  $\frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} < L < R_d$ , which follows from  $P_C^*(R) > \frac{1}{R}$  and the assumption in (4).

If the return revealed in period 1 is low and all depositors withdraw early, then the bank defaults since the maximum liquidity it can supply by paying out of its liquid assets  $L$  is less than the demand  $R_d$ .<sup>32</sup> Therefore the best response for an individual normal depositor is to withdraw early since the payment from withdrawing early, which is the total liquidation value of the bank  $L$  since the bank experiences a run, is greater than the payment from withdrawing late, which is zero.

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\begin{aligned} E[U_S] = & \eta \left( \mu_g (\alpha R_d \phi + L - R_d \phi + R(1-L)) + (1 - \mu_g)(\alpha \phi + 1 - \phi)L \right) \\ & + (1 - \eta) \left( \mu_b \left( \alpha R_d \phi + \frac{L - R_d \phi}{P_C^*(b)} + R(1-L) \right) + (1 - \mu_b)(\alpha \phi + 1 - \phi)L \right). \end{aligned}$$

The locally optimal  $R_d$  is the upper bound  $\kappa$  since  $\alpha > 1$  and  $\alpha P_B^*(b) > 1$ .

**Case 2:** If  $\left\{ \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} \right\}_{\omega=b,g} \leq R_d \leq L$ , we consider an equilibrium in which liquidity-shocked depositors always withdraw early and normal depositors always withdraw late.<sup>33</sup> Note that this case is well-defined since  $\frac{L/P_C^*(b)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)} < L$  follows from  $P_C^*(R) > \frac{1}{R}$  and the assumption in (4).

If the individual return in period 1 is high and the equilibrium is as described,

<sup>31</sup>To see this, note that  $\frac{L/P_C^*(\omega)+R(1-L)}{1-\phi+\phi/P_C^*(\omega)}$  is between  $L + R(1-L)$  (when evaluated at  $P_C^*(\omega) = 1$ ) and  $\frac{L}{\phi}$  (when evaluated at the limit as  $P_C^*(\omega) \rightarrow 0$ ). Then note that  $\kappa < L + R(1-L)$  by the assumption in (6) and  $\kappa < \frac{L}{\phi}$  by the assumption  $L > \kappa\phi$ .

<sup>32</sup>Recall that the bank cannot sell bonds against its observably worthless assets.

<sup>33</sup>Note that there is no equilibrium in which normal depositors withdraw late when the return is low by a similar argument as in Case 2 for complex bank.

then the best response for an individual normal depositor is to withdraw late since  $R_d \leq \kappa$  implies that the condition in (31) is satisfied. The best response for an individual liquidity-shocked depositor is to withdraw early since  $R_d \geq \frac{L/P_C^*(b)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(b)}$  implies that the condition in (33) is satisfied.

If the return in period 1 is low, the economic state is  $\omega$ , a mass  $\phi$  of liquidity-shocked depositors withdraws in period 1, and a mass  $1 - \phi$  of normal depositors withdraws in period 2, then the expected payment in period 2 for an individual normal depositor is  $\frac{(L-\kappa\phi)/P_C^*(\omega)}{1-\phi}$ . The best response for an individual normal depositor is to withdraw late if and only if

$$\begin{aligned} & \frac{(L - R_d\phi)/P_C^*(\omega)}{1 - \phi} > R_d \\ \iff & \frac{L/P_C^*(b)}{1 - \phi + \phi/P_C^*(\omega)} \geq R_d. \end{aligned} \quad (33)$$

This is satisfied since  $P_C(\omega) \leq 1$ . The best response for an individual liquidity-shocked depositor is to withdraw early if and only if

$$\begin{aligned} & \frac{(L - R_d\phi)/P_C^*(\omega)}{1 - \phi} \leq \alpha R_d \\ \iff & \frac{L/P_C^*(\omega)}{\alpha(1 - \phi) + \phi/P_C^*(b)} \leq R_d. \end{aligned} \quad (34)$$

This is satisfied since  $R_d \geq \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)}$ .

Therefore, there is an equilibrium as described. In this equilibrium, the utility of the bank is

$$\begin{aligned} \mathbf{E}[U_S] = & \eta \left( \alpha R_d \phi + L - R_d \phi + \mu_g R(1 - L) \right) \\ & + (1 - \eta) \left( \alpha R_d \phi + \frac{L - R_d \phi}{P_C^*(b)} + \mu_b R(1 - L) \right). \end{aligned} \quad (35)$$

The locally optimal  $R_d$  is the upper bound  $R_d = L$  since  $\alpha > 1$  and  $\alpha > \frac{1}{P_C^*(b)}$ . This portfolio is dominated by investing in complex investments and setting  $R_d = L + P_C^*(b)\mu_g R(1 - L)$ . This can be seen by observing that the expected utility is the same function of  $R_d$  as Case 2 for a complex bank (equation (28)), this function is increasing in  $R_d$ , and the local optimum from Case 2 for a complex bank  $R_d = L + P_C^*(b)\mu_b R(1 - L)$  is larger than the local optimum for Case 2 of a simple bank  $L$ .

**Case 3:** If  $R_d < \left\{ \frac{L/P_C^*(\omega)+R(1-L)}{\alpha(1-\phi)+\phi/P_C^*(\omega)} \right\}_{\omega=b,g}$ , then there is no equilibrium in which liquidity-shocked depositors withdraw early in both states since at least one of (32) or (34) is violated. If liquidity-shocked depositors withdraw early when the return is low, then the utility of the bank in the low return state,  $\frac{L}{P_C^*(b)}$ , is less than the utility in the low return state from Case 2 in equation (35) since  $\alpha > \frac{1}{P_C^*(b)}$ . Similarly, if liquidity-shocked depositors withdraw early in the high return state then the utility

of the bank in the high return state,  $L + R(1 - L)$ , is less than the utility in the good state from Case 2 in equation (35) since  $\alpha > 1$ . Therefore Case 3 is dominated by Case 2.

Therefore, if a bank invests in simple assets, then the optimal repayment  $R_d$  must correspond to the local maximum from Case 1, which is  $R_d = \kappa$ .  $\square$

**Corollary 1** (Bank-run conditions). *Liquidity-shocked depositors always withdraw early and normal depositors withdraw early if and only if*

- *the bank is complex and the economic state is bad*
- *the bank is simple and the observable return is low*

*Proof.* This follows from the proof of Lemma 2 for the special case where  $R_d = \kappa$ .  $\square$

**Lemma 3** (Bond price). *The bond price satisfies  $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b) < P_C^*(g) = 1$ .*

*Proof.* We first show that  $P_C^*(g) = 1$ . This follows from the fact that, in good times, the only banks that experience a run are simple banks with a low return. However, these banks cannot sell bonds against their long-term assets since the low return is publicly observable. As a result, in good times the supply of bond is zero, which implies that the bond price must be at the maximum possible level,  $P_C^*(g) = 1$ .

Consider the bond price when the economic state is bad  $P_C(b)$ . Recall the relative advantage of investing in complex assets compared to simple assets  $\Delta$  as defined in equation (9). Since banks are ex-ante identical, the price must be such that banks are indifferent between complex and simple assets in an equilibrium. Note that there is no equilibrium in which banks invest in only complex or simple assets. If all banks invested in simple assets, then the bond price  $P_C(b)$  would be equal to 1, but in that case banks would prefer to invest in complex assets. If all banks invested in complex assets, then the bond price would be equal to zero, but in that case banks would prefer to invest in simple assets.

At the maximum possible price  $P_C(b) = 1$ , the second term of equation (9) is equal to  $(1 - \eta)\mu_b\phi(\alpha - 1)[L + R(1 - L) - \kappa]$ , which is positive by the assumption  $\kappa < L + \mu_g R(1 - L)$ . Therefore, complex banks have a higher expected utility.

If the price is very low, then it is easy to see that simple banks have a higher expected utility. Specifically, if the price is as low as  $P_C(b) = \frac{1}{R}$  then the difference in utility  $\Delta$  expressed in equation (9) at  $P_C(b) = \frac{1}{R}$  is equal to

$$\Delta\left(\frac{1}{R}\right) = \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) + (1 - \eta)\mu_b[1 - \phi + \alpha\phi - R(1 - \phi\kappa) - \alpha\phi\kappa]$$

, which is negative by the assumption in (2).

Since the relative benefit of complex assets  $\Delta(P_C(b))$  is increasing and continuous with  $\Delta\left(\frac{1}{R}\right) < 0 < \Delta(1)$ , there is a unique equilibrium bond price that satisfies  $\frac{1}{R} < P_C^*(b) < 1$ .

Finally, note that  $\alpha > R$  from the assumption in (4), which implies that  $\frac{1}{\alpha} < \frac{1}{R} < P_C^*(b)$ .  $\square$

## Proof of Proposition 2

**Proposition 2** (Volume of complex banks). *The volume of complex banks is related to the complex-asset price in bad times as follows:*

$$\underbrace{V^*R(1-L)\mu_b}_{\text{complex-asset supply}} = \underbrace{\left(1 - V^*\right)\mu_b \frac{L - \kappa\phi}{P_C^*(b)}}_{\text{complex-asset demand}}. \quad (10)$$

*Proof.* Consider the equilibrium described Proposition 1. As shown in the proof of Lemma 3, no bonds are issued in the good state. In the bad state, however, all complex banks issue bonds. In particular, each complex bank issues  $\mu_b(1-L)R$  bonds. Therefore, the overall supply of bonds is

$$\int_{i:\xi_i=C} S_{B,i}(P_C^*(b))di = \int_{i:\xi_i=C} \mu_b(1-L)Rdi = \mu_b(1-L)R V^*. \quad (36)$$

At the same time, simple banks with a positive return have excess liquidity, which they fully invest in bonds since  $P_C^*(b) < 1$ . In particular, each simple bank with a positive return demands  $\frac{L-\kappa\phi}{P_C^*(b)}$  bonds. Therefore, the overall demand for bonds is

$$\int_{i:\xi_i=S} \frac{L-\kappa\phi}{P_C^*(b)} \mathbf{1}_{\{R_i^S(b)=R\}} di = \frac{L-\kappa\phi}{P_C^*(b)} \underbrace{\mu_b(1-V^*)}_{\text{mass of simple banks with positive return}}. \quad (37)$$

Equating market supply and market demand for bonds (i.e., Eqs. (36) and (37)), implies the result.  $\square$

## Proof of Proposition 3

**Proposition 3.** *If  $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \in \left[1, \frac{1-1/\kappa}{1-\phi}\right]$ , the equilibrium complex-asset price in bad times  $P_C^*(b)$  is increasing in the liquidity level  $L$ .*

*Proof.* Note that  $P_C^*(b)$  is well-defined and differentiable for  $L \in [\kappa\phi, 1]$  since  $P_C^*(b)$  is the positive solution to the equation  $\Delta(P_C(b)) = 0$ , where  $\Delta$  is the relative advantage of complex assets as defined in equation (9). As shown in the proof of Lemma 3, this has a unique positive solution.

