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ECONtribute
Discussion Paper No. 081

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April 2021 www.econtribute.de

UNIVERSITÄT BONN Universität zu Köln

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany´s Excellence Strategy – EXC 2126/1– 390838866 is gratefully acknowledged.

# Optimal Pricing, Private Information and Search for an Outside Offer

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April 8, 2021

**Abstract**

A buyer can either buy a good at a local monopolist or search for it in the market at a market price. The more intensely the buyer searches, the more likely he will find the good in the market, whereas if his search fails, he can still buy it from the local monopolist. We show that a buyer with a higher willingness to pay searches (weakly) more intensely. This skews the distribution of types buying at the local monopolist towards lower valuations and exerts pressure on the local monopolist to reduce his price. Despite this effect, offering the monopoly price remains weakly optimal in equilibrium: depending on the parameters, the local monopolist either chooses the monopoly price with probability one or he randomizes over a set of prices with the monopoly price as the upper bound of the support. Interestingly, a higher market price can make it more likely that the local monopolist prices below the monopoly level.

*JEL Code*: D82

*Keywords*: Optimal Pricing, Search

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‡University of Bergamo, E-mail: salvapiccolo@gmail.com. We would like to thank the editor David Myatt as well as Alp Atakan, Mehmet Ekmekci, Dino Gerardi, Sander Heinsalu, Toomas Hinnosaar, Johannes Hörner, Stephan Lauermann, Ole Jann, Markus Reisinger, Larry Samuelson, Thomas Wiseman and seminar audiences at Frankfurt School of Management, Essex, LSE, 2016 CSEF-IGIER symposium, Workshop on Economic Theory, Graz 2016, Workshop on Advances in Industrial Organization, Bergamo 2017, and University of Ljubljana. Sarah Auster acknowledges funding from the German Research Foundation (DFG) through Germany's Excellence Strategy—EXC 2126/1—390838866 and CRC TR 224 (Project B02). Parts of this papers have previously been circulated in the paper "Optimal Selling Mechanisms with Endogenous Proposal Rights."

# 1 Introduction

Diamond (1971) established that in a large market even a small cost of search prevents buyers from searching for better options and enables sellers to charge the monopoly price. The result posed a problem for much of the general equilibrium literature studying frictionless markets, as the predictions of these models differ starkly from the ones established by Diamond for nearly frictionless markets. Following this paradox, a large body of work has been dedicated to studying the implications of consumer search in markets. This literature provides various explanations for equilibrium pricing below the monopoly price in search models; Stahl (1989), for example, shows that a model with oligopolistic competition leads to consumers search, Wolinsky (1986) and Anderson and Renault (1999) establish that search arises when products are differentiated, to name just a few. The majority of the work on price search relies on the assumption that consumers are symmetric, or that they have the same willingness to pay.[1] Yet, buyers with different valuations for the good will have different benefits from search. A natural question to ask, then, is how the willingness to pay influences buyers' search. Do buyers with a high willingness to pay search more than buyers with a low willingness to pay, and how does this affect the sellers' pricing?

We study a stylized model of price search with a buyer, who is privately informed about his valuation. The buyer can visit a local monopolist or look for the good in the market, where it is sold at a known market price $q$. The more intensely the buyer searches, the more likely he is to find the good at the market price. Search is costly and the success probability is increasing and concave in the buyer's search intensity. If the buyer's initial search fails, the buyer can still visit the local monopolist. The latter cannot observe the buyer's search intensity but draws conclusions from the fact that the buyer visits his store. Based on these conclusions, the local monopolist makes a take-it-or-leave-it offer.

The buyer's search decision is guided by the expectations about the price that the local monopolist will charge. We show that the buyer's optimal search intensity is (weakly) increasing in his valuation for the good. If the buyer's valuation is below or close to the market price $q$, then the search is not beneficial. If it is sufficiently high above the market price but (potentially) below the monopoly price, then the buyer's benefit of finding the good in the market is strictly increasing in his type; the surplus increases with the type while the market price is fixed. Lastly, if the buyer's willingness to pay is higher than the monopolist's offer, the benefit of search stems from the difference in prices at the local monopolist and in the market. The optimal search intensity is therefore independent of

[1] Some notable exceptions are Hörner and Vieille (2009), Zhu (2012), Lauermann and Wolinsky (2016), Kim and Pease (2017). A more thorough overview of related literature follows below.

the type. Hence, there is a range of low types, who do not search, a range of intermediate types, who search with increasing intensity, and a range of the high types, who search with constant intensity.

Given the buyer's optimal search, it is more likely for low types of the buyer to turn up at the local monopolist's door. Indeed, the monopolist's posterior distribution over types is first-order stochastically dominated by the prior. Constructing an equilibrium thus requires one to conjecture the seller's distribution over prices, find the buyer's optimal search given the conjectured price distribution and finally verify that the local monopolist indeed wants to offer the price distribution given his updated posterior. We show that equilibrium of the game exists and that the pricing in every equilibrium takes one of two forms: either the seller offers the monopoly price, or he randomizes over a set of prices with the monopoly price as the highest price in the support of the distribution.

We then provide sufficient condition for the (deterministic) monopoly price to be an equilibrium. A crucial role in the analysis is played by the probability with which the buyer visits the local monopolist when expecting him to charge the monopoly price as a function of the buyer's type, $\psi(\cdot)$. Since the buyer's search intensity is non-decreasing in his type, $\psi(\cdot)$ is non-increasing: flat for the low types, strictly decreasing in the intermediate region, and flat for high valuations. A sufficient condition for the existence of the monopoly price equilibrium is that the density of the prior is non-decreasing and that the negative of the semi-elasticity of $\psi$, $-\psi'(x)/\psi(x)$, is small enough in comparison to the marginal cost of search. These conditions guarantee that the probability with which the buyer comes to the local monopolist does not fall too fast with the type in the candidate equilibrium. If it did, the local monopolist would optimally deviate to a price below the monopoly price.

These results enable us to study how the local monopolist's pricing depends on the market price. One might expect that a higher market price makes it more likely for the seller to charge the monopoly price. We, however, show that the relationship between the optimal price and the market price is much more nuanced and, in particular, depends on the likelihood that a type visits the local monopolist, $\psi(v; q)$. When $\psi(v; q)$ is log-supermodular ($\frac{\partial^2}{\partial v \partial q} \log \psi(v; q) \geq 0$), if there is an equilibrium where the local monopolist charges the monopoly price when the market price is $q$, then there is an equilibrium with the monopoly price for any market price above $q$. Log-supermodularity of $\psi(v; q)$ guarantees that when the market price $q$ increases, the likelihood that the local monopolist is visited by the type equal to the monopoly price grows faster than the likelihood that he is visited by the lower types. The latter, in turn, guarantees that it is optimal for the seller to offer the monopoly price. When $\psi$ is not log-supermodular, there may exist a monopoly price equilibrium for some market price $q$, but not for a higher market price $q'$.

Interestingly, we show that if there are multiple equilibria, the local monopolist prefers the one with the lowest expected price. Since the monopoly price is in the support of every equilibrium, the local monopolist's profit can always be evaluated at the monopoly price. Since it is also the highest point in the support, all the types above it search with an intensity that only depends on the expected price. A smaller expected price provides fewer incentives to search and thus increases the chances that these types end up buying from the monopolist.

We fully characterise the equilibria of the model with binary valuations and $q = 0$. In this case, the equilibrium is always unique. A striking feature is that the local monopolist's expected payoff is non-monotonic in the prior probability of facing a high type. It is flat for low priors, decreasing in the intermediate region and increasing for high priors.

The special case of our model with $q = 0$ can be interpreted as a model of endogenous bargaining power. The buyer invests in the probability of making an offer. Upon investment $e$, he gets to make an offer with probability $\rho(e)$, in which case he offers to the seller the price 0. The seller makes an offer with the remaining probability.[2] This interpretation of the model has implications on so-called probabilistic bargaining models. In such models, the buyer and the seller have exogenously fixed probabilities of making an offer. Moreover, these probabilities tend to be interpreted as bargaining powers; see for example Zingales (1995), Inderst (2001), Krasteva and Yildirim (2012) and Münster and Reisinger (2015). Our results suggest that in some environments it would be more realistic to assume that bargaining power is increasing in the buyer's valuation.

**Related Literature.** Hörner and Vieille (2009) and Zhu (2012) analyze sequential price search in models where search is costless. The papers more closely related to ours are Lauermann and Wolinsky (2016) and Kim and Pease (2017). Lauermann and Wolinsky (2016) analyse a model where a privately informed buyer solicits sellers at a cost. Being sampled reveals information to the seller. However, their paper is concerned with the question of information aggregation with a fixed price mechanism when the sampling cost goes to zero, rather than with optimal pricing at any sampling cost.[3] Kim and Pease

[2]Complete information models where agents compete for proposal rights have been studied in the context of bargaining without commitment; see, for instance, Yildirim (2007), Yildirim (2010), Board and Zwiebel (2012), Ali (2015) and Rachmilevitch (2019). Crawford (1979), Evans (1997) and Pérez-Castrillo and Wettstein (2001), on the other hand, study the effects of competition on coalitional bargaining games. Under the same assumption Karagozoglu and Rachmilevitch (2018) study a model where agents pay a cost to prepare for negotiations. In their framework, costly preparations do not affect the allocation of proposal rights but are instead necessary to remain at the negotiation table.