It's straightforward to see that  $\frac{\partial\Delta}{\partial P_C(b)} > 0$ . Therefore, by the implicit function theorem, we have that  $\frac{\partial P_C^*(b)}{\partial L}$  has the opposite sign as  $\frac{\partial\Delta}{\partial L}$ , which is described in equation (11). Let  $\hat{L} = \kappa \left(1 - \frac{(1-\phi)(1-\eta)\mu_b}{\eta(1-\mu_g)}\right)$ . Note that the assumption  $\frac{\eta(1-\mu_g)}{(1-\eta)\mu_b} \in \left[1, \frac{1-1/\kappa}{1-\phi}\right]$  implies that  $\hat{L} \in [\kappa\phi, 1]$ , which is the applicable range for  $L$ . Note that when  $L = \hat{L}$  we have  $P_C^*(b) = \frac{1}{\alpha\phi+1-\phi}$  and hence  $\frac{\partial\Delta}{\partial L} < 0$ , which implies  $\frac{\partial P_C^*(b)}{\partial L} > 0$ .<sup>34</sup>

<sup>34</sup>Note that the strictness of the inequality follows from  $\eta(1-\mu_g)\phi(\alpha-1) > 0$ .

Now suppose for a contradiction that  $P_C^*(b)$  is non-monotonic in  $L$ . Since  $P_C^*(b)$  is a continuously differentiable function of  $L$ , there must exist two distinct points  $\bar{L}$  and  $\bar{L}'$  that implement the same equilibrium price but for which  $\frac{\partial P_C^*(b)}{\partial L}|_{L=\bar{L}} \neq \frac{\partial P_C^*(b)}{\partial L}|_{L=\bar{L}'}$ . But this contradicts the fact that  $\frac{\partial \Delta}{\partial L}$  only depends on  $L$  though its effect on  $P_C^*(b)$  (see equation (11)).

Since  $\frac{\partial P_C^*(b)}{\partial L} > 0$  at  $L = \hat{L}$  and  $P_C^*(b)$  must be globally monotonic in  $L$ , then  $P_C^*(b)$  must be everywhere increasing in  $L$ . □

## Proof of Proposition 4

**Proposition 4** (Welfare-maximizing volume of complex banks). *Let  $\hat{L} = \kappa \left( 1 - \frac{(1-\phi)(1-\eta)\mu_b}{(1-\mu_g)\eta} \right)$ .*

*When liquidity requirements are tight,  $L > \hat{L}$ , then there is excess investment in complex assets, i.e.,  $V^W < V^*$ . Moreover, the welfare-maximizing complex-asset price in bad times is equal to the maximum level of 1, i.e.,  $P_C^W(b) = 1 > P_C^*(b)$ . When liquidity requirements are loose,  $L < \hat{L}$ , then there is underinvestment in complex assets, i.e.,  $V^W > V^*$ . Moreover, the welfare-maximizing complex-asset price in bad times  $P_C^W(b)$  satisfies  $0 < P_C^W(b) < P_C^*(b)$ .*

*Proof.* Consider the total effect of varying the volume of complex banks on welfare

$$\frac{d\mathcal{W}}{dV^W} = \frac{\partial \mathcal{W}}{\partial V^W} + \frac{\partial \mathcal{W}}{\partial P_C^W(b)} \frac{\partial P_C^W(b)}{\partial V^W}.$$

The first term corresponds to the direct effect and is equal to the relative advantage of investing in complex assets  $\Delta$  as defined in equation (9). The second term corresponds to the indirect effect through the adjustment of price, which is not internalized by the banks in the equilibrium. Since  $P_C^W(b)$  is related to  $V^W$  by the market-clearing condition in equation (10), we have that the volume of complex banks is inversely related to the price:

$$\frac{\partial P_C^W(b)}{\partial V^W} = -\frac{L - \kappa\phi}{(1-L)R(V^W)^2} = -\frac{P_C^W(b)}{V^W(1-V^W)} < 0. \quad (38)$$

The price, in turn, affects welfare as follows:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)} = (1-\eta)\mu_b \left[ V^W(\alpha\phi + 1 - \phi)R(1-L) - (1-V^W) \frac{L - \kappa\phi}{P_C^W(b)^2} \right]$$

The first term represents the marginal benefit of increasing the price in terms of supporting complex banks, which sell bonds, while the second term represents the marginal cost in terms of decreasing the return for simple banks that draw a high return, which buy bonds.

Then, we can write:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)} \frac{\partial P_C^W(b)}{\partial V^W} = (1 - \eta)\mu_b \left[ \frac{1}{1 - V^W} (\alpha\phi + 1 - \phi) P_C^W(b) R(1 - L) - \frac{1}{V^W} \frac{L - \kappa\phi}{P_C^W(b)} \right].$$

Therefore, the total effect on welfare can be written as

$$\begin{aligned} \frac{d\mathcal{W}}{dV^W} &= \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) \\ &\quad + (1 - \eta)\mu_b \left[ (\alpha\phi + 1 - \phi) \left( L + P_C^W(b) R(1 - L) \left( 1 - \frac{1}{1 - V^W} \right) \right) \right] \\ &\quad - (1 - \eta)\mu_b \left[ \alpha\kappa\phi + \frac{L - \kappa\phi}{P_C^W(b)} \left( 1 - \frac{1}{V^W} \right) + R(1 - L) \right]. \end{aligned}$$

This first term shows that the planner's incentive to invest in complex assets is increasing in the liquidity services advantage of complex assets in good times. The second term corresponds to the advantage of complex banks in bad times compared to simple banks that default, which is their ability to issue bonds in the interbank market. This advantage directly increases the planner's incentive to invest in complex assets, but the planner also internalizes the fact that increasing the volume of complex banks leads to a price reduction that offsets this advantage. The third term corresponds to the disadvantage of complex banks in bad times compared to simple banks with a high return, which is the fact that they always experience a run. This disadvantage directly decreases the planner's incentive to invest in complex assets, but the planner also internalizes the fact that increasing the volume of complex banks leads to a price reduction that has the benefit of increasing the return of the simple banks.

Further simplifying by substituting the market-clearing condition in equation (10) obtains the following:

$$\begin{aligned} \frac{d\mathcal{W}}{dV^W} &= \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) - (1 - \eta)\mu_b\phi\kappa(1 - \phi)(\alpha - 1) \\ &= (\alpha - 1)\phi\eta(1 - \mu_g) \left[ \underbrace{\kappa \left( 1 - \frac{(1 - \phi)(1 - \eta)\mu_b}{\eta(1 - \mu_g)} \right)}_{=\hat{L}} - L \right]. \end{aligned} \quad (39)$$

This equation illustrates that the direct and price effects of increasing the volume of complex banks in bad times offset each other such that the net effect is constant in the volume of complex banks and the price. This drives the solution to the boundaries of the choice space depending on the sign of equation (39), which in turn depends on the magnitude of the tightness of liquidity requirements  $L$  relative to  $\hat{L}$ .

**Case 1:** If  $L > \hat{L}$ , the optimal policy is to reduce the volume of complex banks and therefore increase the bond price  $P_C^W(b)$  until it is equal to its upper bound of 1. Note that reducing the volume of complex banks does not create an incentive

to deviate from the debt contract  $R_d = \kappa$  and bank-run conditions as described in Proposition 1. In particular, the proof of Lemma 2 shows that the debt contract and bank-run conditions hold for any  $P_C(b)$  satisfying  $\frac{1}{R} \leq P_C(b) \leq 1$ . Since  $\frac{1}{R} < P_C^*(b) < 1$ , they hold for  $P_C^W(b) \in [P_C^*(b), 1]$ .

Intuitively, increasing the bond price supports complex banks since they experience a run in the bad state. Once the bond price is equal to 1, which is the return on liquid assets, there is no incentive to further reduce the volume of complex banks since the bond price cannot be any higher. Additionally, reducing the volume of complex banks has a cost since complex assets yield a higher expected utility when  $P_C^W(b) = 1$ .<sup>35</sup>

**Case 2:** If  $L < \hat{L}$ , the optimal policy is to increase the volume of complex banks and therefore decrease bond price  $P_C(b)$  until the debt contract of  $R_d = \kappa$  and bank-run conditions as described in Proposition 1 can no longer be maintained.

The minimal  $P_C^W(b)$  such that banks still prefer to offer an early repayment of  $R_d = \kappa$  is strictly greater than 0. To see this, note that Lemma 2 shows that a necessary condition for banks to set  $R_d = \kappa$  is given by (30). Note that the left hand side of (30) has the same sign as when it is multiplied by  $P_C(b) > 0$ , which can be denoted by

$$F(P_C(b)) \equiv P_C(b)\eta\phi(\alpha - 1)(\kappa - \{L + \mu_b P_C(b)R(1 - L)\}) - (1 - \eta)(1 - \phi)(L + P_C(b)\mu_b R(1 - L))(1 - P_C(b)).$$

It is then straightforward to see that  $F(0) < 0$ , and  $0 < F(1)$  follows from the assumption in (5). Since  $F$  is quadratic, this implies there is a unique solution  $P_C^W(b) \in (0, 1)$  to the equation  $F(P_C^W(b)) = 0$ .

**Alternative intuition.** Consider the *local* incentive for the planner to increase the volume of complex bank relative to the equilibrium:

$$\frac{d\mathcal{W}}{dV^W}|_{V^W=V^*} = \frac{\partial\mathcal{W}}{\partial V^W}|_{V^W=V^*} + \frac{\partial\mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)} \frac{\partial P_C^W(b)}{\partial V^W}|_{V^W=V^*}.$$

Note that  $\frac{\partial\mathcal{W}}{\partial V^W}|_{V^W=V^*} = 0$  since banks are indifferent between the two types of assets in the equilibrium. Recall that  $\frac{\partial P_C^W(b)}{\partial V^W} < 0$  from equation (38). Therefore,  $\frac{\partial\mathcal{W}}{\partial V^W}|_{V^W=V^*}$  has the opposite sign as  $\frac{\partial\mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)}$ , which corresponds to the regulator's incentive to adjust the price that is not internalized by the individual banks:

$$\frac{\partial\mathcal{W}}{\partial P_C^W(b)}|_{P_C^W(b)=P_C^*(b)} = (1 - \eta)\mu_b \left[ (\alpha\phi + 1 - \phi)R(1 - L)V^* - \frac{(1 - V^*)(L - \kappa\phi)}{P_C^*(b)^2} \right].$$

<sup>35</sup>This cost can be seen by differentiating the expected utility for  $P_C^W(b) = 1$ :

$$\text{sign}\left(\frac{\partial\mathcal{W}}{\partial V^W}\right) = \text{sign}\left((1 - \eta)\mu_b(L - \kappa + R(1 - L)) + \eta(1 - \mu_g)(\kappa - L)\right) > 0.$$

As described above, the first term corresponds to the marginal benefit of increasing the price in terms of increasing the return of complex banks, which sell bonds, while the second term corresponds to the marginal cost in terms of decreasing the return for simple banks that draw a high return, which buy bonds.

Using the market-clearing condition in equation (10), we can write:

$$\frac{\partial \mathcal{W}}{\partial P_C^W(b)} \Big|_{P_C^W(b)=P_C^*(b)} = \frac{(1-\eta)\mu_b(L-\kappa\phi)R(1-L)}{L-\kappa\phi+P_C^*(b)(1-L)R} \left[ (\alpha\phi+1-\phi) - \frac{1}{P_C^*(b)} \right].$$

Therefore,  $\frac{\partial \mathcal{W}}{\partial V^W} \Big|_{V^W=V^*}$  has the same sign as

$$(\alpha\phi+1-\phi) - \frac{1}{P_C^*(b)}. \quad (40)$$

Note that the equilibrium price at  $\hat{L}$  is equal to  $\frac{1}{\alpha\phi+1-\phi}$ . If the equilibrium price is increasing in  $L$  (see Proposition 3 for a sufficient condition), the expression in (40) is positive (negative) when  $L$  is greater (less) than  $\hat{L}$ . Intuitively,  $L$  determines whether the equilibrium price is large enough that the benefit of further increasing the price in terms of insuring complex banks exceeds the cost in terms of decreasing the returns on asset purchases for simple banks. □

## Proof of Proposition 5

Rearranging equation (14) and supposing that the volume of complex banks is equal to the level corresponding to no tax  $V(0)$ , the price can be expressed as

$$P_C^\tau(b) = \frac{\mu_b(L-\kappa\phi)(1-V(0)) + \tau\delta}{RV(0)(1-L)\mu_b},$$

hence it follows that

$$\frac{\partial P_C^\tau(b)}{\partial \tau} = \frac{\delta}{RV(0)(1-L)\mu_b} > 0. \quad (41)$$

Note also that increasing the tax maintains the debt contract and bank-run conditions as stated in Proposition 1. In particular, the proof of Lemma 2 shows that the debt contract and bank-run conditions hold for any  $P_C(b)$  satisfying  $\frac{1}{R} \leq P_C(b) \leq 1$ . Since the price that would occur in the absence of a tax  $P_C^0(b)$  satisfies  $\frac{1}{R} \leq P_C^0(b) < 1$ , this implies that they hold for  $P_C^\tau(b) \in [P_C^0(b), 1]$ .<sup>36</sup>

<sup>36</sup>Note that we restrict to  $\delta$  small enough such that a similar proof as in Lemma 3 works to show that there exists a unique equilibrium price  $P_C^\tau(b) \in (\frac{1}{R}, 1)$ .

## Proof of Proposition 6

**Proposition 6** (QE without commitment). *If QE is implemented without commitment, then the optimal tax is positive and equal to the minimum of income  $v$  and the minimum tax necessary to increase the complex-asset price in bad times  $P_C^\tau(b)$  to 1.*

*Proof.* Rearranging the market-clearing condition (equation (14)), we substitute

$$\frac{\tau\delta}{P_C^\tau(b)} = V(0)R(1-L)\mu_b - \left(1 - V(0)\right)\mu_b \frac{L - \kappa\phi}{P_C^\tau(b)}$$

into the expression for welfare (equation (17)). Then, if  $P_C^\tau(b) < 1$ , taking the derivative with respect to  $\tau$  and using the expression for  $\frac{\partial P_C^\tau(b)}{\partial \tau}$  from equation (41) in the proof of Proposition 5 obtains

$$\begin{aligned} \frac{\partial \mathcal{W}(\tau)}{\partial \tau} &= V(0)(1-\eta)(\alpha\phi + 1 - \phi) \left[ \underbrace{\frac{\delta}{RV(0)(1-L)\mu_b}}_{=\frac{\partial P_C^\tau(b)}{\partial \tau}} \mu_b R(1-L) - \delta \right] \\ &\quad - (1 - V(0))(1 - \eta)[\mu_b\delta + (1 - \mu_b)(\alpha\phi + 1 - \phi)\delta] \\ &> (1 - \eta)\delta(\alpha\phi + 1 - \phi) \left[ V(0) \left( \frac{1}{V(0)} - 1 \right) - (1 - V(0)) \right] \\ &= 0. \end{aligned}$$

Since welfare is increasing in  $\tau$  as long as  $P_C^\tau(b) < 1$ , the government optimally increases taxes until either  $P_C^\tau(b) = 1$  or  $\tau = v$ .  $\square$

## Proof of Proposition 7

**Proposition 7.** *If QE is undertaken with commitment and  $P_C^\tau(b) < 1$ , then*

(a) *the equilibrium complex-asset price is increasing in the tax  $\tau$ :  $\frac{\partial P_C^\tau(b)}{\partial \tau} > 0$ , and*

(b) *the equilibrium volume of complex banks is increasing in the tax  $\tau$ :  $\frac{\partial V(\tau)}{\partial \tau} > 0$ .*

*Proof. Part (a).* Note that the relative benefit of investing in complex assets is summarized by subtracting equation (16) from equation (15):

$$\begin{aligned} H^\tau(P_C(b)) &\equiv \mathbf{E}[U_C|P_C(b)] - \mathbf{E}[U_S|P_C(b)] \\ &= \eta(1 - \mu_g)\phi(\kappa - L)(\alpha - 1) \\ &\quad + (1 - \eta)\mu_b \left[ (\alpha\phi + 1 - \phi)(L + P_C(b)R(1-L)) - \left( \alpha\kappa\phi + \frac{L - \kappa\phi}{P_C(b)} + R(1-L) \right) \right] \\ &\quad + \delta\phi(\alpha - 1)[(1 - \eta)(v - \tau)\mu_b - \eta v(1 - \mu_g)]. \end{aligned} \tag{42}$$

Since banks are ex-ante identical and anticipate the tax, the equilibrium price must be such that banks are indifferent between complex and simple assets. For  $\delta$  suffi-

ciently small, there is a unique price  $P_C^\tau(b) \in \left(\frac{1}{R}, 1\right)$  satisfying  $H^\tau(P_C(b)) = 0$ .<sup>37</sup> Then, by the implicit function theorem, we have:

$$\begin{aligned} \frac{\partial P_C^\tau(b)}{\partial \tau} &= -\frac{\partial H/\partial \tau}{\partial H/\partial P_C(b)} \\ &= \frac{\delta\phi(\alpha-1)}{\left(\alpha\phi+1-\phi\right)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}} > 0. \end{aligned} \quad (43)$$

Note that increasing the tax maintains the debt contract and bank-run conditions as stated in Proposition 1. In particular, the proof of Lemma 2 shows that the debt contract and bank-run conditions hold for any  $P_C(b)$  satisfying  $\frac{1}{R} \leq P_C(b) \leq 1$ . Since the price that would occur in the absence of a tax  $P_C^0(b)$  satisfies  $\frac{1}{R} \leq P_C^0(b) < 1$ , this implies that they hold for  $P_C^\tau(b) \in [P_C^0(b), 1]$ .

**Part (b).** Rearranging equation (14) implies that

$$V(\tau) = \frac{\mu_b(L - \kappa\phi) + \delta\tau}{P_C^\tau(b)\mu_b R(1-L) + \mu_b(L - \kappa\phi)}. \quad (44)$$

Therefore, using equation (43) we have:

$$\begin{aligned} \frac{\partial V(\tau)}{\partial \tau} &= \frac{\delta(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L - \kappa\phi)) - (\mu_b(L - \kappa\phi) + \delta\tau) \frac{\partial P_C^\tau(b)}{\partial \tau} \mu_b R(1-L)}{\left(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L - \kappa\phi)\right)^2} \\ &= \frac{\delta(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L - \kappa\phi)) - (\mu_b(L - \kappa\phi) + \delta\tau) \frac{\delta\phi(\alpha-1)\mu_b R(1-L)}{(\alpha\phi+1-\phi)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}}}{\left(P_C^\tau(b)\mu_b R(1-L) + \mu_b(L - \kappa\phi)\right)^2} > 0, \end{aligned} \quad (45)$$

where the inequality follows because

$$\frac{\phi(\alpha-1)R(1-L)}{\left(\alpha\phi+1-\phi\right)R(1-L)+\frac{L-\kappa\phi}{P_C^\tau(b)^2}} < 1$$

since  $L > \kappa\phi$ , and

$$P_C^\tau(b)R(1-L) + L - \kappa\phi = (\delta\tau + \mu_b(L - \kappa\phi)) \frac{1}{V(\tau)} \geq \delta\tau + \mu_b(L - \kappa\phi)$$

since  $V(\tau) \leq 1$ . □

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<sup>37</sup>Note that we restrict to  $\delta$  small enough such that a similar proof as in Lemma 3 works to show that there exists a unique equilibrium price  $P_C^\tau(b) \in \left(\frac{1}{R}, 1\right)$ .

## Proof of Proposition 8

**Proposition 8** (QE with commitment). *Under QE with commitment, the optimal tax can in general be either positive or zero. If the liquidity level  $L$  is sufficiently high, then the optimal tax is zero.*

*Proof.* **Proof that the optimal tax is zero when  $L$  is large.** Since banks are ex-ante identical and banks anticipate the tax, the equilibrium price must be such that banks are indifferent between complex and simple assets. Therefore it suffices to consider how the tax affects the expected utility from investing in complex assets. Taking the derivative of the expected utility from investing in complex assets (equation (15)) with respect to  $\tau$  obtains

$$\begin{aligned} \frac{\partial \mathbf{E}[U_C | P_C(b) = P_C^\tau(b)]}{\partial \tau} &= (1 - \eta)(\alpha\phi + 1 - \phi) \left[ \underbrace{\frac{\delta\phi(\alpha - 1)}{(\alpha\phi + 1 - \phi)R(1 - L) + \frac{L - \kappa\phi}{P_C^\tau(b)^2} \mu_b R(1 - L) - \delta}}_{= -\frac{\partial P_C^\tau(b)}{\partial \tau}} \right] \\ &+ (1 - \eta) \left[ \frac{\delta}{P_C^\tau(b)} - \frac{\tau\delta}{P_C^\tau(b)^2} \frac{\partial P_C^\tau(b)}{\partial \tau} \right]. \end{aligned} \quad (46)$$

For  $L$  close to 1, this is approximately equal to

$$\begin{aligned} \frac{\partial \mathbf{E}[U_C | P_C(b) = P_C^\tau(b)]}{\partial \tau} &\approx -(1 - \eta)(\alpha\phi + 1 - \phi)\delta + (1 - \eta) \left[ \frac{\delta}{P_C^\tau(b)} - \frac{\tau\delta}{P_C^\tau(b)^2} \frac{\partial P_C^\tau(b)}{\partial \tau} \right] \\ &< (1 - \eta)\delta \left[ \frac{1}{P_C^\tau(b)} - (\alpha\phi + 1 - \phi) \right]. \end{aligned}$$