[3]Endogenous distribution of types also arises in the models of large matching markets with search as

(2017) analyse a continuous-time adverse selection model with binary types where a seller can affect the arrival rate of buyers. They identify two opposing effects. Since the low type seller benefits more from trade, he searches with higher intensity. Being solicited, thus, conveys negative information about the seller. At the same time, the fact that the seller did not yet sell the good implies he is more likely to be a high type.

Our model also relates to Lauermann and Wolinsky (2017). They study a setting where a privately informed seller gets to select a number of buyers that participate in a first-price auction. The fact that they have been selected reveals something about the common value of the object, similar to a customer's visit to the local monopolist signaling information about the customer's willingness to pay in our model. Lastly, Zryumov (2015) and Heinsalu (2017) analyse models with endogenous entry into a market with adverse selection. Entry is informative about the entrant's type and, therefore, reflected in the market price.

# 2 The Model

There is a consumer who wants to purchase a unit of a good. The consumer's valuation $v$ is drawn from a cumulative distribution function $F$ on $[0, 1]$. $F$ might be continuous, as often assumed, or have atoms. The consumer can either purchase the good from the local monopolist or search for it in the market. Alternative sources are known to offer the good (or a perfect substitute) at a competitive price $q \geq 0$, but finding them is costly. The consumer chooses a search intensity $e \in \mathbb{R}_+$ at a cost $ke$ and finds an alternative source with probability $\rho(e)$, where $\rho : \mathbb{R}_+ \to [0, 1)$ is strictly increasing, strictly concave and differentiable on its support with $\rho'(0) \in \mathbb{R}$.[4] If the search is instead unsuccessful, the consumer must decide whether or not to accept the offer of the local monopolist. The local monopolist—to whom we will refer as the seller—has a value of zero for the good and makes an offer based on the observation that the consumer has not already bought the good elsewhere. After the offer is accepted/rejected the game ends. The timing can be summarized as follows:

1. The consumer observes his type $v$ and decides on the search intensity $e \in \mathbb{R}_+$.
2. The search outcome is realized: if the search is successful and the consumer accepts, the game ends; otherwise we move to stage 3.

in Inderst (2005), Moreno and Wooders (2010) and Guerrieri, Shimer, and Wright (2010).

[4]By assuming that the range of $\rho$ is $[0, 1)$ we avoid rather uninteresting problems brought upon by the possibility of out of equilibrium beliefs. In particular, the assumption implies that the consumer showing up at the local monopolist is always on the equilibrium path.

3. The consumer visits the local monopolist, who makes a take-it-or-leave-it offer.

4. The consumer accepts or rejects.

The equilibrium concept we employ is sequential equilibrium; equilibrium for short. Notice that there are no issues with applying sequential equilibrium here as there are no relevant out-of-equilibrium beliefs.

# 3 Equilibrium Analysis

It is useful to start with some preliminary observations. First, if the consumer searches and finds the good at the competitive price, he buys it. Otherwise, it would have been better not to search. The consumer's strategy can therefore be reduced to the search intensity $e$ and his decision to accept/reject the monopolist's offer in case the initial search was unsuccessful.

**Search choice.** The buyer's search decision is governed by his private valuation and the price he expects the seller to offer. The seller's randomisation over prices can be described by a cumulative distribution function $H$, where $H(p)$ is the probability that the offered price is weakly smaller than $p$.[5] Given $H$, type $v$ of the consumer solves the problem

$$\max_{e\in[0,\infty)} \quad \rho(e)(v-q)+(1-\rho(e))\int_0^v (v-p)dH(p)-ke. \tag{1}$$

With probability $\rho(e)$ the consumer's search is successful and he obtains a payoff equal to $v-q$. With the complementary probability the consumer can only purchase the good at the terms which the seller sets. In this case, the consumer accepts the offer as long as his valuation is not below the price. The following lemma characterizes the solution to the consumer's problem and shows that the optimal search intensity is non-decreasing in the consumer's valuation. All the proofs are collected in the Appendix.

[5] Since there is only one consumer, it is without loss of generality to restrict attention to price offers or randomizations over those. Any (direct) mechanism $(x, t)$ of relevance, where $x$ specifies the probability with which the consumer wins the good and $t$ the transfer can be implemented with randomization over prices. When revenue equivalence applies (continuum of types), the distribution is $H(\cdot) = x(\cdot)$, otherwise a bit more care is required; see Carrasco, Luz, Kos, Messner, Monteiro, and Moreira (2018).

**Lemma 1.** *Let $e^*(v; H)$ denote the buyer's search intensity as a function of $v$ when he expects price distribution $H$; given by (1). Then $e^*(v; H) = 0$ if*

$$\rho'(0)\left((1 - H(v))v - q + \int_0^v p dH(p)\right) \leq k \ .$$

*Otherwise it is*

$$e^*(v; H) = (\rho')^{-1}\left(\frac{k}{(1 - H(v))v - q + \int_0^v p dH(p)}\right) \ . \tag{2}$$

*For all $H$, the function $e^*(v; H)$ is non-decreasing in $v$ and non-increasing in $q$.*

The consumer's expected payoff is strictly concave in $e$, implying that each type has a unique best response. In equilibrium, the consumer thus plays a pure strategy. The consumer's search intensity is equal to zero for types below or close to $q$ and increasing up to the lowest type who always buys when the initial search fails, and constant thereafter; see Figure 1. This is easy to see when the consumer expects the seller to offer a deterministic price $p > q$. For types below the seller's price $p$, the gain of finding another offer is equal to $v - q$, as in the absence of such offer, the good can only be bought at a price above the consumer's valuation. The gain is strictly increasing in $v$ and search is optimal as long as $\rho'(0)(v - q) > k$. Hence, types $v > q + k/\rho'(0)$ find it optimal to search, whereas for types $v \leq q + k/\rho'(0)$ the gain from buying the good at price $q$ is not sufficient to compensate the cost of search. Crucially, types above the seller's price $p$ trade with probability one even if their initial search is unsuccessful. Their gain from searching is thus given by the difference in the price they have to pay. This gain is constant for all types above $p$, as are, therefore, the incentives to search.

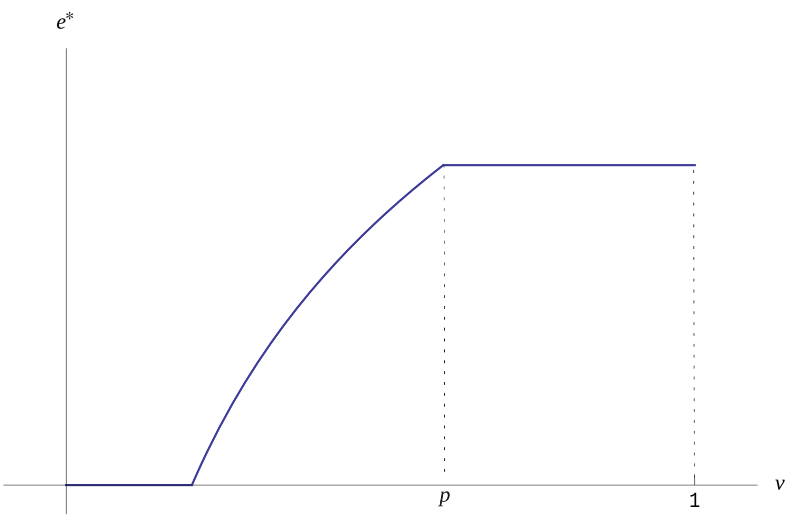


Figure 1: The consumer's optimal search intensity when he expects the seller to offer price $p$.

**Updating.** The monotonicity of the consumer's search intensity implies that higher types are more likely to find an alternative source and, hence, less likely to visit the seller. Given the consumer's search strategy $e(v)$, the probability that type $v$ buys from an alternative source is $\rho(e(v))$. Using this probability, the seller can compute the posterior distribution over the consumer's valuations, conditionally on the consumer not having bought the good already:

$$G(v;e) = \frac{\int_0^v (1-\rho(e(t)))dF(t)}{\int_0^1 (1-\rho(e(t)))dF(t)}. \tag{3}$$

To lighten the notational burden we suppressed the dependence of $G$ on the buyer's expectation of the seller's distribution $H$.

**Lemma 2.** *Let the search intensity $e(\cdot)$ be non-decreasing and non-constant on the support of $F$. Then $F$ first-order stochastically dominates $G$.*

Since higher types of the consumer tend to search more than lower types, the seller is more likely to encounter a consumer with a lower valuation. As a consequence, the prior distribution of types, $F$, first-order stochastically dominates the posterior belief $G$.[6] While the arrival of a consumer is good news for the seller, it is a negative signal about the consumer's search intensity and hence his type, somewhat reminiscent of the winner's curse. Meeting a consumer, therefore, depresses the seller's belief.

**Price choice.** Given the consumer's search strategy $e$, the seller solves the problem

$$\max_{p\in\mathbb{R}_+} \quad (1-G(p;e))p. \tag{4}$$

The revised beliefs dictate the optimal price, leading to a rather interesting fixed point problem. The consumer conjectures the price (or the distribution over prices) that the seller will charge and chooses the search intensity in response. The seller updates his beliefs and charges a price that is optimal with respect to his posterior. In equilibrium, the consumer's conjecture about the price distribution must coincide with the seller's optimal price distribution. The following lemma shows that this problem has a fixed point.

**Proposition 1.** *An equilibrium of the game exists.*

[6] When the buyer's valuations are distributed on an interval with a density $f$, a stronger result obtains. The ratio of the prior and posterior density, $f/g$, is non-decreasing, i.e. the monotone likelihood ratio property holds.

The main obstacle towards showing the existence of equilibrium is the discontinuity of the seller's payoff in the price. If there is a mass point in the consumer's distribution over valuations, the seller's payoff jumps up as the price approaches the mass point from above. Here we rely on the results of Reny (1999) and Carbonell-Nicolau and McLean (2017) on the existence of mixed strategy equilibria in discontinuous games with infinite strategy spaces. Before applying the result we simplify the game by using some of the above derived properties of equilibria. After proving the existence of an equilibrium in the simplified game, we show that it embeds as a sequential equilibrium in the original game.

**Equilibrium pricing.** Having established the existence of an equilibrium, we turn our attention to the question of the seller's equilibrium pricing strategy. If the consumer could not search for outside offers, the seller would offer the price that maximises $p(1-F(p))$, denoted by $p^*$, and termed the monopoly price. We abstract from the non-generic case where $p(1-F(p))$ has multiple maximisers and show the following.

**Proposition 2.** *Fix an equilibrium. Then $p^*$ is the maximum of the support of the equilibrium distribution over the seller's prices. As a consequence, every equilibrium takes one of the following two forms:*

- *it is a pure-strategy equilibrium in which the seller offers price $p^*$;*
- *it is a mixed-strategy equilibrium in which the seller randomises over a set of prices of which $p^*$ is the highest.*

In every equilibrium the monopoly price $p^*$ is among the optimal prices for the seller and the seller never offers a price higher than $p^*$. Given that the seller revizes his beliefs towards the lower types one should expect that the upper boundary of support of the prices is not above the monopoly price. More surprisingly, the upper bound of the set of equilibrium prices cannot be smaller than $p^*$. Namely, all types above the upper bound of the price support choose the same search intensity (see Lemma 1). As a consequence, the relative posterior probabilities across these types are the same as under the prior and, consequently, the maximiser over these prices is the same before and after updating.[7] In particular, if the upper bound of the price support, denoted by $\bar{p}$, was strictly smaller than $p^*$, then the fact that $p^*$ is optimal ex ante would imply that it also yields the highest profit among the prices in $[\bar{p}, 1]$ after the revision of beliefs. Hence, $p^*$ would yield

[7]More precisely, $(1-F(p'))/(1-F(p'')) = (1-G(p'))/(1-G(p''))$ for all $p', p'' \in [\bar{p}, 1]$. Thus if $p'(1-F(p')/(p''(1-F(p'')) > 1$ for some $p', p'' \in [\bar{p}, 1]$, then $p'(1-G(p')/(p''(1-G(p'')) > 1$.

a strictly higher profit than $\bar{p}$, contradicting the assumption that $\bar{p}$ belongs to the set of prices over which the seller randomises.

**Monopoly price equilibrium.** Since the monopoly price $p^*$ is in the support of every equilibrium, the only possible pure-strategy equilibrium is the one where, despite revising his beliefs, the seller offers the monopoly price; termed the monopoly price equilibrium. Existence of the monopoly price equilibrium can be checked by computing the consumer's best response to price $p^*$, as specified in Lemma 1, and verifying that the seller has no incentives to deviate to a lower price after updating his beliefs. To simplify notation, we define

$$\psi(v; q) = 1 - \rho\left((\rho')^{-1}\left(\frac{k}{v-q}\right)\right) \tag{5}$$

as the seller's probability of making an offer against type $v \in (\underline{v}, p^*)$ when the buyer expects price $p^*$, where $\underline{v} \equiv q + k/\rho'(0)$ is the threshold type below which it is never optimal for the buyer to search (see Lemma 1). $\psi(v; q) = 0$ for $v < \underline{v}$. Notice that $\psi$ is non-increasing and differentiable on $(\underline{v}, p^*)$. The next proposition provides a sufficient condition for the monopoly price equilibrium to exist.

**Proposition 3.** *Assume $F$ is differentiable on $[0, 1]$ and let the density function $f$ be non-decreasing. If*

$$-\frac{\psi'(v; q)}{\psi(v; q)} \leq 2k, \tag{6}$$

*for all $v \in [0, 1]$, then the monopoly price equilibrium exists.*

The sufficient condition for the existence of the monopoly price equilibrium requires that the negative of the semi-elasticity, $-\psi'/\psi$, be small enough. That is, the probability that the seller faces a given type should not be falling too fast with the type. If it did, the seller would deduce that the buyer is considerably more likely to be a low type and deviate to a lower price.

Checking the sufficient condition in Proposition 3 requires one to first compute the search intensity and then the probability with which the seller faces each type. A more direct, but less intuitive, sufficient condition is the following. In the proof of Proposition 3, we show that the semi-elasticity can be rewritten as

$$\frac{\psi'(v; q)}{\psi(v; q)} = \frac{1}{k} \cdot \frac{(\rho'(e^*(v; q)))^3}{\rho''(e^*(v; q))(1 - \rho(e^*(v; q)))}, \tag{7}$$

with $e^*$ specified in Lemma 1. A sufficient condition for (6) is therefore that $-(\rho'(x))^3/(\rho''(x)(1-\rho(x))) \leq 2k$ holds for all $x \geq 0$. This is a joint condition on function $\rho$ and the marginal cost of search $k$. An interesting feature of this condition is that it does not depend on the market price $q$. As long as it is satisfied, there is an equilibrium in which the seller offers the monopoly price regardless of whether the competitive price is high or low. The value of $q$ matters, however, for the equilibrium search intensity. If $p^* \leq \underline{v}$, or equivalently,

$$q \geq p^* - \frac{k}{\rho'(0)}, \tag{8}$$

then the consumer does not search and the seller does not learn, so offering $p^*$ is trivially optimal. Notice that the threshold for $q$ in this condition is lower than the monopoly price by the ratio of the marginal cost and the marginal benefit of search at intensity zero. For an illustration of this and the conditions of Proposition 3, consider the following example.

**Example 1.** Let $\rho(e) = e/(s+e)$ and $f$ be non-decreasing. Then $\rho'(e) = s/(s+e)^2$, $\rho''(e) = -2s/(s+e)^3$ and consequently

$$-\frac{(\rho'(e))^3}{\rho''(e)(1-\rho(e))} = \frac{s}{2(s+e)^2}.$$

This term is strictly decreasing in $e$. The sufficient condition for the existence of the monopoly price equilibrium is then satisfied if it holds at search intensity zero, hence if:

$$\frac{1}{2s} \leq 2k.$$

The monopoly price equilibrium thus exists whenever the product $ks$ is above $1/4$. The equilibrium also features search if condition (8) is violated, that is, if $q < p^* - k/\rho'(0) = p^* - ks$. $\square$

In what follows, we study in more detail how the outside price $q$ affects existence of the monopoly price equilibrium *with search*. We thus focus on the case where (8) is violated and thus at least some types search. One might be inclined to believe that a larger competitive price $q$ decreases the value of the consumer's outside option and thus renders the equilibrium with the monopoly price more likely. However, this is not always true. Indeed, to guarantee that an increase in $q$ makes the monopoly price equilibrium more likely, we need to impose some conditions, as the following result shows.

**Proposition 4.** *Suppose* $\frac{\psi'(v;q)}{\psi(v;q)}$ *is non-decreasing in* $q$ *for all* $v < p^*$. *If there exists a monopoly price equilibrium at some* $q > 0$, *then a monopoly price equilibrium exists for all* $q' > q$.

If the probability of facing a buyer type, $\psi(v;q)$, is log-supermodular, and a monopoly price equilibrium exists at $q$, then the monopoly price exists at all $q' > q$.[8] The fact that a higher outside price $q'$ is less attractive for the buyer is not enough for the seller to keep charging the monopoly price. A higher $q$ merely guarantees that the consumer searches for an alternative with lower intensity and therefore visits the seller more often. What matters for the pricing decision of the seller, however, is his posterior belief and hence the *relative likelihood* of facing a high vs. a low type of the consumer. Therefore, it is crucial that when $q$ increases the lower types do not start showing up at the local monopolist proportionally more often then the high types.