For this to be negative, it suffices to show  $P_C^\tau(b) > \frac{1}{\alpha\phi + 1 - \phi}$ . Recall that  $P_C^\tau(b)$  is the unique positive solution  $H^\tau(P_C^\tau(b)) = 0$ , where  $H^\tau(P_C(b))$  in equation (42) is the relative advantage of investing in complex assets for a given price  $P_C(b)$ . Since  $H^\tau$  is increasing in  $P_C(b)$ , to show that  $P_C^\tau(b) > \frac{1}{\alpha\phi + 1 - \phi}$  it suffices to show  $H^\tau\left(\frac{1}{\alpha\phi + 1 - \phi}\right) < 0$ . For  $L$  near 1 and  $\delta$  negligibly small compared to the other terms in  $H^\tau$ , we have:

$$H^\tau\left(\frac{1}{\alpha + 1 - \phi}\right) \approx \phi(\alpha - 1) \left[ \eta(1 - \mu_g)(\kappa - 1) - (1 - \eta)\mu_b\kappa(1 - \phi) \right] < 0,$$

where the inequality follows from the fact that if  $L$  is near 1, then the parametric restriction in 6 implies that  $\kappa$  must also be close to 1.<sup>38</sup>

**Proof that the optimal tax can be positive.** Evaluating equation (46) at  $t = 0$

<sup>38</sup>Note that the maximum  $L$  that is consistent with the parametric restriction in (6) is generally less than 1 and not necessarily large enough for this result to hold assuming the other parameters are held fixed.

and  $L$  close to  $\kappa\phi$  obtains

$$\frac{\partial \mathbf{E}[U_C | P_C(b) = P_C^\tau(b)]}{\partial \tau} \Big|_{\tau=0} \approx (1-\eta)\delta \left[ \frac{1}{P_C^\tau(b)} - (1 + (1-\mu_b)\phi(\alpha-1)) \right].$$

For this to be positive, it suffices to show that  $P_C^\tau(b) < \frac{1}{\alpha\phi+1-\phi}$ . By similar reasoning as above, it suffices to show  $H^\tau\left(\frac{1}{\alpha\phi+1-\phi}\right) > 0$ . Note that at  $L = \kappa\phi$  and for  $\delta$  negligibly small compared to the other terms in  $H^\tau$  we have:

$$H^\tau\left(\frac{1}{\alpha\phi+1-\phi}\right) \approx \kappa\phi(1-\phi)(\alpha-1) \left[ \eta(1-\mu_g) - (1-\eta)\mu_b \right],$$

which is positive if  $\eta(1-\mu_g) > (1-\eta)\mu_b$ .<sup>39</sup>

□

## Proof of Proposition 9

**Proposition 9** (Ex-ante insurance). *Implementing the ex-ante insurance policy (i) increases the equilibrium complex-asset price in bad times, (ii) decreases the volume of complex banks, and (iii) increases overall welfare.*

*Proof.* Denote the volume of complex banks when the redistributive policy is in place by  $V^\tau$  and the equilibrium price by  $P_C^\tau(b)$ . Note that the optimal debt contract is still given by  $R_d = \kappa$ . In particular, complex banks offer  $R_d = \kappa$  as long as (30) holds. Since it holds with  $P_C^*(b)$  and  $P_C^\tau(b) > P_C^*(b)$ , it also holds with  $P_C^\tau(b)$ . It is straightforward to see that the incentive for simple banks to offer  $R_d = \kappa$  is strengthened when they no longer experience a risk of a run.

**Bond price in bad times.** First, we show that the policy leads to an increase in the equilibrium price, or  $P_C^\tau(b) > P_C^*(b)$ . To see this, first note that the expected utility of a complex bank is

$$\begin{aligned} \mathbf{E}[U_C | P_C^\tau(b)] &= \eta \left( \phi\alpha\kappa + L - \phi\kappa + \mu_g R(1-L) \right) \\ &\quad + (1-\eta) \left( \phi\alpha + 1 - \phi \right) \left( L + P_C^\tau(b)\mu_b R(1-L) \right). \end{aligned} \quad (47)$$

The redistributive tax changes the period-2 income for all simple banks  $\mu_g R(1-L)$ . A similar argument as in the proof of Lemma 1 shows that this is high enough for simple banks to avoid a run. The expected utility for a simple bank is therefore

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<sup>39</sup>Note that the minimum  $L$  that is consistent with the parametric restrictions in Proposition 1 is generally greater than  $\kappa\phi$  and not necessarily small enough for this result to hold assuming the other parameters are held fixed.

given by

$$\begin{aligned} \mathbf{E}[U_S|P_C^\tau(b)] &= \eta \left( \phi \alpha \kappa + L - \phi \kappa + \mu_g R(1-L) \right) \\ &\quad + (1-\eta) \left( \mu_b \left( \phi \alpha \kappa + \frac{L-\phi \kappa}{P_C^\tau(b)} + R(1-L) \right) + (1-\mu_b)(\phi \alpha + 1 - \phi)L \right). \end{aligned} \quad (48)$$

The bond price is determined by the indifference condition:

$$0 = \mathbf{E}[U_C|P_C^\tau(b)] - \mathbf{E}[U_S|P_C^\tau(b)] \quad (49)$$

$$= \left( \frac{1}{P_C^\tau(b)} - (\phi \alpha + 1 - \phi) \right) \left( L - \phi \kappa + P_C^\tau(b) R(1-L) \right) + \phi \left( \alpha - (\phi \alpha + 1 - \phi) \right). \quad (50)$$

Rearranging the right hand side of (50) obtains

$$\begin{aligned} \Upsilon(P_C^\tau(b)) &:= \underbrace{(\phi \alpha + 1 - \phi) R(1-L)}_{:=a_2} P_C^\tau(b)^2 \\ &\quad + \underbrace{\left( (\phi \alpha + 1 - \phi)L - R(1-L) - \phi \alpha \kappa \right)}_{:=a_1} P_C^\tau(b) \\ &\quad - \underbrace{(L - \phi \kappa)}_{:=a_0}. \end{aligned} \quad (51)$$

Clearly,  $\Upsilon(0) < 0 < \Upsilon(1)$ , meaning that  $\Upsilon(\cdot)$  has a unique root in  $(0, 1)$ .<sup>40</sup> So,  $P_C^\tau(b) = \frac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$ . Recall that  $P_C^*(b) = \frac{-\tilde{a}_1 + \sqrt{\tilde{a}_1^2 - 4a_0a_2}}{2a_2}$  where  $\tilde{a}_1 > a_1$  (see Proposition 1). Since  $\frac{\partial P_C^*(b)}{\partial \tilde{a}_1} < 0$  thus  $P_C^\tau(b) > P_C^*(b)$ , finishing the proof of the lemma.

**Volume of complex banks.** Since the equilibrium price increases, the market-clearing condition in equation (10) implies that the volume of complex banks decreases, or  $V^\tau < V^*$ .

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<sup>40</sup>Note that  $\Upsilon(1) = \phi(L + R(1-L) - \kappa)(\alpha - 1) > 0$  and  $\Upsilon(0) = -(L - \phi \kappa) < 0$ .

**Welfare.** Finally, we show the policy improves welfare. Recall that welfare is defined as the expected utility of representative depositor, which is given by

$$\begin{aligned}
\mathcal{W}^\tau &= V^\tau \mathbf{E}[U_C | P_C^\tau(b)] + (1 - V^\tau) \mathbf{E}[U_S | P_C^\tau(b)] \\
&= (1 - V^\tau) \left[ (1 - \eta) \left( \mu_b \left( \phi \alpha \kappa + \frac{L - \phi \kappa}{P_C^\tau(b)} + R(1 - L) \right) + (1 - \mu_b) (\phi \alpha + 1 - \phi) L \right) \right] \\
&\quad + V^\tau \left[ (1 - \eta) (\phi \alpha + 1 - \phi) \left( L + (1 - L) P_C^\tau(b) \mu_b R \right) \right] \\
&\quad + \eta (\phi \alpha \kappa + L - \phi \kappa + \mu_g R(1 - L)). \tag{52}
\end{aligned}$$

Substituting the market-clearing condition in (10) into (52) obtains

$$\begin{aligned}
\mathcal{W}^\tau &= R(1 - L) \left( \eta \mu_g + (1 - \eta) \mu_b \right) + \left( \eta + (1 - \eta) (\phi \alpha + 1 - \phi) \right) L + \eta \mu_g \phi (\alpha - 1) \kappa \\
&\quad + \phi (\alpha - 1) \kappa \left[ \eta (1 - \mu_g) + (1 - V^\tau) \mu_b (1 - \eta) (1 - \phi) \right]. \tag{53}
\end{aligned}$$

Now, recall that welfare in the original equilibrium is

$$\begin{aligned}
\mathcal{W} &= R(1 - L) \left( \eta \mu_g + (1 - \eta) \mu_b \right) + \left( \eta + (1 - \eta) (\phi \alpha + 1 - \phi) \right) L + \eta \mu_g \phi (\alpha - 1) \kappa \\
&\quad + \phi (\alpha - 1) \left[ V^* \eta \kappa (1 - \mu_g) + (1 - V^*) \left( \eta (1 - \mu_g) L + (1 - \eta) \mu_g (1 - \phi) \kappa \right) \right]. \tag{54}
\end{aligned}$$

Then finally, equation (53) minus equation (54) is

$$\left( (1 - V^*) \eta (\kappa - L) (1 - \mu_g) + \underbrace{(V^* - V^\tau) (1 - \phi) \mu_b (1 - \eta) \kappa}_{>0} \right) (\alpha - 1) \phi > 0.$$

Thus, the redistributive policy always improves welfare compared to the original equilibrium.  $\square$

## Proof of Proposition 10

**Proposition 10.** *If  $L < \hat{L}$  and  $v$  is sufficiently large, then the constrained-efficient volume of complex banks can be implemented via QE with commitment. However, the tax that implements the constrained-efficient volume of complex banks may not be welfare-optimizing. If  $L > \hat{L}$ , then neither QE nor the ex-ante insurance policy can implement the constrained-efficient volume of complex banks.*

*Proof.* **Case 1:**  $L < \hat{L}$ . In this case, the constrained-efficient volume of complex banks is greater than the equilibrium (Proposition 4). Note that the expression for the volume of complex banks  $V(\tau)$  in equation (44) is increasing and unbounded in the tax  $\tau$ . Therefore, there is a tax level for which  $V(\tau)$  is equal to the volume of

complex banks in the planner solution  $V^W$ , which can be implemented as long as  $\nu$  is sufficiently high.