A sufficient condition for the log-supermodularity of $\psi$ is that $\frac{\psi'(v;q)}{\psi(v;q)}$ is decreasing in $v$.[9] Equation (7) implies that $\frac{\psi'(v;q)}{\psi(v;q)}$ is decreasing in $v$ if $\frac{(\rho'(x))^3}{\rho''(x)(1-\rho(x))}$ is decreasing (recall that $e^*$ is increasing). A decreasing ratio $\frac{(\rho'(x))^3}{\rho''(x)(1-\rho(x))}$ is sufficient to guarantee that the seller's equilibrium probability of meeting a high type increases relative to that of meeting a low type.[10] Notice however that this property is violated for several natural specifications of $\rho$; for instance, for that of Example 1. An example where the condition is satisfied is $\rho(x) = x^\alpha$ with $\alpha \in (1/2, 1)$.[11]

Apart from the fact that higher values of $q$ do not necessarily make the monopoly price more likely, it is interesting to notice that the monopoly price equilibrium may actually not be desirable from the seller's point of view.

**Proposition 5.** *If there are multiple equilibria, the seller prefers the one with the lowest expected price.*

Since the monopoly price is in the support of every equilibrium, we can evaluate the seller's profit in any equilibrium at the monopoly price; see Proposition 2. Moreover, since the monopoly price is the highest price in any equilibrium, the types above the monopoly price base their search intensity only on the expected price. The lower the expected price, the lower is the intensity with which these types search for the alternative. The seller, therefore, sells more often at the monopoly price when the expected price is smaller, i.e. when there is mixing in equilibrium. This is the case we consider next.

[8] A differentiable function $f$ is log-supermodular if $\frac{\partial^2}{\partial x \partial y} log f(x,y) \geq 0$, that is, if $\frac{\partial}{\partial y}\left(\frac{f_x(x,y)}{f(x,y)}\right) \geq 0$; see Athey (2002). $\psi$ is continuous and differentiable almost everywhere.

[9] Notice that in $d\psi(v;q)/dq = -d\psi(v;q)/dv$.

[10] This result is not an immediate implication of Proposition 3, as we do not impose assumptions on the distribution here.

[11] Notice that this example violates our assumption $\rho'(0) \in \mathbb{R}$. Our results can be easily extended to the case where $\rho$ satisfies the Inada conditions. In this case, we have $\underline{v} = 0$, which means that all types above $q$ choose a positive search intensity.

**Mixed strategy equilibria.** First, we provide a condition under which the monopoly price equilibrium fails to exist. By implication the seller randomizes across prices in any equilibrium.

**Proposition 6.** *Let $q < p^* - \frac{k}{\rho'(0)}$. If*

$$\frac{\psi'(p^*)}{\psi(p^*)} < -\frac{2 + \frac{f'(p^*)}{f(p^*)}}{p^*} \tag{9}$$

*then the monopoly price equilibrium does not exist.*

The seller's expected payoff after updating beliefs continues to have a critical point at $p^*$. However, this point may no longer be a maximizer. Indeed, if the optimal search intensity $e^*$ varies sufficiently across types below $p^*$, the seller's expected payoff after updating has an inflection point at $p^*$. Condition (9) formalises this property. It is satisfied if $\psi'(p^*)/\psi(p^*)$ is sufficiently negative, that is, if the seller's probability of meeting the consumer falls sufficiently fast for types below $p^*$. Proposition 9 thus provides a sufficient condition for the existence of a profitable deviation to prices just below $p^*$ in a candidate monopoly price equilibrium. In such a case any equilibrium is in mixed strategies. It is possible to construct examples for mixed strategy equilibria in which the seller randomizes over a discrete set of prices. It remains an open question, whether there can be equilibria in which the seller randomizes over a continuum of prices.

We argued above that a higher competitive price $q$ does not necessarily make the monopoly price equilibrium more likely. In fact, under certain conditions the opposite happens: if there is a $q$ such that (9) is satisfied and hence the monopoly price equilibrium fails to exist, then it fails to exist for all higher values of $q$, unless it is the no-search equilibrium.

**Corollary 1.** *Suppose that $\frac{\psi'(v;q)}{\psi(v;q)}$ is decreasing in $q$ for $q$ such that $v > \underline{v}(q)$. If condition (9) is violated for some $q < p^* - \frac{k}{\rho'(0)}$, so that the monopoly price equilibrium fails to exist, then it fails to exists for all $q' \in (q, p^* - \frac{k}{\rho'(0)})$.*

Condition (9) is violated when $\psi'/\psi$ is too low. In that case the seller's payoff is convex in a neighborhood below $p^*$, with the minimum at $p^*$. If $q$ further reduces the semi-elasticity $\psi'/\psi$, the monopoly price equilibrium will continue to fail to exist. Corollary 1 points towards conditions under which the monopoly price equilibrium with search exists only if the market price is sufficiently low. The following example illustrates this idea: for low values of $q$ there is a monopoly price equilibrium with search, while for intermediate values of $q$ such an equilibrium does not exist.

**Example 2.** Suppose $\rho(e) = 1 - \exp(-\alpha e)$, $F$ is uniform on $[0, 1]$, and $q < p^* - \frac{k}{\rho'(0)} = p^* - \frac{k}{\alpha}$. It is easy to verify that this specification satisfies the conditions of Corollary 4. Using Lemma 1 one obtains

$$e^*(v; q) = \begin{cases} 0 & \text{if } v \leq q + k/\alpha \\ -\frac{1}{\alpha} \ln\left(\frac{k}{\alpha(v-q)}\right) & \text{if } v \in (q + k/\alpha, p^*) \\ -\frac{1}{\alpha} \ln\left(\frac{k}{\alpha(p^*-q)}\right) & \text{if } p \geq p^*, \end{cases}$$

and

$$\psi(v; q) = \begin{cases} 1 & \text{if } v \leq q + k/\alpha \\ \frac{k}{\alpha(v-q)} & \text{if } v \in (q + k/\alpha, p^*) \\ \frac{k}{\alpha(p^*-q)} & \text{if } p \geq p^*. \end{cases}$$

With $p^* = 0.5$, the second-order condition (9) boils down to $q > 1/4$. This, together with $q < p^* - k/\alpha$ implies that, provided $k/\alpha < 1/4$, the monopoly price equilibrium fails to exist for all $q \in (1/4, 1/2 - k/\alpha)$. On the other hand, one can verify that at $q = 0$ deviating to any price $p < 1/2$ in the candidate monopoly price equilibrium is unprofitable. Therefore, for $q$ close to 0, this equilibrium does exist. $\square$

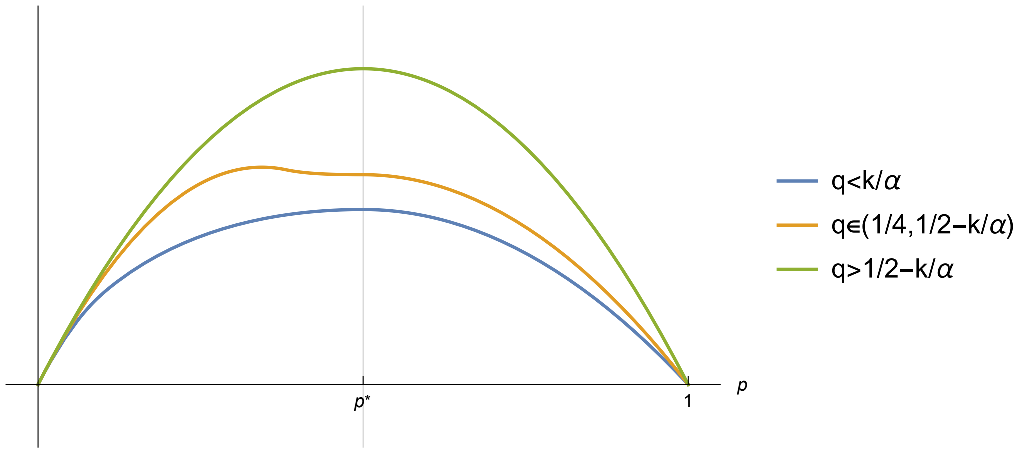


Figure 2: Seller's expected payoff after updating

# 4 Binary Types

To gain further insights on how search affects the trading outcome and profits in our environment, we now focus on the case where the consumer's valuation is binary. This allows us to characterize the equilibrium strategies explicitly and prove uniqueness. Let the consumer's valuation belong to the set $\{v_L, v_H\}$, with $0 < v_L < v_H$, and let $\mu$ denote the ex-ante probability that the consumer's valuation is high. For simplicity, we assume that the alternative price $q$ is zero and the marginal cost of search is low enough such that the low type chooses a positive search intensity: $k < \rho'(0)v_L$.

In equilibrium the seller offers price $v_L$ or price $v_H$, or some randomization over the two. Proposition 2 implies that when the monopoly price is $v_L$—equivalently, when $\mu \leq v_L/v_H$—there is a unique equilibrium in which the seller offers price $v_L$. The more interesting case is when the monopoly price is $v_H$, that is, $\mu > v_L/v_H$. Letting $\sigma$ denote the probability which the consumer attaches to the seller charging price $v_H$, the consumer's optimal search intensity is:

$$e^*(v;\sigma) = \begin{cases} (\rho')^{-1}(\frac{k}{\sigma v_H + (1-\sigma)v_L}) & \text{if } v = v_H \\ (\rho')^{-1}(\frac{k}{v_L}) & \text{if } v = v_L. \end{cases}$$

Whenever the initial search was unsuccessful and the good can only be bought from the local monopolist, the low type's payoff is 0—either the price is too high to trade or the seller extracts the full surplus. Consequently, the low type's payoff does not depend on the probability of the seller charging the high price. The high type's payoff, on the other hand, does. The larger is $\sigma$, the higher is the expected price the high type has to pay when he does not find the good in the market. Concavity of $\rho$ implies that $(\rho')^{-1}$ is a decreasing function and, hence, that the high type's search intensity $e^*(v_H;\sigma)$ is increasing in $\sigma$. In other words, the higher is the expected price the seller charges, the larger is the consumer's benefit of finding a different offer and, thus, the incentives to search.

When $\sigma$ is strictly positive, the high type searches with higher intensity than the low type and the seller updates his beliefs accordingly. The seller's posterior belief about the consumer's type being high, conditional on the consumer not having found a cheaper offer, is

$$\hat{\mu} := \frac{\mu\left(1-\rho\left((\rho')^{-1}(\frac{k}{\sigma v_H+(1-\sigma)v_L})\right)\right)}{\mu\left(1-\rho\left((\rho')^{-1}(\frac{k}{\sigma v_H+(1-\sigma)v_L})\right)\right)+(1-\mu)\left(1-\rho\left((\rho')^{-1}(\frac{k}{v_L})\right)\right)}.$$

The seller's posterior $\hat{\mu}$ is strictly decreasing in $\sigma$ and takes value $\mu$ (equal to the prior) when $\sigma = 0$. Given the restriction $\mu > v_L/v_H$, two cases need to be considered. If $\hat{\mu}$ evaluated at $\sigma = 1$ is weakly greater than $v_L/v_H$, the monopoly price $p^* = v_H$ is optimal after updating for all values of $\sigma$. In this case, the unique equilibrium is in pure strategies, where the seller offers the price $v_H$. On the other hand, if $\hat{\mu}$ evaluated at $\sigma = 1$ is smaller than $v_L/v_H$, there is a unique value of $\sigma$ at which the seller's posterior $\hat{\mu}$ coincides with $v_L/v_H$, rendering him indifferent between the low and the high price. This value of $\sigma$ is the seller's randomisation strategy in the mixed-strategy equilibrium that obtains. Letting $m$ be the value of $\mu$ at which $\hat{\mu}$ evaluated at $\sigma = 1$ is exactly $v_L/v_H$, the discussion can be summarised as follows.

**Proposition 7.** *In the environment with two types, the equilibrium is generically unique. The seller charges price $v_H$ with probability*

$$\sigma = \begin{cases} 0 & \text{if } \mu < \frac{v_L}{v_H} \\ \sigma^* & \text{if } \mu \in (\frac{v_L}{v_H}, m) \\ 1 & \text{if } \mu \geq m, \end{cases}$$

*where $\sigma^*$ is the unique solution to*

$$\frac{1}{1 + \frac{1-\mu}{\mu} \frac{1-\rho((\rho')^{-1}(\frac{k}{v_L}))}{1-\rho(\rho')^{-1}(\frac{k}{\sigma v_H + (1-\sigma)v_L})}} = \frac{v_L}{v_H}, \tag{10}$$

*and $v_L$ with the remaining probability.*

The equilibrium is most readily described as a function of the seller's prior. When $\mu$ is below $v_L/v_H$ so that the monopoly price is $v_L$, this price remains optimal after updating. Instead, when $\mu$ is just above $v_L/v_H$, the prior favors the high price only slightly. If the consumer expects the high price and chooses the search intensity accordingly, the seller's posterior would drop below $v_L/v_H$, making the low price optimal. However, if the consumer expects the seller to charge the low price, both types of the consumer would search with the same intensity and the seller's posterior would stay above $v_L/v_H$, rendering the high price optimal. The seller, thus, randomises over the two prices. Finally, when $\mu$ is higher than $m$, the posterior belief $\hat{\mu}$ is always above $v_L/v_H$ and, therefore, the high price $v_H$ is optimal.

The case where the seller is indifferent between the high and the low price is non-generic in the standard monopoly problem. Proposition 7 shows that when the seller's customer base is endogenously determined through search (or the lack of it), there is always an interval of values of $\mu$ where both prices are optimal for the seller in equilibrium. Moreover, the seller's equilibrium probability of offering the high price as a function of the prior $\mu$ has two jumps, taking a value in $(0, 1)$ in the intermediate region.

**Comparative Statics.** It is of interest to study how the seller's payoff changes with the proportion of the high types.

**Proposition 8.** *The seller's equilibrium payoff, $u_s$, as a function of the prior $\mu$ is non-monotonic:*

$$u_s'(\mu)\begin{cases} = 0 & \text{if } \mu \le v_L/v_H \\ < 0 & \text{if } \mu \in (v_L/v_H, m) \\ > 0 & \text{if } \mu \ge m. \end{cases}$$

*Proof.* See Appendix for the proof. $\square$

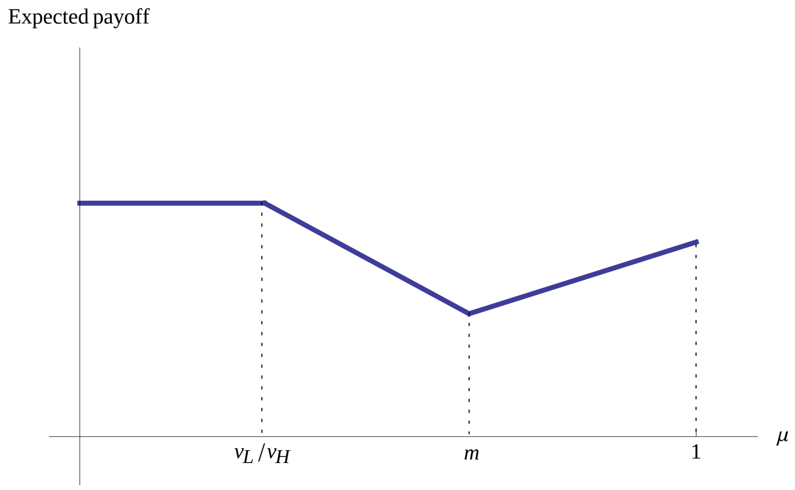


Figure 3: Seller's equilibrium expected payoff

The striking property of the seller's payoff is that it is non-monotonic in the probability of the consumer's type being high; it is constant for low priors, decreasing for intermediate priors and increasing for high priors (see Figure 3).

In the first region, $\mu \le v_L/v_H$, the seller invariably offers the pooling price $v_L$ and both types of the consumer search with the same intensity for outside offers. The seller's payoff in this region is therefore independent of the prior. In the intermediate region of priors, $\mu \in (v_L/v_H, m)$, the seller is indifferent between the two price offers. His payoff conditional on being visited is equal to $v_L$—the seller is randomising over the two prices and the low price is accepted with certainty—and as such constant in $\mu$. As to the probability of meeting the consumer, it is useful to recall that in the mixed-strategy equilibrium, the seller's posterior belief $\hat{\mu}$ is fixed to $v_L/v_H$. When the prior rises, this entails that the high type distinguishes himself from the low type by increasing his search intensity, thereby keeping the seller's posterior belief constant. Hence, the probability of the consumer visiting the seller is falling for two reasons: as $\mu$ increases, 1)