**Case 2:**  $L > \hat{L}$ . In this case, the constrained-efficient volume of complex banks is less than the equilibrium (Proposition 4). QE without commitment cannot implement the constrained-efficient volume of complex banks because it has no effect on the volume of complex banks. QE with commitment cannot implement the constrained-efficient volume of complex banks because the investment in complex assets is increasing in the tax (Proposition 7). The redistributive transfers policy described in Section 4.2 also cannot implement the constrained-efficient volume of complex banks. To see this, note that the redistributive tax only affects the bond price in bad times through the volume of complex banks. Therefore it suffices to check whether it can implement the price in the planner solution, which is equal to  $P_C^W(b) = 1$  for  $L > \hat{L}$ . However, the proof of Proposition 9 shows that the price determined by the tax is strictly between 0 and 1. □

## Proof of Proposition 11

**Proposition 11.** Denote by  $\Delta(P_C(b)) = \mathbf{E}[U_C|P_C(b)] - \mathbf{E}[U_S|P_C(b)]$  the relative benefit of investing in complex assets without the tax as expressed in equation (9), by  $V^W$  the constrained-efficient volume of complex banks, and by  $P_C^W(b)$  the complex-asset price in bad times for the constrained-efficient allocation. Then the following hold:

- If  $L < \hat{L}$  and  $\frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)} < \frac{R(1-L) - (\kappa - L)}{R(1-L)}$ , then the constrained-efficient volume of complex banks can be implemented by transferring from simple to complex banks via a tax at the rate  $\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}$ .
- If  $L > \hat{L}$  and  $\frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)} < \frac{\mu_g R(1-L) - (L - \kappa)}{\mu_g R(1-L)}$ , then the constrained-efficient volume of complex banks can be implemented by transferring from complex to simple banks via a tax at the rate  $\tau^* = \frac{\Delta(P_C^W(b))(1-V^W)}{\eta\mu_g R(1-L)}$ .

Additionally, the tax level that implements the constrained-efficient volume of complex banks also maximizes welfare.

*Proof.* Note that the tax only affects the bond price in bad times through the volume of complex banks. Therefore it suffices to check whether it can implement the price in the planner solution  $P_C^W(b)$ .

First consider the case where  $L < \hat{L}$ . For a tax level  $\tau$ , bond price  $P_C^\tau(b)$ , and volume of complex banks  $V^\tau$ , the expected return for a complex bank is

$$\begin{aligned} \mathbf{E}[U_C|P_C^\tau(b), V^\tau] &= \eta \left( \phi \alpha \kappa + L - \phi \kappa + \mu_g R(1-L) \left( 1 + \frac{1 - V^\tau}{V^\tau} \tau \right) \right) \\ &\quad + (1 - \eta) \left( \phi \alpha + 1 - \phi \right) \left( L + P_C^\tau(b) \mu_b R(1-L) \right), \end{aligned}$$

and the expected return for a simple bank is

$$\begin{aligned} \mathbf{E}[U_S|P_C^\tau(b), V^\tau] &= \eta \left( \mu_g(\alpha\kappa\phi + L - \kappa\phi + R(1-L)(1-\tau)) + (1-\mu_g)(\alpha\phi + 1 - \phi)L \right) \\ &\quad + (1-\eta) \left( \mu_b \left( \phi\alpha\kappa + \frac{L-\phi\kappa}{P_C^\tau(b)} + R(1-L) \right) + (1-\mu_b)(\phi\alpha + 1 - \phi)L \right). \end{aligned}$$

The bond price is determined by the indifference condition:

$$\begin{aligned} 0 &= \mathbf{E}[U_C|P_C^\tau(b), V^\tau] - \mathbf{E}[U_S|P_C^\tau(b), V^\tau] \\ &\quad + \eta \left[ (1-\mu_g)\phi(\kappa-L)(\alpha-1) - \frac{\mu_g R(1-L)}{V^\tau} \tau \right] \\ &\quad + (1-\eta)\mu_b \left[ (\alpha\phi + 1 - \phi)(L + P_C(b)\mu_b R(1-L)) - \left( \alpha\kappa\phi + \frac{L-\kappa\phi}{P_C(b)} + R(1-L) \right) \right] \\ &\quad + (1-\eta)(1-\mu_b)(\alpha\phi + 1 - \phi)P_C(b)\mu_b R(1-L) \\ &= \Delta(P_C^\tau(b)) + \frac{\eta\mu_g R(1-L)}{V^\tau} \tau. \end{aligned} \tag{55}$$

Therefore, the tax level that is consistent with the constrained-efficient price  $P_C^W(b)$  and volume  $V^W$  is given by

$$\tau^* = \frac{-\Delta(P_C^W(b))V^W}{\eta\mu_g R(1-L)}.$$

Note that the parametric assumptions for this proposition ensure that this tax can be implemented while maintaining an equilibrium of the form described in Proposition 1. To show this, we have to check that the tax does not induce a run for banks that invest in the taxed asset.

The incentive-compatibility condition for normal depositors of simple banks to withdraw late requires

$$\begin{aligned} \frac{L - \kappa\phi + R(1-L)(1-\tau)}{1-\phi} &> \kappa \\ \iff \tau &< \frac{R(1-L) - (\kappa-L)}{R(1-L)}. \end{aligned}$$

This is satisfied by  $\tau^*$  by assumption in the proposition. It is simple to show that it is incentive compatible for both simple and complex banks to continue to offer  $R_d = \kappa$ . Specifically, the local optima for all of the cases in the proof of Lemma 2 are unaffected by the tax as well as the inequality in (30).

It is straightforward to check that welfare only depends on the tax through its effect on the volume of complex banks and the bond price in bad times. Therefore, the welfare-optimizing tax coincides with the tax that implements the constrained-efficient volume of complex banks.

The case where  $L > \hat{L}$  follows analogously.  $\square$

# ONLINE APPENDIX—NOT FOR PUBLICATION

## A Repo-market Interpretation

The interbank market for direct asset sales can also be interpreted as a repo market. To see this, suppose that, instead of selling assets, banks can sell bonds. Denote the state-dependent price for a bond backed by complex assets by  $P_C(\omega)$  and the price of a bond backed by simple assets with a high return by  $P_S(\omega)$ . Note that simple assets with a low return cannot be used as collateral since they are publicly observed to be worthless. The *repo rate* is the rate of return  $\frac{1}{P_\theta(\omega)}$ . The *haircut*  $h_\theta(\omega)$  is defined as the percentage difference between the market value of collateral and the cash that is exchanged at the start of a repo. We assume that the haircut is equal to  $h_\theta(\omega) = 1 - P_\theta(\omega)$ , which implies that a lender holds 1 dollar of collateral for each bond purchased. In that case, the return from investing in either type of repo is equal to the repo rate, regardless of whether the borrower defaults. The rest of the model follows as in the original presentation except that the repo interpretation has the additional feature of a haircut.

## B Calibration to the COVID-19 Crisis

This section describes the results when the model is calibrated to the COVID-19 crisis rather than the GFC.

We calibrate the parameters of the model in a manner analogous to the description in Section 3:

- The long-term return  $R$  is calibrated to match 1.039, which is approximately the mean of the 30-year fixed-rate mortgage rate in March 2020 (1.0327)<sup>41</sup> and Moody’s medium-grade corporate bond yield in March 2020 (1.0429).<sup>42</sup>
- The short-term interest rate  $R_D = \kappa$  is calibrated to match 1.0065, which is the federal funds rate in March 2020.<sup>43</sup>
- The liquidity level  $L$  is calibrated to match 0.243, which is approximately the ratio of total liquid assets to total assets based on 2019Q4 FR Y-9C filings for bank holding companies. Liquid assets include cash and balances due from depository institutions, federal funds sold, securities purchased under agreement to resell, Treasury securities, and government agency debt and mortgage-backed securities (not including government-sponsored agency (GSE) debt and MBS).
- The bond price in bad times  $P_C^*(b)$ , which is also the ratio of the bond price in bad times to the bond price in good times, is calibrated to match 0.988, which corresponds to the ratio of the 3-month U.S. dollar LIBOR-FF spread at

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<sup>41</sup>Data: FRED series MORTGAGE30US

<sup>42</sup>Data: FRED series BAA.

<sup>43</sup>Data: FRED series FEDFUNDS.

its peak in April 1, 2020 ( $1/1.0138 \approx 0.986$ ) to its level just before the onset of the COVID-19 crisis on February 3, 2020 ( $1/1.0015 \approx 0.9985$ ).

- The fraction of complex assets  $V^*$  is calibrated to match 0.149, which is approximately the ratio of complex assets to total illiquid assets based on 2019Q4 FR Y-9C filings. Illiquid assets are defined as assets minus liquid assets, as given above. Complex assets include GSE MBS, non-agency MBS, asset-backed securities, and structured financial products.

Table B.1 presents the calibrated parameters, and Table B.2 compares the empirical and model-generated values for the observables. Unlike the calibration of the model for the GFC (see Section 3), the threshold level of liquidity  $\hat{L} = 0.206$  is less than  $L = 0.243$ , which implies that there is overinvestment in complex assets in equilibrium.

Table B.1: Calibrated parameters.

Parameter	Value
High return ( $R$ )	1.039
Liquidity ratio ( $L$ )	0.243
Short-term return ( $\kappa$ )	1.006
Probability of good state ( $\eta$ )	0.999
Probability of high return in good state ( $\mu_g$ )	0.999
Probability of high return in bad state ( $\mu_b$ )	0.8
Fraction of liquidity-shocked depositors ( $\phi$ )	0.007
Marginal utility from liquidity shock ( $\alpha$ )	5.7

Table B.2: Comparison of empirical and model-generated variables.

Variable	Empirical	Model
High return	1.039	1.039
Liquidity ratio	0.243	0.243
Short-term return	1.006	1.006
Price in bad times	0.988	0.97
Fraction of complex assets	0.149	0.236

The remaining results are qualitatively similar to the version of the model calibrated to the GFC:

- Figure B.1 shows that the comparative statics with respect to  $L$  are similar (as compared to Figure 2)
- Figure B.2 shows that the variation in the optimal  $L$  with respect to the long-run return  $R$  and the probability of normal times  $\eta$  is similar (as compared to Figure 3)

- Figure B.3 shows that the effect of QE without commitment is similar (as compared to Figure 4)
- Figure B.4 shows that the effect of QE with commitment is similar (as compared to Figure 5)
- Figure B.5 shows that the effect of a redistributive policy is similar (as compared to Figure 6)
- Figure B.6 shows that the comparison of the welfare effects of the different policies is similar (as compared to Figure 8).

Figure B.1: Variation in  $L$  (COVID-19 crisis calibration). This figure shows how the bond price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in the equilibrium, the planner solution, and the difference between them.