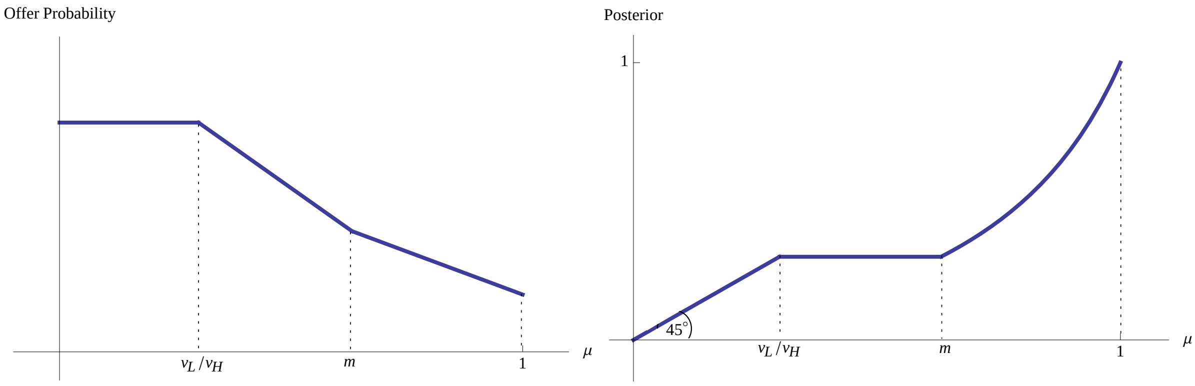


Figure 4: Seller's probability of making an offer and posterior belief in equilibrium

the probability that the high type finds an outside offer increases, and 2) the consumer's valuation is high with a higher probability. This, together with the fact that the seller's payoff conditional on the consumer not having bought the good elsewhere, implies that the seller's expected payoff is decreasing in the prior probability of the consumer's valuation being high. For high priors, $\mu \geq m$, the seller optimally offers price $v_H$. The high type's search intensity is constant in $\mu$, which means the seller's probability of meeting the high type is constant as well. Since the high price $v_H$ is only accepted by the high type, this implies that above $m$ the seller's payoff is strictly increasing in $\mu$. The non-monotonicity is connected to the seller's randomization over prices. The fact that randomization can occur in equilibrium even in environments where the consumers valuation is drawn from a distribution without atoms (as we show in Section 3) suggests that the non-monotonicity is not a mere consequence of discrete values.

Whether for high priors the seller's payoff increases above the value it attains for low priors depends on the parameters of the problem. The seller's expected payoff is smaller at $\mu = 1$ than at $\mu = 0$ when the difference in the consumer's valuation is small relative to the difference in the search intensities. This suggests that the case for the seller's payoff being decreasing in the fraction of high types could have been made in an environment with perfect information. Our result is stronger: even when the seller's payoff at $\mu = 1$ is larger than at $\mu = 0$, there is an intermediate region of priors under which the seller is strictly worse off than when he faces the low-value consumer with certainty.

The fact that the seller's payoff can be decreasing in the probability of the high type has the following interesting application. If the seller could cater to two different groups of consumers, the members of the first group predominantly of the low value type, the second group with a higher incidence of the high value type —the seller might prefer to target the group whose members value the good less. On the one hand, there is less surplus to be extracted, on the other hand, consumers will not find it profitable to search intensely for different sources. The seller prefers, so to say, to pick the low hanging fruit.

# 5 Concluding Remarks

We introduce a model in which a privately informed buyer can search for a good in a market at a cost. If he does not find the good in the market, or if he does not search, he can visit a local monopolist who then offers a price. We show that consumers with higher valuations search with a (weakly) higher intensity. The fact that the search intensity, and therefore the likelihood that they find an alternative, is increasing in the the consumer's valuation prompts the seller to revise his beliefs towards lower types when a buyer appears at his doorstep. This leads to the following implication regarding the equilibria of the game: the only possible pure-strategy equilibrium is the one where the seller's learning does not lead him to revise the price, that is, where the seller offers the monopoly price. If the latter does not constitute an equilibrium, the seller randomises over prices in equilibrium, with the monopoly price being the maximum of the support of the seller's randomisation.

Interestingly, the likelihood by which the local monopolist offers the monopoly price is not necessarily monotonic in the market price. There can be an equilibrium with the local monopolist offering the monopoly price with probability one when the market price is low, but not when the market price is high. Although an increase in the market price leads the buyer to search with a lower intensity and thus to visit the local monopolist more often, the crucial question is how the relative likelhood of visiting the local monopolist changes across types. We provide sufficient conditions on the environment that guarantee that if there is a monopoly price equilibrium for some market price, then such an equilibrium exists for any higher market price.

# A Appendix

**Proof of Lemma 1.** The first-order condition of (1) is given by

$$\rho'(e)\left(v(1-H(v))-q+\int_0^v pdH(p)\right)\leq k. \tag{11}$$

Let $\underline{v}$ be the value of $v \in [0,1]$ for which (11) holds as an equality at $e = 0$. The optimal search function is described by (2) if $v > \underline{v}$ and by $e^*(v;H) = 0$ otherwise. Concavity of $\rho$ implies that $\rho'$ and its inverse are decreasing. Together with the fact that $v(1-H(v))-q+\int_0^v pdH(p)$ is non-decreasing in $v$, it follows that $e^*(v;H)$ is non-decreasing in $v$. $\square$

**Proof of Lemma 2.** The seller's posterior

$$G(v;e)=\int_0^v \frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}dF(t).$$

Since the search function $e(\cdot)$ is non-decreasing, $\frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}$ is non-increasing in $t$. The fact that the search function is not constant and that $G$ is a distribution function further imply

$$\frac{1-\rho(e(0))}{\int_0^1(1-\rho(e(s)))dF(s)}>1>\frac{1-\rho(e(1))}{\int_0^1(1-\rho(e(s)))dF(s)}.$$

Let $t^*$ be such that $\frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}>1$ for all $t\in[0,t^*)$ and $\frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}\leq 1$ for $t\geq t^*$. For $v\in[0,t^*)$

$$\begin{aligned}G(v;e)&=\int_0^v \frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}dF(t)\\&\geq\int_0^v \frac{1-\rho(e(v))}{\int_0^1(1-\rho(e(s)))dF(s)}dF(t)\\&=\frac{1-\rho(e(v))}{\int_0^1(1-\rho(e(s)))dF(s)}F(v)\\&>F(v),\end{aligned}$$

where the first inequality follows from $\rho$ and $e$ being non-decreasing. On the other hand, for $v\in[t^*,1]$

$$\begin{aligned}1-G(v;e)&=\int_v^1 \frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}dF(t)\\&\leq\int_v^1 \frac{1-\rho(e(v))}{\int_0^1(1-\rho(e(s)))dF(s)}dF(t)\\&=\frac{1-\rho(e(v))}{\int_0^1(1-\rho(e(s)))dF(s)}(1-F(v))\\&\leq 1-F(v),\end{aligned}$$

where the first inequality follows from $\rho$ and $e$ being non-decreasing and the second from $\frac{1-\rho(e(t))}{\int_0^1(1-\rho(e(s)))dF(s)}\leq 1$ for $t\geq t^*$. □

**Proof of Proposition 1.** The proof proceeds in two steps: first we argue existence of a Bayesian Nash equilibrium in an auxiliary game, then we show the existence of a sequential equilibrium in the original game.

In the auxiliary game we fix the consumer's acceptance strategy to $x_b(v,p) = 1$ if $p \leq v$ and $x_b(v,p) = 0$ otherwise. The seller's strategy is then the price offer $p_s$, and the consumer's strategy is the search intensity $e$. Given a pair $(e, p_s)$ the consumer's and seller's payoffs are

$$\rho(e)(v-q) + (1-\rho(e))1_{v \geq p_s}(v - p_s) - ke,$$

and

$$(1-\rho(e))1_{v \geq p_s} p_s,$$

respectively. In what follows we verify conditions needed for Theorem 1 in Carbonell-Nicolau and McLean (2017); which itself builds on existence results in Reny (1999) and Monteiro and Page Jr (2007). Uniform payoff security of the game is implied by the continuity of the seller's payoff in $e$ and the consumer's payoff in $p_s$. In addition, the sum of the two players' payoffs, $\rho(e)(v-q) + (1-\rho(e))1_{v \geq p_s} v - ke$, is upper semicontinuous for every $v$ in the profile of strategies $(e, p_s)$ due to the assumption that the consumer buys the good when indifferent. Finally, since only the consumer is privately informed, the absolute continuity of the distribution is automatically satisfied. The only remaining requirement is compactness of the strategy sets. The seller's optimal price is never above 1, therefore we can restrict his set of prices to $[0,1]$. Likewise, we confine the consumer's search intensity to the set $[0, 1/k]$; provided that prices are non-negative, the most the consumer can gain from searching is his highest possible valuation: 1. Restricting $e$ to $[0, 1/k]$ and $p_s$ to $[0,1]$ renders the strategy sets compact. Theorem 1 of Carbonell-Nicolau and McLean (2017), then, delivers the existence of equilibrium.

Having established the existence of an equilibrium of the auxiliary game, we return to the original game. The seller's strategy consists of the price distribution he charges. The consumer's strategy consists of the search intensity and the acceptance decision when the seller is offering the price. The seller's strategy is, therefore, a best response to the consumer's behavior. Likewise, one can argue that if the consumer had a profitable deviation, then he would have a profitable deviation where he plays the assumed acceptance strategy. Such a profitable deviation does not exist. □

**Proof of Proposition 2.** Let the seller's equilibrium randomisation over prices be described by the cumulative distribution function $H$ and let $\bar{p}$ denote the maximum of the support of $H$. We want to argue that $\bar{p} = p^*$.