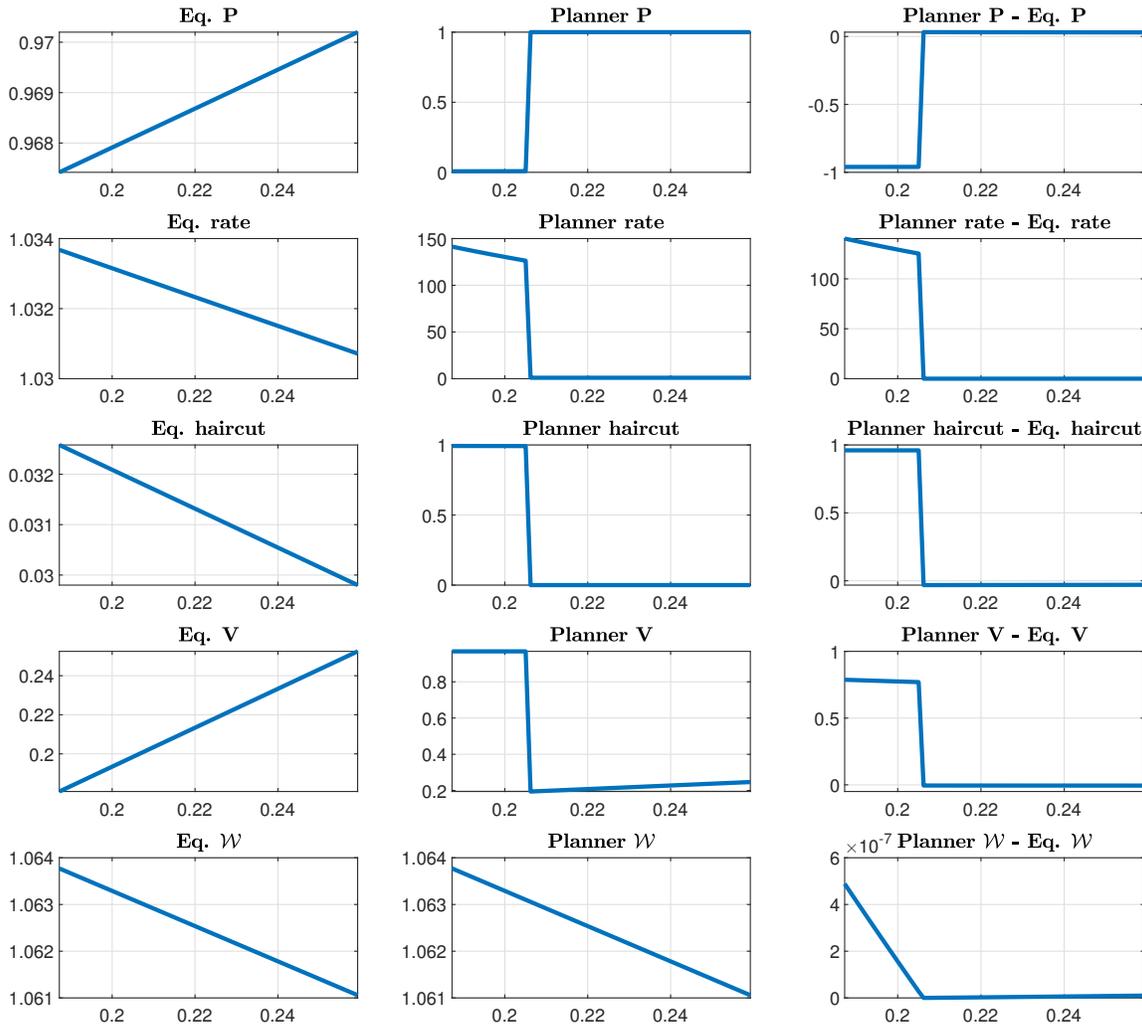
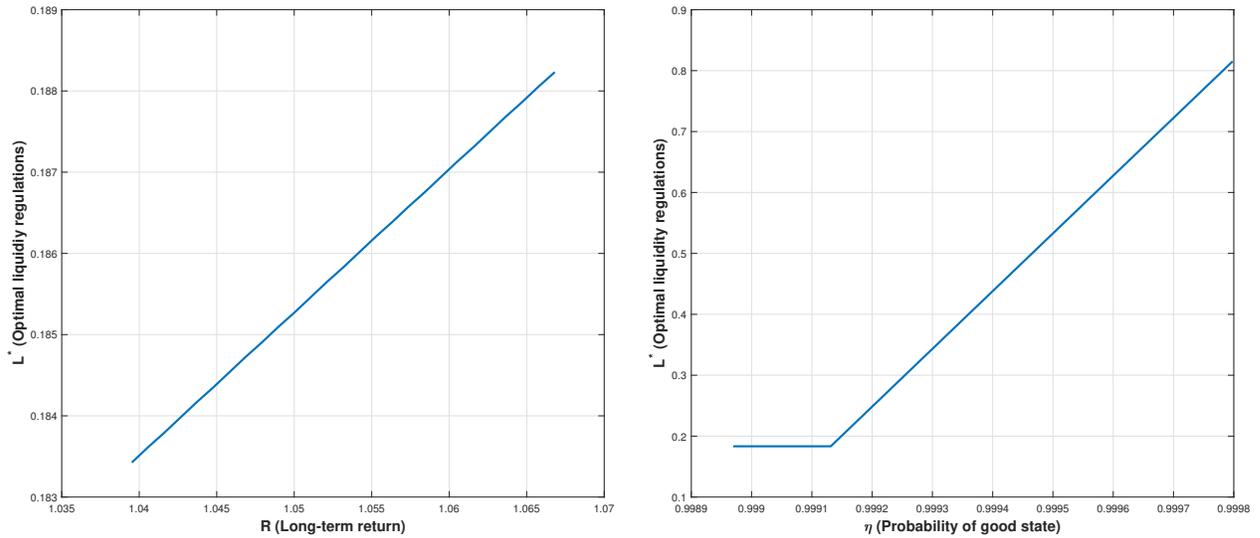


Figure B.2: This panel shows the optimal liquidity level that maximizes welfare in the planner solution  $L^*$  as a function of the long-term return  $R$  (left) and the probability of the good state  $\eta$  (right) (COVID-19 crisis calibration).



## C Additional Empirical Findings

This section presents evidence that the LCR has been associated with higher interbank lending prices during crises by comparing the great financial crisis (GFC), which occurred before the introduction of the LCR, with the COVID-19 crisis, which occurred afterwards. Finally, we show that the LCR was associated with an amplified effect of QE on MBS prices.

### C.1 The Effect of Liquidity Regulation on Interbank Lending Prices

Recall that the model shows that higher liquidity requirements are associated with higher complex-asset prices in bad times (see Proposition 3 and Figure 2). Consistent with this result, it can easily be seen from Figure C.1 that the GFC in 2008, the last crisis preceding the introduction of the LCR, was associated with a more dramatic increase in the LIBOR-FF spread compared to the COVID-19 crisis in 2020, the first crisis following the introduction of the LCR in the U.S. This is consistent with the model. However, we acknowledge that it is difficult to disentangle the effect of the LCR from that of various other differences between the two crises, such as the origins of the crises arising from either the financial system or the real economy, their magnitudes, and other policy responses.

The remainder of this section presents suggestive evidence that is consistent with this result by comparing the 3-month U.S. dollar LIBOR to the effective federal funds rate (EFFR) spread, which is a measure of interbank loan prices, during stock market corrections, which is a proxy for turbulent times in financial markets, before versus after the introduction of the LCR. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

Figure B.3: Variation in  $L$  in QE without commitment (COVID-19 crisis calibration). This figure shows how the bond price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in equilibrium, under optimal QE without commitment, and the difference between them.