In equilibrium the probability with which consumer type $v$ finds an outside offer is $\rho(e^*(v; H))$. Given this probability, the seller's posterior when the consumer shows up is described by $G(v; e^*(\cdot; H))$. Any price in the support of $H$ must maximise

$$p(1 - G(p; e^*(\cdot; H))) = p\frac{\int_p^1 (1 - \rho(e^*(t; H)))dF(t)}{\int_0^1 (1 - \rho(e^*(t; H)))dF(t)},$$

and therefore $p\int_p^1 (1 - \rho(e^*(t; H)))dF(t)$. By Lemma 1, $e^*(v; H)$ is non-decreasing in $v$, which means that $\rho(e^*(v; H))$ is non-decreasing in $v$, and therefore that $1 - \rho(e^*(v; H))$ is non-increasing in $v$. Towards a contradiction, suppose now $\bar{p} \neq p^*$. Then

$$\begin{aligned} p^* \int_{p^*}^1 (1 - \rho(e^*(t; H)))dF(t) &\geq p^* \int_{p^*}^1 (1 - \rho(e^*(\bar{p}; H)))dF(t), \\ &= (1 - \rho(e^*(\bar{p}; H)))p^*(1 - F(p^*)), \\ &> (1 - \rho(e^*(\bar{p}; H)))\bar{p}(1 - F(\bar{p})), \\ &= \bar{p}\int_{\bar{p}}^1 (1 - \rho(e^*(t; H)))dF(t). \end{aligned}$$

The first inequality is due to the fact that $1 - \rho(e^*(v; H))$ is non-increasing in $v$ and constant above $\bar{p}$: if $\bar{p} < p^*$, then $1 - \rho(e^*(t; H)) = 1 - \rho(e^*(\bar{p}; H))$ for all $t \geq p^*$, whereas if $\bar{p} > p^*$, then $1 - \rho(e^*(t; H)) > 1 - \rho(e^*(\bar{p}; H))$ for all $t \in (p^*, \bar{p})$ and $1 - \rho(e^*(t; H)) = 1 - \rho(e^*(\bar{p}; H))$ for all $t \geq \bar{p}$. The second inequality is due to the ex-ante optimality of $p^*$. Given the inequalities, the price $\bar{p}$ cannot maximise $p(1 - G(p; e^*(\cdot; H)))$ and, hence, cannot belong to the support of $H$.

This proves that $\bar{p} = p^*$ holds. Since $p^*$ always belongs to the support of $H$, the only possible pure-strategy equilibrium is the one where the seller offers $p^*$. □

**Proof of Proposition 3.** Consider the candidate equilibrium where the seller offers $p^*$ with probability one and let $\underline{v} = q + k/\rho'(0)$ be the highest type who does not search conditional on not accepting the seller's price offer, as defined in the main text. If $p^* \leq \underline{v}$, then no type of the consumer has incentives to search, so the seller does not learn and the equilibrium exists. Assuming then $p^* > \underline{v}$, consider the seller's deviation payoff. For prices in the interval $[p^*, 1]$, this payoff is given by

$$\frac{\psi(p^*; q)}{\int_0^1 \psi(v; q)dF(v)}(1 - F(p))p.$$

The deviation payoff is equal to the ex-ante payoff multiplied by a constant. Since $p^*$ maximizes the ex-ante payoff on $[p^*, 1]$, it also maximizes the seller's payoff after updating on this domain. Deviating to prices above $p^*$ is thus not profitable.

Next, consider the seller's deviation payoff for prices in the interval $[\underline{v}, p^*)$:

$$\frac{\int_p^{p^*} \psi(v;q)dF(v) + (1-F(p^*))\psi(p^*;q)}{\int_0^1 \psi(v;q)dF(v)} p.$$

Ignoring the term in the denominator, the first derivative with respect to $v$ is:

$$\int_p^1 \psi(v;q)dF(v) + (1-F(p^*))\psi(p^*;q) - \psi(p;q)f(p)p. \tag{12}$$

Evaluated at $p = p^*$ this derivative is:

$$\psi(p^*;q)(1-F(p^*) - f(p^*)p^*).$$

Since the monopoly price $p^*$ solves $1 - F(p^*) - f(p^*)p^* = 0$, the term above equals zero. The seller's updated payoff, therefore, has a critical point at the monopoly price $p^*$. We check for the second derivative:

$$-\psi(p;q)(f'(p)p + 2f(p)) - \psi'(p;q)f(p)p. \tag{13}$$

If (13) is non-positive for all $p \in (\underline{v}, p^*)$, then the seller's payoff is concave on that interval. Since the derivative of the seller's profit is zero at $p^*$, this condition assures that $p^*$ maximizes the seller's payoff on $p \in (\underline{v}, p^*)$. The second derivative is non-positive as long as

$$-\frac{\psi'(p;q)}{\psi(p;q)} \leq \frac{2 + \frac{f'(p)}{f(p)}}{p},$$

for which it is enough that

$$-\frac{\psi'(p;q)}{\psi(p;q)} \leq 2, \tag{14}$$

since $f'(p) \geq 0$ and $p \in (0,1)$.

It remains to consider prices in the interval $[0, \underline{v}]$. The seller's deviation payoff for such prices is

$$\frac{(1-F(p^*))\psi(p^*;q) + \int_{\underline{v}}^{p^*} \psi(v;q)dF(v) + F(\underline{v}) - F(p)}{\int_0^1 \psi(v;q)dF(v)} p$$

Ignoring again the constant in the denominator, the derivative of this payoff is

$$(1-F(p^*))\psi(p^*;q) + \int_{\underline{v}}^{p^*} \psi(v;q)dF(v) + F(\underline{v}) - F(p) - f(p)p.$$

Notice that this term evaluated at $\underline{v}$ is the same as (12) evaluated at $\underline{v}$. The seller's expected payoff after updating is thus differentiable at $\underline{v}$. The second derivative of the seller's payoff for prices below $\underline{v}$ is given by $-(2f(p)+f'(p))$. By the assumption $f' \geq 0$, this term is negative. Together with the previous results, this implies that the seller's expected payoff after updating is concave on $[0, p^*]$ and thus maximized at $p^*$.

Finally, to show condition (7), suppose that the buyer expects the seller to charge the monopoly price $p^*$. For $v \in (\underline{v}, p^*)$, we have

$$e^*(v;q) = (\rho')^{-1}\left(\frac{k}{v-q}\right).$$

with

$$\frac{de^*(v;q)}{dv} = -\frac{\rho'(e^*(v;q))}{\rho''(e^*;q)}(v-q).$$

We can thus write

$$\begin{aligned}\frac{\psi'(v;q)}{\psi(v;q)} &= \frac{-\rho'(e^*(v;q))\frac{de^*(v;q)}{dv}}{1-\rho(e^*(v;q))} \\ &= \frac{1}{k}\cdot\frac{(\rho'(e^*(v;q)))^3}{\rho''(e^*(v;q))(1-\rho(e^*(v;q)))},\end{aligned}$$

for all $v \in (\underline{v}, p^*)$.

$\square$

**Proof of Proposition 4.** Fix a $q$, and suppose that the monopoly price equilibrium exists. Hence, the seller has no incentives to deviate to lower prices, which implies that the following inequality holds for all $p < p^*$:

$$\psi(p^*;q)(1-F(p^*))p^* \geq \psi(p^*;q)(1-F(p^*))p + \int_p^{p^*}\psi(v;q)dF(v)p,$$

or equivalently

$$(1-F(p^*))p^* \geq (1-F(p^*))p + \int_p^{p^*}\frac{\psi(v;q)}{\psi(p^*;q)}dF(v)p.$$

We want to argue that this inequality implies

$$(1-F(p^*))p^* \geq (1-F(p^*))p + \int_p^{p^*}\frac{\psi(v;q')}{\psi(p^*;q')}dF(v)p.$$

for all $q' > q$ and all $p < p^*$. A sufficient condition is that $\frac{\psi(v;q)}{\psi(p^*;q)} \geq \frac{\psi(v;q')}{\psi(p^*;q')}$ holds for all

$v \le p^*$ or equivalently that this ratio is non-increasing in $q$.

Several cases need to be considered. First, notice that $\underline{v}(q') > \underline{v}(q)$. If some type does not search for an alternative, when the market price is $q$, the same type does not search when the market price is $q'$. By a similar logic: $\psi(p^*; q') > \psi(p^*; q)$. The seller faces type $p^*$ when the market price is $q'$ with a higher likelihood than when the market price is $q < q'$. When the market price is smaller type $p^*$ searches with a higher intensity.

When $v < \underline{v}(q)$, $\psi(v; q) = \psi(v; q') = 1$ and therefore $\frac{\psi(v;q)}{\psi(p^*;q)} > \frac{\psi(v;q')}{\psi(p^*;q')}$ holds since $\psi(p^*; q') > \psi(p^*; q)$.