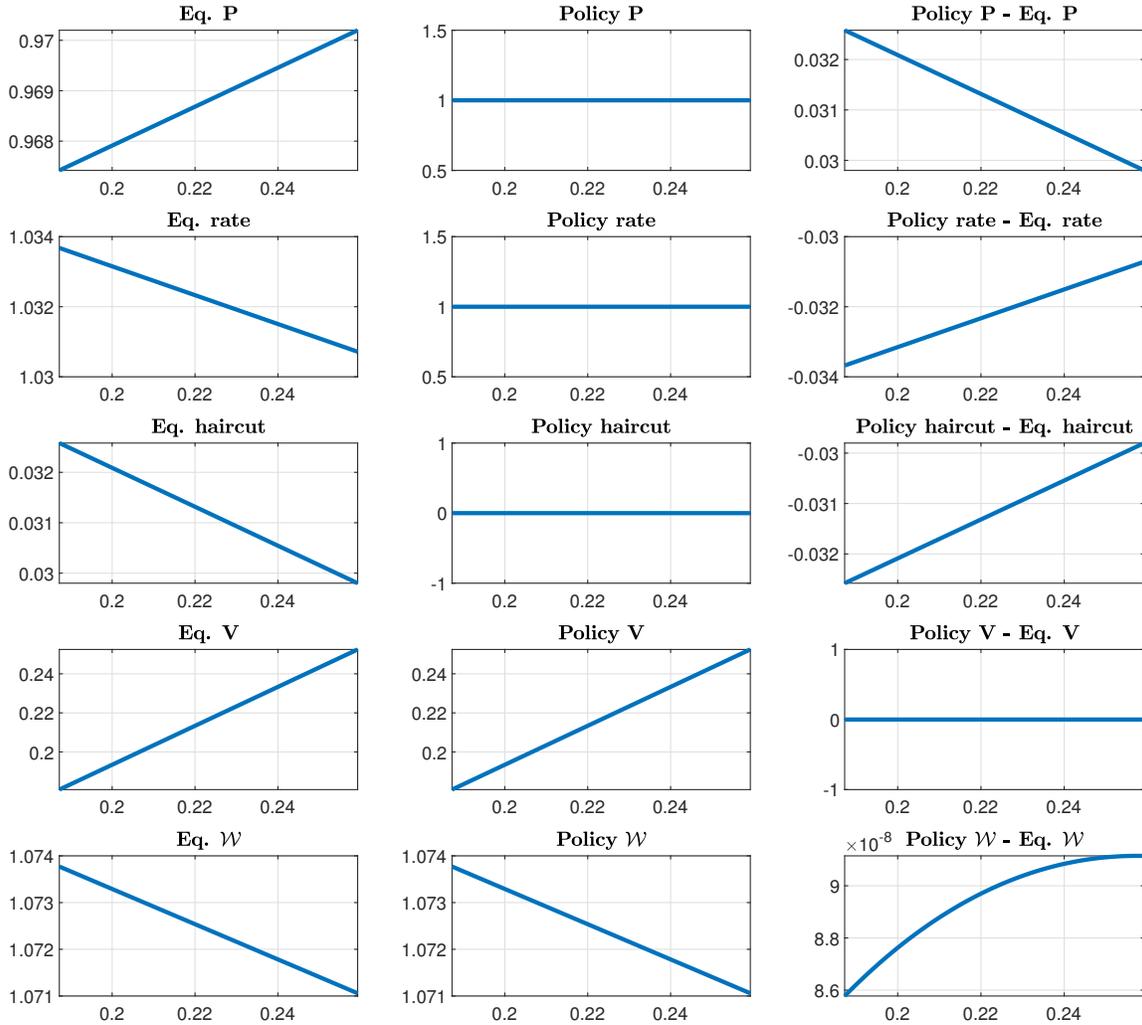
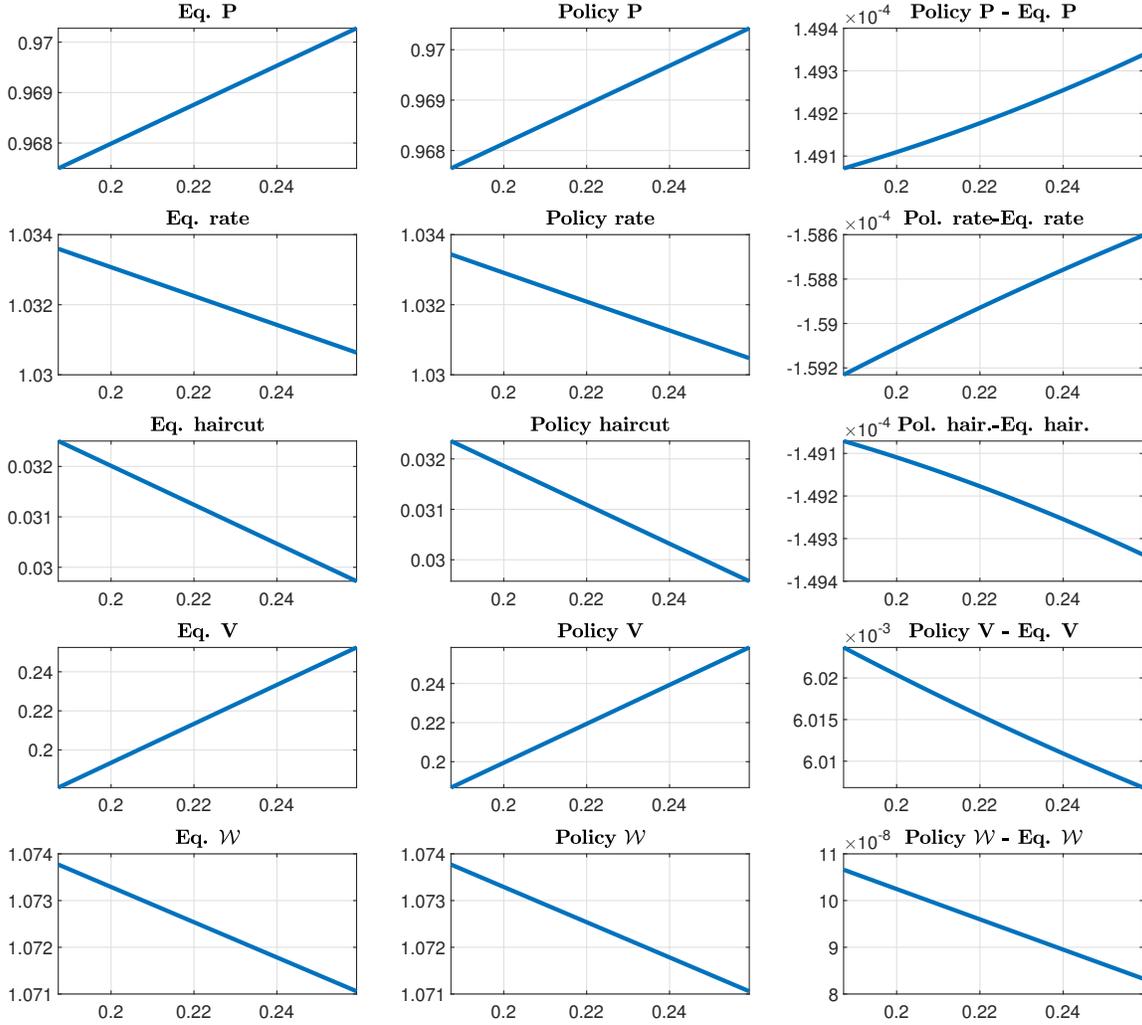


Figure C.1 shows the LIBOR-EFFR spread from January 2005 to April 2020 as well as periods with stock market corrections. Table C.1 shows the mean LIBOR-EFFR spread during stock market corrections before versus after the introduction of the LCR, as well as the  $t$ -statistic for a difference-of-means test. The average LIBOR-EFFR spread during stock market corrections experiences a drop after the introduction of the LCR that is statistically significant at the 5% level. This finding is consistent with the calibrated model, although caveats about interpreting this observation causally still apply because we cannot rule out confounding effects due to other changes in the financial system that have occurred during this time period.

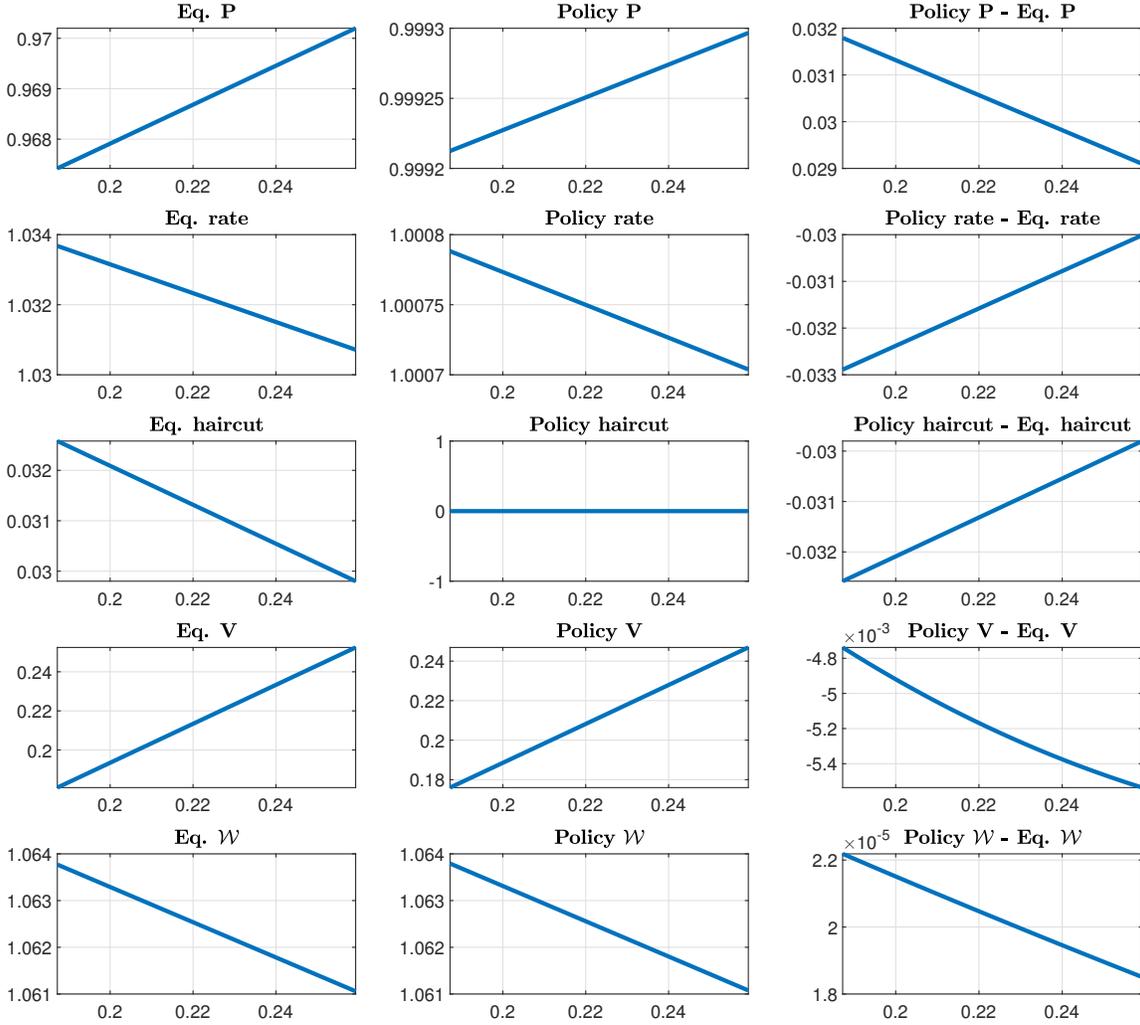
Figure B.4: Variation in  $L$  in QE with commitment (COVID-19 crisis calibration). This figure shows how the bond price in bad times, the gross rate of return on complex-asset purchases, the haircut (or “hair.”), the volume of complex banks, and welfare vary with  $L$  in equilibrium, under optimal QE with commitment (“Policy” or “Pol.”), and the difference between them.



## C.2 The Effect of Liquidity Regulation on QE

Recall that the model predicts that quantitative easing (QE) increases interbank lending prices (Proposition 5 and Proposition 7). This is consistent with empirical evidence showing that QE has been associated with decreased rates of return on assets used as collateral in interbank loans, such as mortgage-backed securities (see [Krishnamurthy and Vissing-Jørgensen, 2011](#), and Figure C.2a for the GFC, and Figure C.2b for the COVID-19 crisis). The calibrated model shows that tightening liquidity requirements can amplify the effect of QE on interbank lending prices, depending on whether it is implemented without commitment (Figure 4) or with commitment (Figure 5). This subsection presents evidence that MBS yields were more responsive to QE announcements after the implementation of the LCR compared to before the implementation of the LCR, consistent with QE with commitment.

Figure B.5: Variation in  $L$  in the redistributive policy (COVID-19 crisis calibration). This figure shows how the bond price in bad times, the gross rate of return on complex-asset purchases, the haircut, the volume of complex banks, and welfare vary with  $L$  in equilibrium, under the redistributive policy, and the difference between them.



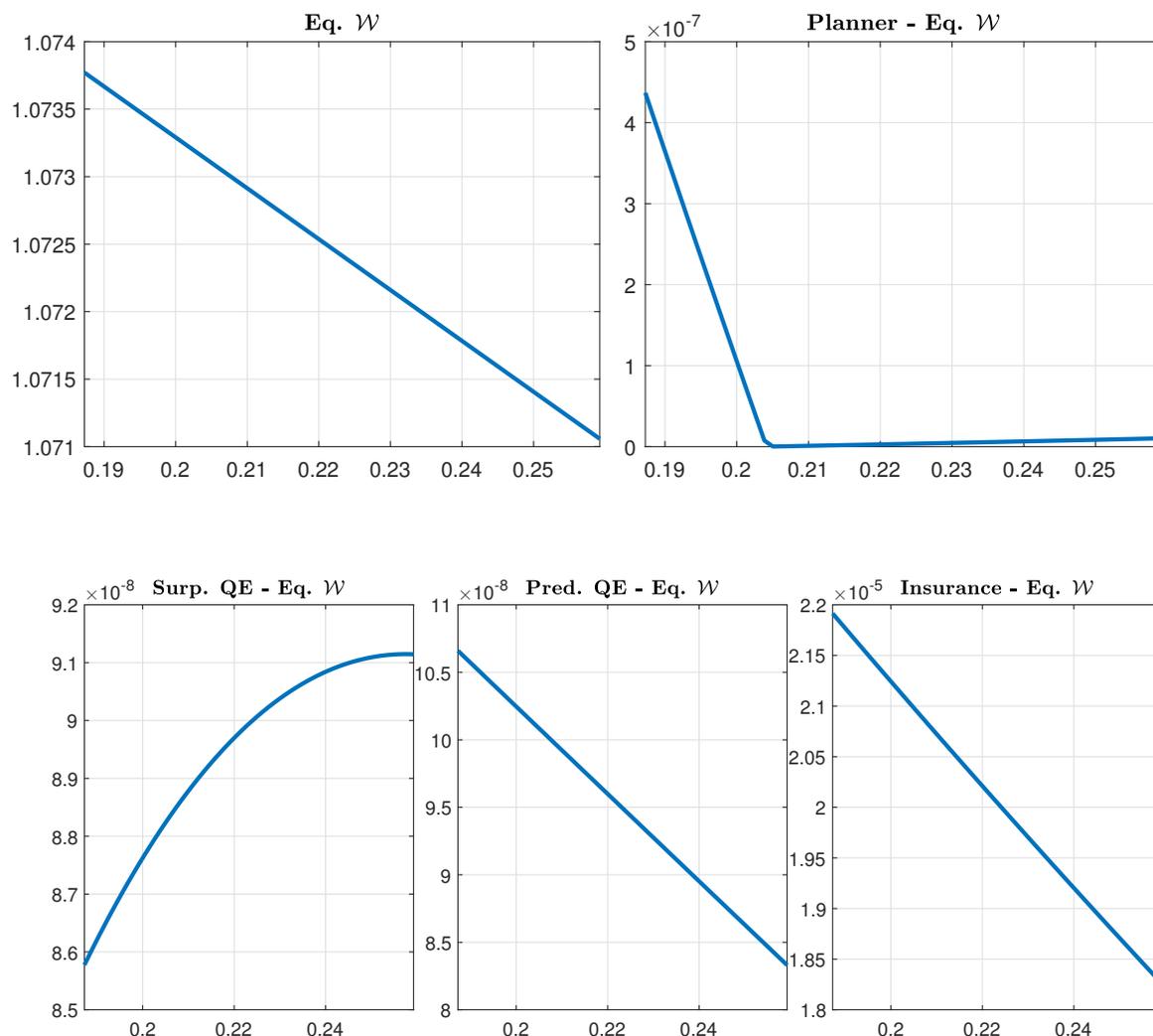
However, we acknowledge that it is difficult to attribute this difference solely to the LCR since it could also reflect other differences between the two crises, such as the magnitude of the QE response and the market's confidence in the efficacy of QE.

Following the methodology in [Krishnamurthy and Vissing-Jørgensen \(2011\)](#), we measure the effect of QE using the change in MBS yields within a 2-day window around QE announcement dates.<sup>44</sup> We average yields for 15-year and 30-year current-coupon MBS backed by Ginnie Mae, Fannie Mae, and Freddie Mac.<sup>45</sup> To as-

<sup>44</sup>Specifically, for each announcement date, we consider the difference in the last price on the trading day after the announcement date minus the last price on the trading day before the announcement date.

<sup>45</sup>Specifically, the 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCI, and MTGEFGCI. The 30-year yield is similarly the average of the follow-

Figure B.6: Comparison of welfare gains from policy (COVID-19 crisis calibration). This figure shows welfare as a function of the liquidity level  $L$  in the baseline equilibrium with income shocks as well as the improvement in utility associated with QE without commitment (“Surp. QE”), QE with commitment (“Pred. QE”), and the ex-ante insurance policy.



to assess the effect of QE on MBS yields before the implementation of the LCR, we focus specifically on QE1 since it included purchases of MBS. We consider the same five dates as in [Krishnamurthy and Vissing-Jørgensen \(2011\)](#). To assess the effect of QE on MBS yields after the implementation of the LCR, we consider the QE announcements on March 15, 2020 and March 23, 2020 in response to the COVID-19 crisis. On March 15, the Federal Reserve announced that it would purchase \$500 billion in Treasuries and \$200 billion in MBS. On March 23, the Federal Reserve revised this plan, and announced that it would buy an indefinite volume of Treasuries and MBS in order to support the smooth functioning of the markets.

ing fields: MTGEGNSF, MTGEFNCL, MTGEFGLM.

Figure C.1: The LIBOR-EFFR spread. This figure shows the 3-month U.S. dollar LIBOR to effective federal funds rate (EFFR) spread from January 2000 to April 2020. Periods exhibiting stock market corrections are indicated by grey shading, and the proposal of the LCR in 2013Q4 is indicated by the dashed line. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

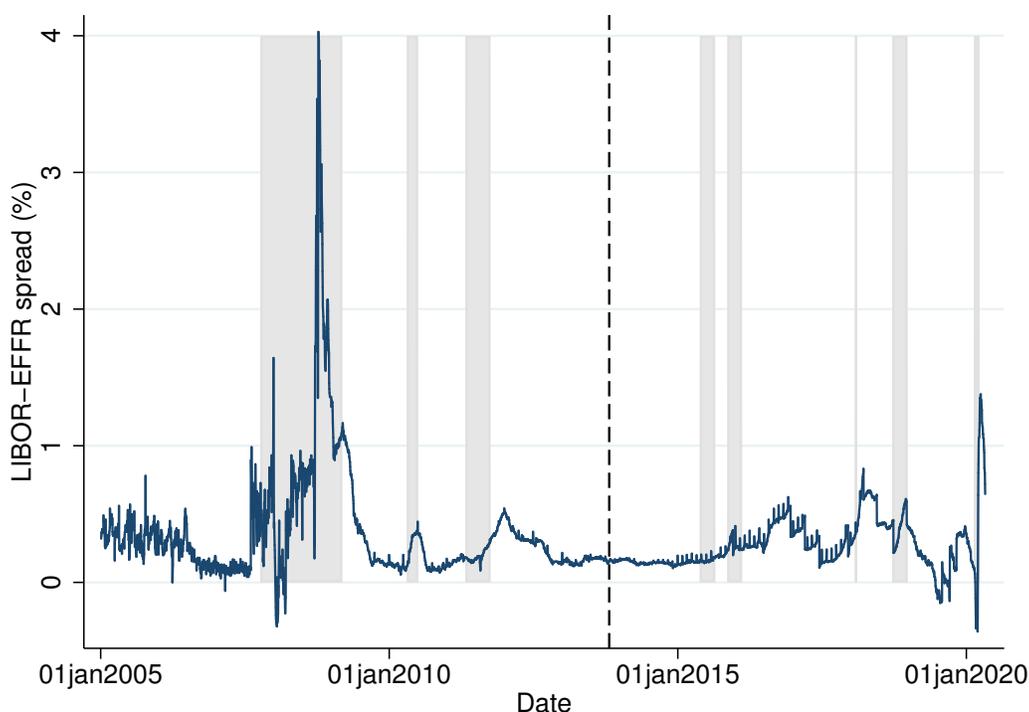
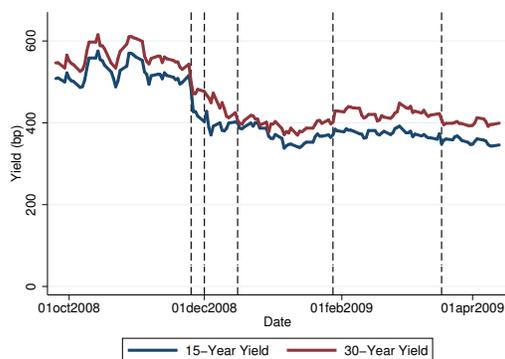


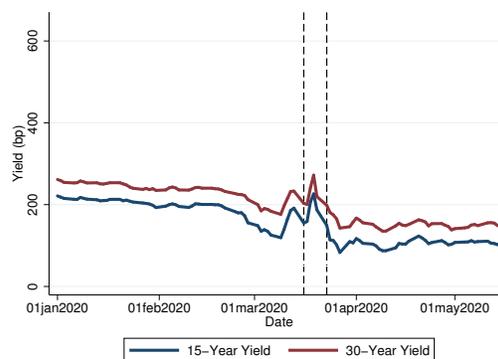
Table C.1: Average LIBOR-FF spread. This table shows the average 3-month U.S. dollar LIBOR to effective federal funds rate spread during stock market corrections since the year 2005 that occurred either before or after the proposal of the LCR in 2013Q4. It also shows the  $t$ -statistic from a difference in means test comparing observations before vs after the introduction of the LCR. Stock market corrections are periods over which the S&P 500 declines by at least 10% from peak to trough. Precise dates are obtained from Yardeni Research, Inc.

Before LCR	After LCR	$t$ -statistic
0.709	0.270	-8.936

Figure C.2: The effect of QE announcement dates on MBS yields. Figure (a) shows 15- and 30-year yields, in basis points, of mortgage-backed securities (MBSs) around announcement dates for QE1 during the great financial crisis, as indicated by the dashed lines. The 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCL, and MTGEFGCI. The 30-year yield is similarly the average of the following fields: MTGEGNSF, MTGEFNCL, MTGE-FGLM. Figure (b) similarly shows MBS yields around QE announcement dates during the COVID-19 crisis.



(a) Great financial crisis



(b) COVID-19 crisis

Table C.2 presents the findings. The average effect of QE on MBS yields was greater during the COVID-19 crisis than during the GFC. This is consistent with our result that the effect of QE with commitment on the interbank complex-asset price in bad times is increasing in the tightness of liquidity requirements (Figure 5).

Table C.2: Effect of QE on MBS yields. This table shows the change in 30-year and 15-year mortgage-backed securities (MBS) yields in basis points for a 2-day window around each QE announcement date. For each announcement date, we consider the difference in the last price in the trading day after the announcement date minus the last price in the trading day before the announcement date. The 15-year yield is the average of the following MBS yield indices from Bloomberg: MTGEGNJO, MTGEFNCL, and MTGEFGCI. The 30-year yield is similarly the average of the following fields: MTGEGNSF, MTGEFNCL, MTGEFGLM.

Date	30-Year	15-Year
Before LCR		
Nov. 25, 2008	-72	-88
Dec. 1, 2008	-14	12
Dec. 16, 2008	-26	-16
Jan. 28, 2009	31	20
Mar. 18, 2009	-27	-16
Average	-21.6	-19.8
After LCR		
Mar. 15, 2020	-32	-33
Mar. 23, 2020	-39	-73
Average	-35.5	-53