When $v > \underline{v}(q')$, the requirement that $\frac{\psi(v;q)}{\psi(p^*;q)}$ is decreasing in $q$ can be written in the derivative form as:

$$\frac{\psi_q(v; q)\psi(p^*; q) - \psi_q(p^*; q)\psi(v; q)}{\psi(p^*; q)^2} = \frac{\psi(v; q)}{\psi(p^*; q)} \left( \frac{\psi_q(v; q)}{\psi(v; q)} - \frac{\psi_q(p^*; q)}{\psi(p^*; q)} \right) \le 0,$$

where $\psi_q$ denotes the partial derivative of $\psi(v; q)$ with respect to $q$. This condition, in turn, is satisfied for all $v \le p^*$ if $\psi_q(v; q)/\psi(v; q)$ is non-decreasing in $v$, or alternatively if $\psi_v(v; q)/\psi(v; q)$ is non-decreasing in $q$.

Finally, if $v \in (\underline{v}(q), \underline{v}(q')$, $\psi(v; q) < 1$ and $\psi(v; q') = 1$. Let $\tilde{q}$ be the smallest $q$ such that $\psi(v; q) = 1$. Then, $q < \tilde{q} \le q'$, since $\psi(v; q)$ is nondecreasing in $q$. Furthermore, $\frac{\psi(v;q)}{\psi(p^*;q)} > \frac{\psi(v;\tilde{q})}{\psi(p^*;\tilde{q})}$ by the argument considered above and $\frac{\psi(v;\tilde{q})}{\psi(p^*;\tilde{q})} = \frac{1}{\psi(p^*;\tilde{q})} \ge \frac{1}{\psi(p^*;q')}$ since $\psi(p^*; q') \ge \psi(p^*; \tilde{q})$. The two inequalities together yield

$$\frac{\psi(v; q)}{\psi(p^*; q)} > \frac{1}{\psi(p^*; q')} = \frac{\psi(v; q')}{\psi(p^*; q')}.$$

We are left to verify that at $q'$ the seller does not want to deviate to prices above $p^*$. By Lemma 1 all type above $p^*$ search with the same intensity, therefore if $p > p^*$, the requirement that the seller does not want to deviate reads

$$\psi(p^*; q')p^*(1 - F(p^*)) \ge \psi(p^*; q')p(1 - F(p)),$$

which reduces to $p^*(1 - F(p^*)) \ge p(1 - F(p))$, and the latter holds because $p^*$ is the monopoly price. □

**Proof of Proposition 5.** Suppose the game has multiple equilibria and consider two of these equilibria with price distributions $H_1, H_2$ such that $\int_0^1 p dH_1(p) < \int_0^1 p dH_2$. Since

$p^*$ is the upper bound of $H_1$ and $H_2$, this inequality implies

$$\int_0^{p^*} p dH_1(p) < \int_0^{p^*} p dH_2,$$

and hence, for all $v \geq p^*$,

$$e^*(v; H_1) < e^*(v; H_2),$$

by Lemma 1. Since the monopoly price is in the support of every equilibrium, we can evaluate the seller's profit in any equilibrium at the monopoly price; see Proposition 2. In the equilibrium with price distribution $H_i, i = 1, 2$, the expected profit is

$$(1 - F(p^*))(1 - \rho(e^*(p^*; H_i)))p^*.$$

Since $e^*(v; H_1) < e^*(v; H_2)$, we have $1 - \rho(e^*(p^*; H_1)) > 1 - \rho(e^*(p^*; H_2))$. Hence, the seller's expected payoff is strictly greater in the equilibrium with price distribution $H_1$ than in the equilibrium with price distribution $H_2$. $\square$

**Proof of Proposition 6.** In the proof of Proposition 3 we showed that the seller's expected payoff after updating has a critical point at $p^*$ and that the second derivate for prices in $[\underline{v}, p^*]$ is given by (13). Condition (9) says that this derivative evaluated at $p^*$ is strictly positive. Hence, in a candidate pure-strategy equilibrium, there is a profitable deviation for the seller to a price in a left neighbourhood of $p^*$, so the monopoly price equilibrium does not exist. $\square$

**Proof of Corollary 1.** Suppose that (9) holds at $p^*$ for some $q < p^* - \frac{k}{\rho'(0)}$:

$$-(2f(p^*) + f'(p^*)p^*) - \frac{\psi_v(p^*; q)}{\psi(p^*; q)} f(p^*)p^* > 0.$$

Since $\psi_v(p^*; q)/\psi(p^*; q)$ is assumed decreasing in $q$, the same condition holds for higher $q$ too; as long as there is investment. $\square$

**Proof of Proposition 7.** The assumption $k < \rho'(0)v_L$ assures that $e^*(v_i; \sigma) > 0, i = L, H$ for all $\sigma \in [0, 1]$. We distinguish three cases:

- $\sigma = 0$: suppose in equilibrium the seller offers price $v_L$ with probability one. In this case the search intensity of both consumers is the same ($e^*(v_L; 0) = e^*(v_H; 0)$), so the seller does not learn in equilibrium. Offering price $v_L$ is optimal for the seller if and only if it is optimal according to the prior, that is: $\mu \leq v_L/v_H$.

- $\sigma = 1$: suppose that in equilibrium the seller offers price $v_H$ with probability one. The high-type consumer searches with a strictly higher intensity than the low-type consumer and the seller's posterior is

$$\hat{\mu} = \frac{\mu(1 - \rho(e^*(v_H; 1)))}{\mu(1 - \rho(e^*(v_H; 1))) + (1 - \mu)(1 - \rho(e^*(v_L; 1)))}.$$

This posterior is strictly increasing in $\mu$ and equal to $v_L/v_H$ at $\mu = m$. The equilibrium exists if and only if price $v_H$ is optimal after updating. This is satisfied if $\hat{\mu}$ is greater than $v_L/v_H$ or, equivalently, $\mu \geq m$.

- $\sigma \in (0, 1)$ : suppose the seller randomises across prices in equilibrium. This requires that the high-type consumer's search intensity, $e^*(v_H; \sigma)$, is such that after updating the seller is indifferent between both prices:

$$\frac{\mu(1 - \rho(e^*(v_H; \sigma)))}{\mu(1 - \rho(e^*(v_H; \sigma))) + (1 - \mu)(1 - \rho(e^*(v_L; \sigma)))} = v_L/v_H. \tag{15}$$

Since $e^*(v_H; \sigma)$ is continuous and strictly increasing on $[0, 1]$ and $e^*(v_L; \sigma)$ is constant, the term on the left-hand side of (15) is continuous and strictly decreasing on $[0, 1]$. At $\sigma = 0$, the left hand side is equal to $\mu$. Substituting for $e^*(v_i; \sigma), i = L, H$, it follows that the unique solution of (15) is implicitly defined by (10) if and only if $\mu \in (v_L/v_H, m)$. Hence, there exists an equilibrium where the seller randomises over both prices if and only if $\mu \in (v_L/v_H, m)$. Substituting for $e^*(v_i; \sigma), i = L, H$ in (15) yields (10), which implicitly defines the seller's equilibrium randomisation, $\sigma^*$.

□

**Proof of Proposition 8.** Given the consumer's equilibrium search strategy, as described in Proposition 7, the seller's payoff is given by the product of the probability with which the consumer does not find a different source and his conditional payoff in this event. His expected payoff as a function of the prior belief $\mu$ can thus be written as

$$u_s(\mu) = \begin{cases} (1 - \rho(e_L^*)v_L & \text{if } \mu < \frac{v_L}{v_H} \\ [\mu(1 - \rho(e_H^*(\sigma^*))) + (1 - \mu)(1 - \rho(e_L^*)]v_L & \text{if } \mu \in (\frac{v_L}{v_H}, m) \\ \mu(1 - \rho(e_H^*(1))v_H & \text{if } \mu \geq m. \end{cases}$$

where $\sigma^*$ is defined by (10).

- For $\mu \leq v_L/v_H$, the payoff function $u_s$ does not depend on $\mu$ and hence we have

$u_s'(\mu) = 0$.

- For $\mu \in (v_L/v_H, m)$, taking the first derivative of the seller's payoff with respect to $\mu$ yields

$$u_s'(\mu) = \left[ -(\rho(e_H^*(\sigma^*)) - \rho(e_L^*)) - \mu\rho'(e_H^*(\sigma^*)) {e_H^*}'(\sigma^*) \frac{\mathrm{d}\sigma^*}{\mathrm{d}\mu} \right] v_L.$$

  The first term in the square bracket is strictly negative. Since $\rho$ and $e_H^*$ are increasing functions, the second term is negative if $\mathrm{d}\sigma^*/\mathrm{d}\mu$ is positive. Given that the seller's posterior on the left-hand side of (10) is increasing in $\mu$ and decreasing in $\sigma$, $\mathrm{d}\sigma^*/\mathrm{d}\mu > 0$ is indeed satisfied. Hence, the second term in the square bracket is negative as well and the seller's expected payoff is strictly decreasing in $\mu$ in the parameter region $\mu \in (v_L/v_H, \hat{m})$.

- Finally, for $\mu \geq m$ we have $u_s'(\mu) = (1 - \rho(e_H^*(1))v_H$, which is strictly positive.

□

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